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GEARS AND GEARING

External spur gears are cylindrical gears with straight teeth cut parallel to the axes. Gears transmit drive between parallel shafts. Tooth loads produce no axial thrust. Excellent at moderate speeds but tend to be noisy at high speeds. Shafts rotate in opposite directions.

Internal spur gears provide compact drive arrangements for transmitting motion between parallel shafts rotating in the same direction.

Helical gears are cylindrical gears with teeth cut at an angle to the axes. Provide rive between shafts rotating in opposite directions, with superior load carrying capacity and quietness than spur gears. Tooth loads produce axial thrust.

Crossed helical gears are helical gears that mesh together on non-parallel axes.

Straight bevel gears have teeth that are radial toward the apex and are of conical form. Designed to operate on intersecting axes, bevel gears are used to connect two shafts on intersecting axes. The angle between the shafts equals the angle between the two axes of the meshing teeth. End thrust developed under load tends to separate the gears.

Spiral bevel gears have curved oblique teeth that contact each other smoothly and gradually from one end of a tooth to the other. Meshing is similar to that of straight bevel gears but is smoother and quieter in use. Left hand spiral teeth incline away from the axis in an anti-clockwise direction looking on small end of pinion or face of gear, right-hand teeth incline away from axis in clockwise direction. The hand of spiral of the pinion is always opposite to that of the gear and is used to identify the hand of the gear pair. Used to connect two shafts on intersecting axes as with straight bevel gears. The spiral angle does not affect the smoothness and quietness of operation or the efficiency but does affect the direction of the thrust loads created. A left-hand spiral pinion driving clockwise when viewed from the large end of the pinion creates an axial thrust that tends to move the pinion out of mesh.

Zerol bevel gears have curved teeth lying in the same general direction as straight bevel teeth but should be considered to be spiral bevel gears with zero spiral angle.

Hypoid bevel gears are a cross between spiral bevel gears and worm gears. The axes of hypoid bevel gears are non-intersecting and non-parallel. The distance between the axes is called the offset. The offset permits higher ratios of reduction than is practicable with other bevel gears. Hypoid bevel gears have curved oblique teeth on which contact begins gradually and continues smoothly from one end of the tooth to the other.

Worm gears are used to transmit motion between shafts at right angles, that do not lie in a common plane and sometimes to connect shafts at other angles. Worm gears have line tooth contact and are used for power transmission, but the higher the ratio the lower the efficiency.

Definitions of Gear Terms.— The following terms are commonly applied to the various classes of gears:

Active face width is the dimension of the tooth face width that makes contact with a mating gear.

Addendum is the radial or perpendicular distance between the pitch circle and the top of the tooth.

Arc of action is the arc of the pitch circle through which a tooth travels from the first point of contact with the mating tooth to the point where contact ceases.

Arc of approach is the arc of the pitch circle through which a tooth travels from the first point of contact with the mating tooth to the pitch point.

Arc of recession is the arc of the pitch circle through which a tooth travels from its contact with a mating tooth at the pitch point until contact ceases.

Axial pitch is the distance parallel to the axis between corresponding sides of adjacent teeth.

Axial plane is the plane that contains the two axes in a pair of gears. In a single gear the axial plane is any plane containing the axis and any given point.

Axial thickness is the distance parallel to the axis between two pitch line elements of the same tooth.

Backlash is the shortest distance between the non-driving surfaces of adjacent teeth when the working flanks are in contact.

Base circle is the circle from which the involute tooth curve is generated or developed.

Base helix angle is the angle at the base cylinder of an involute gear that the tooth makes with the gear axis.

Base pitch is the circular pitch taken on the circumference of the base circles, or the distance along the line of action between two successive and corresponding involute tooth profiles. The *normal base pitch* is the base pitch in the normal plane and the *axial base pitch* is the base pitch in the axial plane.

Base tooth thickness is the distance on the base circle in the plane of rotation between involutes of the same pitch.

Bottom land is the surface of the gear between the flanks of adjacent teeth.

Center distance is the shortest distance between the non-intersecting axes of mating gears, or between the parallel axes of spur gears and parallel helical gears, or the crossed axes of crossed helical gears or worm gears.

Central plane is the plane perpendicular to the gear axis in a worm gear, which contains the common perpendicular of the gear and the worm axes. In the usual arrangement with the axes at right angles, it contains the worm axis.

