1.72, Groundwater Hydrology
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Lecture Packet #8: Pump Test Analysis

The idea of a pump test is to stress the aquifer by pumping or injecting water and to note the drawdown over space and time.

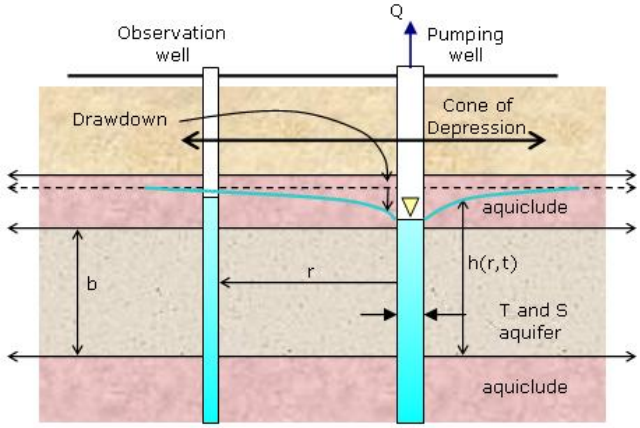
History

- The earliest model for interpretation of pumping test data was developed by Thiem (1906)
  (Adolf and Gunther) for
  - Constant pumping rate
  - Equilibrium conditions
  - Confined and unconfined conditions
- Theis (1935) published the first analysis of transient pump test for
  - Constant pumping rate
  - Confined conditions
- Since then, many methods for analysis of transient well tests have been designed for increasingly complex conditions, including
  - Aquitard leakage (study of hydrogeology has become more a study of aquitards and less of aquifers)
  - Aquitard storage
  - Wellbore storage
  - Partial well penetration
  - Anisotropy
  - Slug tests
  - Recirculating well tests (water is not removed)

It is important to note the assumptions for a given analysis.

Steady Radial Flow in a Confined Aquifer

Assume:
- Aquifer is confined (top and bottom)
- Well is pumped at a constant rate
- Equilibrium is reached (no drawdown change with time)
- Wells are fully screened and is only one pumping



Consider Darcy's law through a cylinder, radius r, with flow toward well.

$Q = K\dfrac{dh}{dr}2\pi rb$ and rearrange as $dh = \dfrac{Q}{2\pi Kb}\dfrac{dr}{r}$

Integrate from r_1, h_1 to r_2, h_2

$$\int_{h_1}^{h_2} dh = \frac{Q}{2\pi Kb}\int_{r_1}^{r_2}\frac{dr}{r}$$

$$h_2 - h_1 = \frac{Q}{2\pi Kb}\frac{r_2}{r_1}$$ or noting that T = Kb

$$\boxed{T = \frac{Q}{2\pi(h_2 - h_1)}\ln\frac{r_2}{r_1}}$$ this is the Thiem equation

Notes on the Thiem equation:

- Good with any self consistent units L and t
- If drawdown has stabilized can use any two observation wells
- Water is not coming from storage (S doesn't appear) cannot get S from this test
- Commonly used in USGS units and $\log_{10}$, T in gpd/ft (gallons per day per foot), Q in gpm (gallons per minute), r and h in ft.

