Discrete Structures EPFL, Fall 2014

Problem Set 1

Date: 19.09.2014 Not graded

Problem 1. Determine the truth value of the following propositions.

a) $12 + 1 = 13$ if and only if $6 + 2 = 8$.

b) $17 + 2 = 13$ if and only if $8 + 2 = 8$.

c) If it is raining, then $\sqrt{2} < 2$.

d) If the earth is flat, then this course is going to be wonderful.

e) If $2 + 1 = 3$ or $2 + 2 = 5$, then $5 + 6 = 11$ and $7 + 1 = 9$.

Problem 2. Consider the proposition $\neg(r \to \neg q) \vee (p \wedge \neg r)$, depending on propositional variables p, q and r.

a) Write down the truth table for this proposition.

b) If a fair coin is flipped to determine the truth value of each of the variables p, q and r, how likely is it that the whole proposition is true? How likely is it that the proposition is false?

Problem 3. Find a proposition with three variables p, q, and r that

a) is true when p, r and q are false, and false otherwise;

b) is false when at least one of the three variables is true, and true otherwise;

c) is true when exactly two of the three variables are true, and false otherwise;

d) is always false.

Note that there might be many possible solutions to this problem.

Problem 4. A set of propositions is *consistent* if there exists a set of truth assignments of the involved variables so that all propositions are simultaneously true. Is the following set of propositions consistent? It may help to write the propositions using logic symbols.

- The system is is operating normally if and only if it is in multiuser state.
- If the kernel is not functioning, then the system is not operating normally.
- The kernel is not functioning and the system is in interrupt mode.
- The system is not in multiuser state or the system is in interrupt mode.

Problem 5. Show the following without writing down the truth tables.

a) $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent.

b) $(p \to q) \wedge (\neg p \to (\neg q \to (p \wedge \neg p)))$ and q are logically equivalent.

Problem 6. You are interviewing 2014 people (good luck ☺). The n-th person tells you "exactly n of the people will lie to you." How many people tell the truth and how many lie to you? What happens if we replace "exactly" by "at least"?

Problem 7. *(Bonus)* Each inhabitant of a remote village always tells the truth or always lies. A villager will give only a "Yes" or a "No" response to a question a tourist asks. Suppose you are a tourist visiting this area and come to a fork in the road. One branch leads to the ruins you want to visit; the other branch leads deep into the jungle. A villager is standing at the fork in the road. What *one* question can you ask the villager to determine which branch to take?