Chordal addendum is the radial distance from the circular thickness chord to the top of the tooth, or the height from the top of the tooth to the chord subtending the circular thickness arc.

Chordal thickness is the length of the chord subtended by the circular thickness arc. The dimension obtained when a gear tooth caliper is used to measure the tooth thickness at the pitch circle.

Circular pitch is the distance on the circumference of the pitch circle, in the plane of rotation, between corresponding points of adjacent teeth. The length of the arc of the pitch circle between the centers or other corresponding points of adjacent teeth.

Circular thickness is the thickness of the tooth on the pitch circle in the plane of rotation, or the length of arc between the two sides of a gear tooth measured on the pitch circle.

Clearance is the radial distance between the top of a tooth and the bottom of a mating tooth space, or the amount by which the dedendum in a given gear exceeds the addendum of its mating gear.

Contact diameter is the smallest diameter on a gear tooth with which the mating gear makes contact.

Contact ratio is the ratio of the arc of action in the plane of rotation to the circular pitch, and is sometimes thought of as the average number of teeth in contact. This ratio is obtained most directly as the ratio of the length of action to the base pitch.

Contact ratio – face is the ratio of the face advance to the circular pitch in helical gears.

Contact ratio – total is the ratio of the sum of the arc of action and the face advance to the circular pitch.

Contact stress is the maximum compressive stress within the contact area between mating gear tooth profiles. Also called the Hertz stress.

Cycloid is the curve formed by the path of a point on a circle as it rolls along a straight line. When such a circle rolls along the outside of another circle the curve is called an *epicycloid*, and when it rolls along the inside of another circle it is called a *hypocycloid*. These curves are used in defining the former American Standard composite Tooth Form.

Dedendum is the radial or perpendicular distance between the pitch circle and the bottom of the tooth space.

Diametral pitch is the ratio of the number of teeth to the number of inches in the pitch diameter in the plane of rotation, or the number of gear teeth to each inch of pitch diameter. Normal diametral pitch is the diametral pitch as calculated in the normal plane, or the diametral pitch divided by the cosine of the helix angle.

Efficiency is the torque ratio of a gear set divided by its gear ratio.

Equivalent pitch radius is the radius of curvature of the pitch surface at the pitch point in a plane normal to the pitch line element.

Face advance is the distance on the pitch circle that a gear tooth travels from the time pitch point contact is made at one end of the tooth until pitch point contact is made at the other end.

Fillet radius is the radius of the concave portion of the tooth profile where it joins the bottom of the tooth space.

Fillet stress is the maximum tensile stress in the gear tooth fillet.

Flank of tooth is the surface between the pitch circle and the bottom land, including the gear tooth fillet.

Gear ratio is the ratio between the numbers of teeth in mating gears.

Helical overlap is the effective face width of a helical gear divided by the gear axial pitch.

Helix angle is the angle that a helical gear tooth makes with the gear axis at the pitch circle, unless specified otherwise.

Hertz stress, see *Contact stress*.

Highest point of single tooth contact (HPSTC) is the largest diameter on a spur gear at which a single tooth is in contact with the mating gear.

Interference is the contact between mating teeth at some point other than along the line of action.

Internal diameter is the diameter of a circle that coincides with the tops of the teeth of an internal gear.

Internal gear is a gear with teeth on the inner cylindrical surface.

Involute is the curve generally used as the profile of gear teeth. The curve is the path of a point on a straight line as it rolls along a convex base curve, usually a circle.

Land: The top land is the top surface of a gear tooth and the *bottom land* is the surface of the gear between the fillets of adjacent teeth.

Lead is the axial advance of the helix in one complete turn, or the distance along its own axis on one revolution if the gear were free to move axially.

Length of action is the distance on an involute line of action through which the point of contact moves during the action of the tooth profile.

Line of action is the portion of the common tangent to the base cylinders along which contact between mating involute teeth occurs.

Lowest point of single tooth contact (LPSTC) is the smallest diameter on a spur gear at which a single tooth is in contact with its mating gear. Gear set contact stress is determined with a load placed on the pinion at this point.

Module is the ratio of the pitch diameter to the number of teeth, normally the ratio of pitch diameter in mm to the number of teeth. Module in the inch system is the ratio of the pitch diameter in inches to the number of teeth.