$$T = \frac{527.7Q}{(h_2 - h_1)}\log\frac{r_2}{r_1}$$

Specific Capacity of a Well – Roughly estimating T

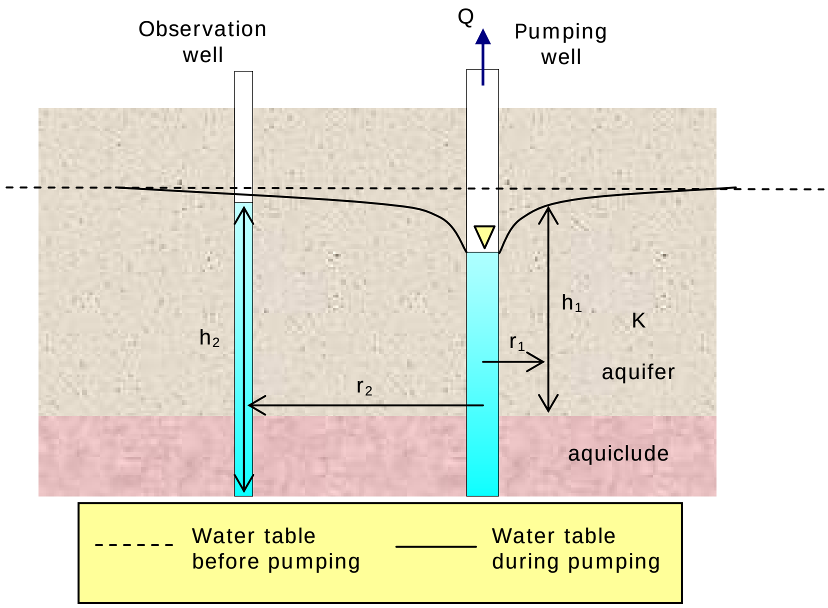
Specific Capacity = Discharge Rate/Drawdown in the well

1. A well is pumped to approximate equilibrium.
2. A good well would be 50 gpm per foot of drawdown, or 20 feet of drawdown at 1,000 gpm.
3. h_e and r_e are the head and corresponding distance from a well where drawdown is effectively zero.
4. Specific capacity = $T = \dfrac{Q}{(h_e - h_w)} = \ln\dfrac{T}{527.7\log\dfrac{r_e}{r_w}}$
5. Rule of Thumb – T ~ 1,800 x Specfic Capacity
6. What is r_e? It doesn't matter that much.
   $r_e = 1{,}000\ r_w$ $\quad\log r_e/r_w = 3$
   $r_e = 10{,}000\ r_w$ $\quad\log r_e/r_w = 4$
7. Case A → T = Specific Capacity [527 x 3] = 1,581 x SC
   Case B → T = Specific Capacity [527 x 4] = 2,108 x SC
8. If you use T ~ 1,800 x Specific Capacity you are not too far off. SC is gpm/ft and T is gpd/ft.

Steady Radial Flow in an Unconfined Aquifer

Assume:

- Aquifer is unconfined but underlain by an impermeable horizontal unit.
- Well is pumped at a constant rate
- Equilibrium is reached (no drawdown change with time)
- Wells are fully screened and
- There is only one pumping well



Radial flow in the unconfined aquifer is given by

$Q = K(2\pi r h)\dfrac{dh}{dr}$ and rearrange as $hdh = \dfrac{Q}{2\pi K}\dfrac{dr}{r}$

Integrate from r_1, h_1 to r_2, h_2

$$\int_{h_1}^{h_2} jdh = \frac{Q}{2\pi K}\int_{r_1}^{r_2}\frac{dr}{r}$$

$$\frac{h_2{}^2 - h_1{}^2}{2} = \frac{Q}{2\pi K}\ln\frac{r_2}{r_1}$$ or noting that T = Kb

$$\boxed{K = \frac{Q}{\pi(h_2{}^2 - h_1{}^2)}\ln\frac{r_2}{r_1}}$$

this is the Thiem equation for unconfined conditions (K not T, h^2 not h, no 2)

- If the greatest difference in head in the system is < 2%, then you can use the confined equation for an unconfined system.
- For r < $1.5h_{max}$ (the full saturated thickness) there will be errors using this equation because of vertical flow near the well.
- If difference in head is > 2% but < 25% use the confined equation with the following correction for drawdown

$(h_2 - h_1)_{new} = \Delta h - \dfrac{\Delta h^2}{2b}$ "measured" reduced compared to confined aquifer case

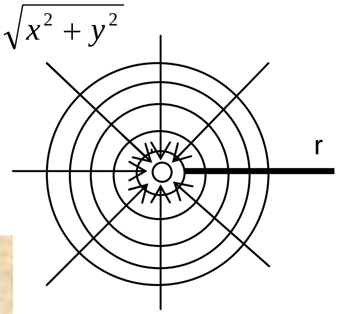
Transient Pumping Tests

$$S\frac{\partial h}{\partial t} = T\left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right]$$

