Discrete Structures EPFL, Fall 2014

Problem Set 3

Date: 3.10.2014 Not graded

In the problems that follow we denote by $\mathbb{N}$ the set of natural numbers including 0.

Problem 1. Prove or disprove:

a) $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by giving a containment proof (that is, prove that the left side is a subset of the right side and that the right side is a subset of the left side).

b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by Venn diagram.

Problem 2. Let $A = \{a, b, c\}$ and $B = \{b, \{c\}\}$. Mark the following statements TRUE or FALSE:

a) $c \in A \setminus B$

b) $|\mathcal{P}(A \times B)| = 64$

c) $\varnothing \in \mathcal{P}(B)$

d) $B \subseteq A$

e) $\{c\} \subseteq B$

f) $\{a, b\} \in A \times A$

Problem 3. Determine whether the set is finite or infinite. If the set is finite, find its size.

a) $\{x \mid x \in \mathbb{Z} \text{ and } x^2 < 10\}$

b) $\{x \mid x \in \mathbb{N} \text{ and } x \text{ odd}\}$

c) $\{x \mid x \in \mathbb{N} \text{ and } 9x^2 - 1 = 0\}$

d) $A \times B$, where $A = \{a, b, c\}$ and $B = \varnothing$

e) $\{x \mid x \in \mathbb{Z} \text{ and } x^2 = 2\}$

f) $\{x \mid x \in \mathbb{N} \text{ and } x = 10^t \text{ for some } t \in \mathbb{N}\}$

g) $\{x \mid x \in \mathbb{Z} \text{ and } x^2 < 8\}$

Problem 4. Let A be the set of integers between 1 and 100. How many numbers in A are multiples of 2, 3, *and* 5? How many numbers in A are multiples of 2, 3, *or* 5?

Problem 5. Determine whether the rule describes a function. If it does so, find if the function is injective, surjective, bijective or none of the previous.

a) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = x^{2014}$.

b) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = x^{1/2014}$.

c) $f : \mathbb{N} \to \mathbb{R}$ where $f(x) = \sin x$.

d) $f : \mathbb{Z} \to \mathbb{Q}$ where $f(x) = x + 2014$.

e) $f : \mathbb{Z} \to \mathbb{Z}$ where $f(x) = x + 2014$.

f) $f : \mathbb{R} \to [-1, 1]$ where $f(x) = \cos x$.

g) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = \begin{cases} \frac{1}{x} & x > 1 \\ \\ 6x & x < 2 \end{cases}$.

h) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = \begin{cases} 4x & x > 1 \\ \\ (1+x)^2 - (1-x)^2 & x < 2 \end{cases}$.

i) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = x^{-2014}$.

j) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = \dfrac{1}{3 - e^{-e^{-x}}}$.

Problem 6. Let $A = \{1, 2, 3, 4\}$. Consider $f : A \to A$ and $g : A \to A$ with $f = \{(1, 3), (2, 2), (3, 4), (4, 2)\}$ and $g = \{(1, 4), (4, 1), (2, 3), (3, 2)\}$.

1. Find $f \circ g$.
2. Find $g \circ f$.
3. Find g^{-1}.
4. Find $g \circ g$.
5. Find $f^{-1} \circ g$.