Normal plane is a plane normal to the tooth surfaces at a point of contact and perpendicular to the pitch plane.

Number of teeth is the number of teeth contained in a gear.

Outside diameter is the diameter of the circle that contains the tops of the teeth of external gears.

Pitch is the distance between similar, equally-spaced tooth surfaces in a given direction along a given curve or line.

Pitch circle is the circle through the pitch point having its center at the gear axis.

Pitch diameter is the diameter of the pitch circle. The operating pitch diameter is the pitch diameter at which the gear operates.

Pitch plane is the plane parallel to the axial plane and tangent to the pitch surfaces in any pair of gears. In a single gear, the pitch plane may be any plane tangent to the pitch surfaces.

Pitch point is the intersection between the axes of the line of centers and the line of action.

Plane of rotation is any plane perpendicular to a gear axis.

Pressure angle is the angle between a tooth profile and a radial line at its pitch point. In involute teeth, the pressure angle is often described as the angle between the line of action and the line tangent to the pitch circle. *Standard pressure angles* are established in connection with standard tooth proportions. A given pair of involute profiles will transmit smooth motion at the same velocity ratio when the center distance is changed. Changes in center distance in gear design and gear manufacturing operations may cause changes in pitch diameter, pitch and pressure angle in the same gears under different conditions. Unless otherwise specified, the pressure angle is the *standard pressure angle at the standard pitch diameter*. The *operating pressure angle* is determined by the center distance at which a pair of gears operate. In oblique teeth such as helical and spiral designs, the pressure angle is specified in the transverse, normal or axial planes.

Principle reference planes are pitch plane, axial plane and transverse plane, all intersecting at a point and mutually perpendicular.

Rack: A rack is a gear with teeth spaced along a straight line, suitable for straight line motion. A basic rack is a rack that is adopted as the basis of a system of interchangeable gears. Standard gear tooth dimensions are often illustrated on an outline of a basic rack.

Roll angle is the angle subtended at the center of a base circle from the origin of an involute to the point of tangency of a point on a straight line from any point on the same involute. The radian measure of this angle is the tangent of the pressure angle of the point on the involute.

Root diameter is the diameter of the circle that contains the roots or bottoms of the tooth spaces.

Tangent plane is a plane tangent to the tooth surfaces at a point or line of contact.

Tip relief is an arbitrary modification of a tooth profile where a small amount of material is removed from the involute face of the tooth surface near the tip of the gear tooth.

Tooth face is the surface between the pitch line element and the tooth tip.

Tooth surface is the total tooth area including the flank of the tooth and the tooth face.

Total face width is the dimensional width of a gear blank and may exceed the effective face width as with a double-helical gear where the total face width includes any distance separating the right-hand and left-hand helical gear teeth.

Transverse plane is a plane that is perpendicular to the axial plane and to the pitch plane. In gears with parallel axes, the transverse plane and the plane of rotation coincide.

Trochoid is the curve formed by the path of a point on the extension of a radius of a circle as it rolls along a curve or line. A trochoid is also the curve formed by the path of a point on a perpendicular to a straight line as the straight line rolls along the convex side of a base curve. By the first definition, a trochoid is derived from the *cycloid*, by the second definition it is derived from the *involute*.

True involute form diameter is the smallest diameter on the tooth at which the point of tangency of the involute tooth profile exists. Usually this position is the point of tangency of the involute tooth profile and the fillet curve, and is often referred to as the TIF diameter.

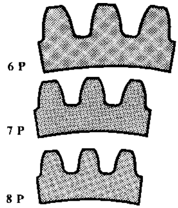
Undercut is a condition in generated gear teeth when any part of the fillet curve lies inside a line drawn at a tangent to the working profile at its lowest point. Undercut may be introduced deliberately to facilitate shaving operations, as in pre-shaving.

Whole depth is the total depth of a tooth space, equal to the addendum plus the dedendum and equal to the working depth plus clearance.

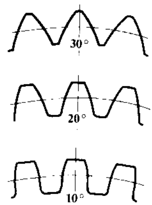
Working depth is the depth of engagement of two gears, or the sum of their addendums. The standard working distance is the depth to which a tooth extends into the tooth space of a mating gear when the center distance is standard.