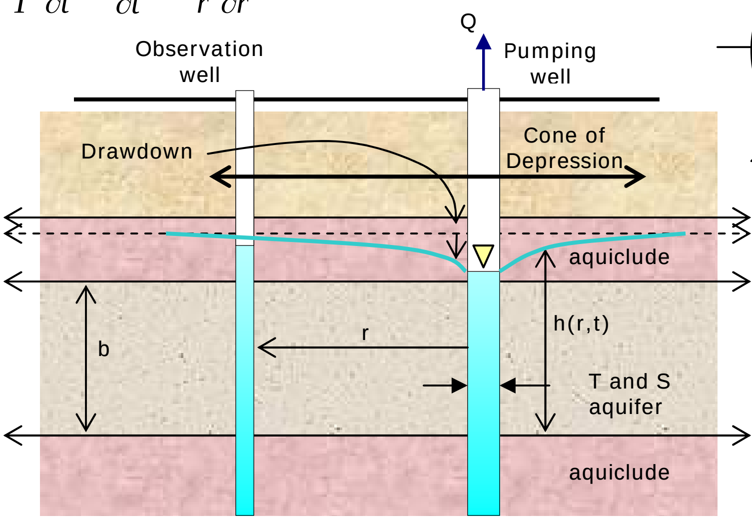
S = S_sb = Storativity [L^3/L^3]
T = Kb = Transmissivity [L^2/T]
b = aquifer thickness [L]
h is head [L]

Assume:

- Aquifer is horizontal, confined both top and bottom, infinite in horizontal extent, constant thickness, homogeneous, isotropic.
- Potentiometric surface is horizontal before pumping, is not changing with time before pumping, all changes due to one pumping well
- Darcy's law is valid and groundwater has constant properties
- Well is fully screened and 100% efficient
- Constant pumping from a well in such a situation is radial, and horizontal, where only 1 space dimension is needed $r = \sqrt{x^2 + y^2}$



$$\frac{S}{T}\frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial t^2} + \frac{1}{r}\frac{\partial h}{\partial r}$$



We want a solution that gives h(r,t) after we start pumping. To solve equation we need initial conditions (ICs) and boundary conditions (BCs)

IC: $h(r,0) = h_0$

BCs: $h(\infty,t)=h_0$ and $\lim_{r\to 0}\left(r\frac{\partial h}{\partial r}\right)=\frac{Q}{2\pi T}$ for $t > 0$
which is just the application of Darcy's law at the well

Theis Equation – transient radial flow

1935 – C.V. Theis solves this equation (with C.I. Lubin from heat conduction)

$$h_0 - h(r,t) = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-u}du}{u}$$

exponential integral in math tables but for our case it is "well function"

←

where $u = \frac{r^2 S}{4Tt}$

$h_0 - h(r,t) = \frac{Q}{4\pi T}W(u)$, well function is W(u)

Determining T and S from Pumping Test

Inverse method: Use solution to the PDE to identify the parameter values by matching simulated and observed heads (dependent variables); e.g., measure aquifer drawdown response given a known pumping rate and get T and S.

1. Identify pumping well and observation wells and their conditions (e.g., fully screened).
2. Determine aquifer type and make a quick estimate to predict what you think will happen during pumping test.
3. Theis Method: Arrange Theis equation as follows:

   $\Delta h = \left[\frac{114.6Q}{T}\right]W(u)$ (in USGS units) and

   $$\frac{r^2}{t} = \left[\frac{T}{1.87S}\right]u \rightarrow t = \left[\frac{1.87\,Sr^2}{T}\right]\frac{1}{u}$$

   $$\log \Delta h = \log\left[\frac{114.6Q}{T}\right] + \log W(u)$$

   $$\log t = \log\left[\frac{1.87Sr^2}{T}\right] + \log\frac{1}{u}$$

4. Plot the well function W(u) versus 1/u on log-log paper. (this is called a type curve)

5. Plot drawdown vs. time on log-log paper of same scale. (this is from data at a single observation well)
6. Superimpose the field curve on the type curve, keeping the axes parallel. Adjust the curves so that most of the data fall on the type curve. You trying to get the constants (bracketed terms) that make the type curve axes translate into your axes.
7. Select a match point (any convenient point will do like W(u) = 1.0 and 1/u = 1.0), and read off the values for W(u) and 1/u. Then read off the values for drawdown and t.
8. Compute the values of T and S from:

Using USGS Units	Using Self-Consistent Units
$T = \dfrac{114.6QW(u)}{(h_0 - h)}$	$T = \dfrac{QW(u)}{4\pi(h_0 - h)}$
$S = \dfrac{Ttu}{1.87r^2}$	$S = \dfrac{4Ttu}{r^2}$

If in USGS units of drawdown (ft), Q (gpm), T (gpd/ft), r (ft), t (days), S (decimal fraction).

$$h_0 - h(r,t) = \frac{Q}{4\pi T} W(u)$$

$$u = \frac{1.87r^2 S}{Tt}$$

To predict drawdown – drawdown vs. distance or time

- Put in r, S, T, t and solve for u
- Find W(u) based on u and table
- Multiply Q/T and factor

The analytic solution describes:

- Geometric characteristics to the cone of depression, steepening toward the well
- For a given aquifer cone of depression increases in depth and extent with time
- Drawdown at a time and location increases linearly with pumping rate
- Drawdown at a time and location is greater for smaller T
- Drawdown at a time and location is greater for lower S

Modified Nonequilibrium Solution – Jacob Method

Recall from Theis solution:

$$h_0 - h(r,t) = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-u}du}{u}$$

C.E. Jacob noted that the well function can be represented by a series.

$$h_0 - h(r,t) = \frac{Q}{4\pi T}\left[-0.5772 - \ln u + u - \frac{u^2}{2\cdot 2!} + \frac{u^3}{3\cdot 3!} - \frac{u^4}{4\cdot 4!} + ...\right]$$

For small values of r and large values of t, u becomes small. (valid when u < 0.01).

Then most terms can be dropped leaving:

$$h_0 - h(r,t) = \frac{Q}{4\pi T}\left[-0.5772 - \ln\frac{r^2 S}{4Tt}\right]$$

Noting that –ln u = ln 1/u and ln 1.78 = 0.5772

$$h_0 - h(r,t) = \frac{Q}{4\pi T}\left[\ln\frac{2.25Tt}{r^2 S}\right]$$

And ln u = 2.3 log u

$$h_0 - h(r,t) = \frac{2.30Q}{4\pi T}\left[\log_{10}\frac{2.25Tt}{r^2 S}\right]$$

Since Q, r, T, and S are constants, drawdown vs. log t should plot as a straight line.

In USGS units:

$$h_0 - h(r,t) = \frac{264Q}{4\pi T}\left[\log_{10}\frac{0.3Tt}{r^2 S}\right]$$

Over one log cycle you get change in drawdown

From t_1 to t_2 : $\Delta[h_0 - h] = \frac{264Q}{T}\left[\log_{10}\frac{t_2}{t_1}\right]$

$T = \frac{264Q}{\Delta[h_0 - h]}\left[\log_{10} 10\right]$ (USGS units)

$$T = \frac{2.3Q}{4\pi\Delta[h_0 - h]}$$ (self-consistent units)

Procedure

1. Plot on semi-log paper – t on log scale and drawdown on arithmetic scale.
2. Pick off two values of time and the corresponding values of drawdowns (over one log cycle make it easy).
3. Solve for T (just a function of Q and Δh)
4. Consider basic solution when drawdown is zero.

$$0 = h_0 - h = \frac{2.30Q}{4\pi T}\left[\log_{10}\frac{2.25Tt_0}{r^2S}\right]$$

$$0 = \left[\log_{10}\frac{2.25Tt_0}{r^2S}\right] \rightarrow \frac{2.25Tt_0}{r^2S} = 1 \text{ or}$$

$$S = \frac{2.25Tt_0}{r^2}$$ where t_0 is the time intercept at which drawdown is zero in USGS units.

$$S = \frac{2.25Tt_0}{r^2}$$ where t_0 is the time intercept in days and the straight line intersects the zero-drawdown axis