Definitions of gear terms are given in AGMA Standards 112.05, 115.01, and 116.01 entitled “Terms, Definitions, Symbols and Abbreviations,” “Reference Information—Basic Gear Geometry,” and “Glossary—Terms Used in Gearing,” respectively; obtainable from American Gear Manufacturers Assn., 1500 King. St., Alexandria, VA 22314.

Comparative Sizes and Shape of Gear Teeth

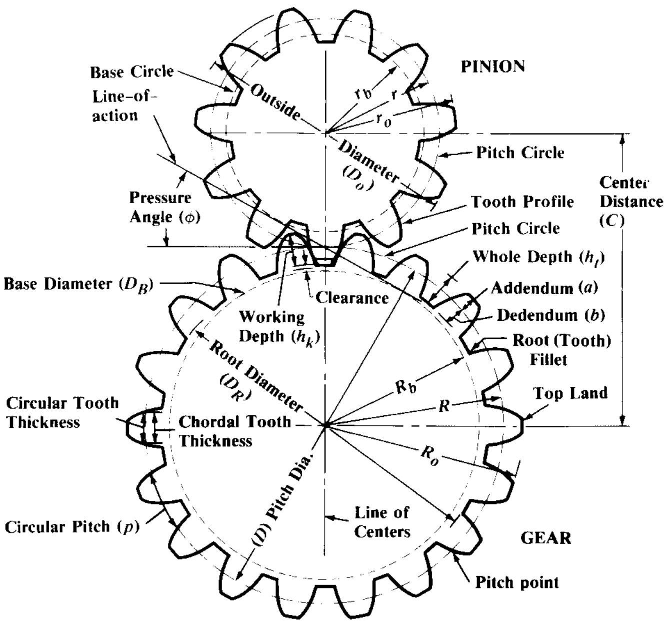


Gear Teeth of Different Diametral Pitch



Shapes of Gear Teeth of Different Pressure Angles

Nomenclature of Gear Teeth



Terms Used in Gear Geometry from Table 1 on page 2004

Properties of the Involute Curve.—The involute curve is used almost exclusively for gear-tooth profiles, because of the following important properties.

1) The form or shape of an involute curve depends upon the diameter of the base circle from which it is derived. (If a taut line were unwound from the circumference of a circle—the *base circle* of the involute—the end of that line or any point on the unwound portion, would describe an involute curve.)

2) If a gear tooth of involute curvature acts against the involute tooth of a mating gear while rotating at a uniform rate, the angular motion of the driven gear will also be uniform, even though the center-to-center distance is varied.

3) The relative rate of motion between driving and driven gears having involute tooth curves is established by the diameters of their base circles.

4) Contact between intermeshing involute teeth on a driving and driven gear is along a straight line that is tangent to the two base circles of these gears. This is the *line of action*.

5) The point where the line of action intersects the common center-line of the mating involute gears, establishes the radii of the pitch circles of these gears; hence true pitch circle diameters are affected by a change in the center distance. (Pitch diameters obtained by dividing the number of teeth by the diametral pitch apply when the center distance equals the total number of teeth on both gears divided by twice the diametral pitch.)

6) The pitch diameters of mating involute gears are directly proportional to the diameters of their respective base circles; thus, if the base circle of one mating gear is three times as large as the other, the pitch circle diameters will be in the same ratio.

7) The angle between the line of action and a line perpendicular to the common center-line of mating gears, is the *pressure angle*; hence the pressure angle is affected by any change in the center distance.

8) When an involute curve acts against a straight line (as in the case of an involute pinion acting against straight-sided rack teeth), the straight line is tangent to the involute and perpendicular to its line of action.

9) The pressure angle, in the case of an involute pinion acting against straight-sided rack teeth, is the angle between the line of action and the line of the rack's motion. If the involute pinion rotates at a uniform rate, movement of the rack will also be uniform.

Nomenclature:

ϕ = Pressure Angle
a = Addendum a_G = Addendum of Gear a_P = Addendum of Pinion
b = Dedendum
c = Clearance
C = Center Distance
D = Pitch Diameter D_G = Pitch Diameter of Gear D_P = Pitch Diameter of Pinion
D_B = Base Circle Diameter D_O = Outside Diameter D_R = Root Diameter
F = Face Width
h_k = Working Depth of Tooth h_t = Whole Depth of Tooth
m_G = Gear Ratio
N = Number of Teeth N_G = Number of Teeth in Gear N_P = Number of Teeth in Pinion
p = Circular Pitch P = Diametral Pitch

Diametral and Circular Pitch Systems.—Gear tooth system standards are established by specifying the tooth proportions of the basic rack. The diametral pitch system is applied to most of the gearing produced in the United States. If gear teeth are larger than about one diametral pitch, it is common practice to use the circular pitch system. The circular pitch system is also applied to cast gearing and it is commonly used in connection with the design and manufacture of worm gearing.

Pitch Diameters Obtained with Diametral Pitch System.—The diametral pitch system is arranged to provide a series of standard tooth sizes, the principle being similar to the standardization of screw thread pitches. Inasmuch as there must be a whole number of teeth on each gear, the increase in pitch diameter per tooth varies according to the pitch. For example, the pitch diameter of a gear having, say, 20 teeth of 4 diametral pitch, will be 5 inches; 21 teeth, $5\frac{1}{4}$ inches; and so on, the increase in diameter for each additional tooth being equal to $\frac{1}{4}$ inch for 4 diametral pitch. Similarly, for 2 diametral pitch the variations for successive numbers of teeth would equal $\frac{1}{2}$ inch, and for 10 diametral pitch the variations would equal $1/10$ inch, etc. Where a given center distance must be maintained and no standard diametral pitch can be used, gears should be designed with reference to the gear set center distance procedure discussed in *Gears for Given Center Distance and Ratio* starting on page 2012.

Table 1. Formulas for Dimensions of Standard Spur Gears

To Find	Formula		To Find	Formula	
Base Circle Diameter	$D_B = D\cos\phi$	(1)	Number of Teeth	$N = P \times D$	(6a)
Circular Pitch	$p = \frac{3.1416D}{N}$	(2a)	Number of Teeth	$N = \frac{3.1416D}{p}$	(6b)
Circular Pitch	$p = \frac{3.1416}{P}$	(2b)	Outside Diameter (Full-depth Teeth)	$D_O = \frac{N+2}{P}$	(7a)
Center Distance	$C = \frac{N_P(m_G+1)}{2P}$	(3a)	Outside Diameter (Full-depth Teeth)	$D_O = \frac{(N+2)p}{3.1416}$	(7b)
Center Distance	$C = \frac{D_P + D_G}{2}$	(3b)	Outside Diameter (American Standard Stub Teeth)	$D_O = \frac{N+1.6}{P}$	(8a)
Center Distance	$C = \frac{N_G + N_P}{2P}$	(3c)	Outside Diameter (American Standard Stub Teeth)	$D_O = \frac{(N+1.6)p}{3.1416}$	(8b)
Center Distance	$C = \frac{(N_G + N_P)p}{6.2832}$	(3d)	Outside Diameter	$D_O = D + 2a$	(9)
Diametral Pitch	$P = \frac{3.1416}{p}$	(4a)	Pitch Diameter	$D = \frac{N}{P}$	(10a)
Diametral Pitch	$P = \frac{N}{D}$	(4b)	Pitch Diameter	$D = \frac{Np}{3.1416}$	(10b)
Diametral Pitch	$P = \frac{N_P(m_G+1)}{2C}$	(4c)	Root Diameter[a]	$D_R = D - 2b$	(11)
Gear Ratio	$m_G = \frac{N_G}{N_P}$	(5)	Whole Depth	$a + b$	(12)
			Working Depth	$a_G + a_P$	(13)

[a] See also formulas in Tables 2 and 4 on pages 2004 and 2008.

Table 2. Formulas for Tooth Parts, 20-and 25-degree Involute Full-depth Teeth ANSI Coarse Pitch Spur Gear Tooth Forms *ANSI B6.1-1968 (R1974)*

To Find	Diametral Pitch, P, Known	Circular Pitch, p, Known
Addendum	$a = 1.000 \div P$	$a = 0.3183 \times p$
Dedendum (Preferred)	$b = 1.250 \div P$	$b = 0.3979 \times p$
(Shaved or Ground Teeth)[a]	$b = 1.350 \div P$	$b = 0.4297 \times p$
Working Depth	$h_k = 2.000 \div P$	$h_k = 0.6366 \times p$

Table 2. *(Continued)* **Formulas for Tooth Parts, 20-and 25-degree Involute Full-depth Teeth ANSI Coarse Pitch Spur Gear Tooth Forms** *ANSI B6.1-1968 (R1974)*

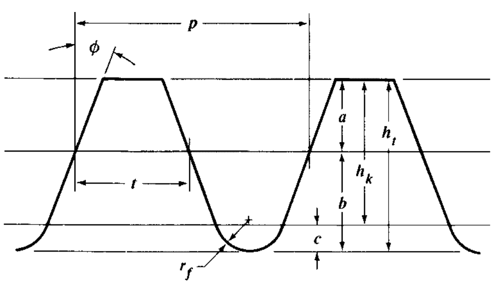
To Find	Diametral Pitch, P, Known	Circular Pitch, p, Known
Whole Depth (Preferred)	$h_t = 2.250 \div P$	$h_t = 0.7162 \times p$
(Shaved or Ground Teeth)	$h_t = 2.350 \div P$	$h_t = 0.7480 \times p$
Clearance (Preferred)[b]	$c = 0.250 \div P$	$c = 0.0796 \times p$
(Shaved or Ground Teeth)	$c = 0.350 \div P$	$c = 0.1114 \times p$
Fillet Radius (Rack)[c]	$r_f = 0.300 \div P$	$r_f = 0.0955 \times p$
Pitch Diameter	$D = N \div P$	$D = 0.3183 \times N_p$
Outside Diameter	$D_O = (N + 2) \div P$	$D_O = 0.3183 \times (N + 2)p$
Root Diameter (Preferred)	$D_R = (N - 2.5) \div P$	$D_R = 0.3183 \times (N - 2.5)p$
(Shaved or Ground Teeth)	$D_R = (N - 2.7) \div P$	$D_R = 0.3183 \times (N - 2.7)p$
Circular Thickness—Basic	$t = 1.5708 \div P$	$t = p \div 2$

[a] When gears are preshave cut on a gear shaper the dedendum will usually need to be increased to 1.40/P to allow for the higher fillet trochoid produced by the shaper cutter. This is of particular importance on gears of few teeth or if the gear blank configuration requires the use of a small diameter shaper cutter, in which case the dedendum may need to be increased to as much as 1.45/P. This should be avoided on highly loaded gears where the consequently reduced J factor will increase gear tooth stress excessively.

[b] A minimum clearance of 0.157/P may be used for the basic 20-degree and 25-degree pressure angle rack in the case of shallow root sections and use of existing hobs or cutters. However, whenever less than standard clearance is used, the location of the TIF diameter should be determined by the method shown in *True Involute Form Diameter* starting on page 2030. The TIF diameter must be less than the Contact Diameter determined by the method shown on page 2028.

[c] The fillet radius of the basic rack should not exceed 0.235/P for a 20-degree pressure angle rack or 0.270/P for a 25-degree pressure angle rack for a clearance of 0.157/P. The basic rack fillet radius must be *reduced* for teeth with a 25-degree pressure angle having a clearance in excess of 0.250/P.

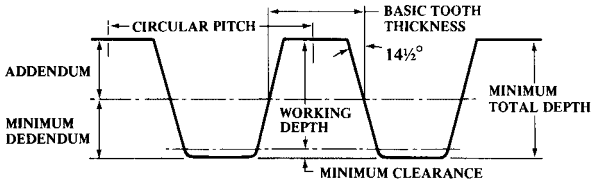
American National Standard and Former American Standard Gear Tooth Forms *ANSI B6.1-1968, (R1974)* and *ASA B6.1-1932*



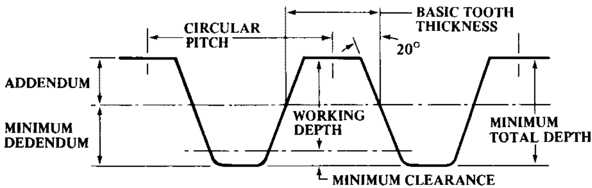
a = addendum
b = dedendum
c = clearance
h_k = working depth
h_t = whole depth
p = circular pitch
r_f = fillet radius of basic rack
t = circular tooth thickness — basic
ϕ = pressure angle

Basic Rack of the 20-Degree and 25-Degree Full-Depth Involute Systems

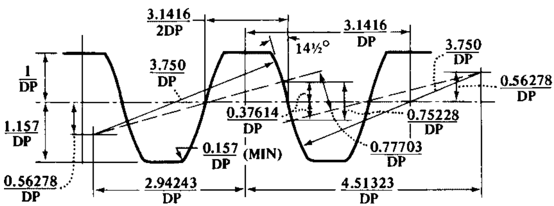
American National Standard and Former American Standard Gear Tooth Forms
ANSI B6.1-1968, (R1974) **and** *ASA B6.1-1932 (Continued)*



Basic Rack of the 14½-Degree Full-Depth Involute System



Basic Rack of the 20-Degree Stub Involute System



Approximation of Basic Rack for the 14½-Degree Composite System

American National Standard Coarse Pitch Spur Gear Tooth Forms.—The American National Standard (ANSI B6.1-1968, R1974) provides tooth proportion information on two involute spur gear forms. These two forms are identical except that one has a pressure angle of 20 degrees and a minimum allowable tooth number of 18 while the other has a pressure angle of 25 degrees and a minimum allowable tooth number of 12. (For pinions with fewer teeth, see tooth proportions for long addendum pinions and their mating short addendum gears in Tables 1 through 3d starting on page 2019.) A gear tooth standard is established by specifying the tooth proportions of the basic rack. Gears made to this standard will thus be conjugate with the specified rack and with each other. The basic rack forms for the 20-degree and 25-degree standard are shown on the following page; basic formulas for these proportions as a function of the gear diametral pitch and also of the circular pitch are given in Table 2. Tooth parts data are given in Table 3.

In recent years the established standard of almost universal use is the ANSI 20-degree standard spur gear form. It provides a gear with good strength and without fillet undercut in pinions of as few as eighteen teeth. Some more recent applications have required a tooth form of even greater strength and fewer teeth than eighteen. This requirement has stimulated the establishment of the ANSI 25-degree standard. This 25-degree form will give greater tooth strength than the 20-degree standard, will provide pinions of as few as twelve teeth without fillet undercut and will provide a lower contact compressive stress for greater gear set surface durability.

Table 3. Gear Tooth Parts for American National Standard Coarse Pitch 20- and 25-Degree Pressure Angle Gears

Dia. Pitch	Circ. Pitch	Stand. Addend.[a]	Stand. Dedend.	Spec. Dedend.[b]	Min. Dedend.	Stand. F. Rad.	Min. F. Rad.
P	p	a	b	b	b	r_f	r_f
0.3142	10.	3.1831	3.9789	4.2972	3.6828	0.9549	0.4997
0.3307	9.5	3.0239	3.7799	4.0823	3.4987	0.9072	0.4748
0.3491	9.	2.8648	3.5810	3.8675	3.3146	0.8594	0.4498
0.3696	8.5	2.7056	3.3820	3.6526	3.1304	0.8117	0.4248
0.3927	8.	2.5465	3.1831	3.4377	2.9463	0.7639	0.3998
0.4189	7.5	2.3873	2.9842	3.2229	2.7621	0.7162	0.3748
0.4488	7.	2.2282	2.7852	3.0080	2.5780	0.6685	0.3498
0.4833	6.5	2.0690	2.5863	2.7932	2.3938	0.6207	0.3248
0.5236	6.	1.9099	2.3873	2.5783	2.2097	0.5730	0.2998
0.5712	5.5	1.7507	2.1884	2.3635	2.0256	0.5252	0.2749
0.6283	5.	1.5915	1.9894	2.1486	1.8414	0.4775	0.2499
0.6981	4.5	1.4324	1.7905	1.9337	1.6573	0.4297	0.2249
0.7854	4.	1.2732	1.5915	1.7189	1.4731	0.3820	0.1999
0.8976	3.5	1.1141	1.3926	1.5040	1.2890	0.3342	0.1749
1.	3.1416	1.0000	1.2500	1.3500	1.1570	0.3000	0.1570
1.25	2.5133	0.8000	1.0000	1.0800	0.9256	0.2400	0.1256
1.5	2.0944	0.6667	0.8333	0.9000	0.7713	0.2000	0.1047
1.75	1.7952	0.5714	0.7143	0.7714	0.6611	0.1714	0.0897
2.	1.5708	0.5000	0.6250	0.6750	0.5785	0.1500	0.0785
2.25	1.3963	0.4444	0.5556	0.6000	0.5142	0.1333	0.0698
2.5	1.2566	0.4000	0.5000	0.5400	0.4628	0.1200	0.0628
2.75	1.1424	0.3636	0.4545	0.4909	0.4207	0.1091	0.0571
3.	1.0472	0.3333	0.4167	0.4500	0.3857	0.1000	0.0523
3.25	0.9666	0.3077	0.3846	0.4154	0.3560	0.0923	0.0483
3.5	0.8976	0.2857	0.3571	0.3857	0.3306	0.0857	0.0449
3.75	0.8378	0.2667	0.3333	0.3600	0.3085	0.0800	0.0419
4.	0.7854	0.2500	0.3125	0.3375	0.2893	0.0750	0.0392
4.5	0.6981	0.2222	0.2778	0.3000	0.2571	0.0667	0.0349
5.	0.6283	0.2000	0.2500	0.2700	0.2314	0.0600	0.0314
5.5	0.5712	0.1818	0.2273	0.2455	0.2104	0.0545	0.0285
6.	0.5236	0.1667	0.2083	0.2250	0.1928	0.0500	0.0262
6.5	0.4833	0.1538	0.1923	0.2077	0.1780	0.0462	0.0242
7.	0.4488	0.1429	0.1786	0.1929	0.1653	0.0429	0.0224
7.5	0.4189	0.1333	0.1667	0.1800	0.1543	0.0400	0.0209
8.	0.3927	0.1250	0.1563	0.1687	0.1446	0.0375	0.0196
8.5	0.3696	0.1176	0.1471	0.1588	0.1361	0.0353	0.0185
9.	0.3491	0.1111	0.1389	0.1500	0.1286	0.0333	0.0174
9.5	0.3307	0.1053	0.1316	0.1421	0.1218	0.0316	0.0165
10.	0.3142	0.1000	0.1250	0.1350	0.1157	0.0300	0.0157
11.	0.2856	0.0909	0.1136	0.1227	0.1052	0.0273	0.0143
12.	0.2618	0.0833	0.1042	0.1125	0.0964	0.0250	0.0131
13.	0.2417	0.0769	0.0962	0.1038	0.0890	0.0231	0.0121
14.	0.2244	0.0714	0.0893	0.0964	0.0826	0.0214	0.0112
15.	0.2094	0.0667	0.0833	0.0900	0.0771	0.0200	0.0105
16.	0.1963	0.0625	0.0781	0.0844	0.0723	0.0188	0.0098
17.	0.1848	0.0588	0.0735	0.0794	0.0681	0.0176	0.0092
18.	0.1745	0.0556	0.0694	0.0750	0.0643	0.0167	0.0087
19.	0.1653	0.0526	0.0658	0.0711	0.0609	0.0158	0.0083
20.	0.1571	0.0500	0.0625	0.0675	0.0579	0.0150	0.0079

[a] When using equal addendums on pinion and gear the minimum number of teeth on the pinion is 18 and the minimum total number of teeth in the pair is 36 for 20-degree full depth involute tooth form and 12 and 24, respectively, for 25-degree full depth tooth form.

[b] The dedendum in this column is used when the gear tooth is shaved. It allows for the higher fillet cut by a protuberance hob.

The working depth is equal to twice the addendum.

The whole depth is equal to the addendum plus the dedendum.

Table 4. Tooth Proportions for Fine-Pitch Involute Spur and Helical Gears of 14½-, 20-, and 25-Degree Pressure Angle *ANSI B6.7-1977*

Item	Spur	Helical
Addendum, a	$\frac{1.000}{P}$	$\frac{1.000}{P_n}$
Dedendum, b	$\frac{1.200}{P} + 0.002$ (min.)	$\frac{1.200}{P_n} + 0.002$ (min.)
Working Depth, h_k	$\frac{2.000}{P}$	$\frac{2.000}{P_n}$
Whole Depth, h_t	$\frac{2.200}{P} + 0.002$ (min.)	$\frac{2.200}{P_n} + 0.002$ (min.)
Clearance, c (Standard)	$\frac{0.200}{P} + 0.002$ (min.)	$\frac{0.200}{P_n} + 0.002$ (min.)
(Shaved or Ground Teeth)	$\frac{0.350}{P} + 0.002$ (min.)	$\frac{0.350}{P_n} + 0.002$ (min.)
Tooth Thickness, t At Pitch Diameter	$t = \frac{1.5708}{P}$	$t_n = \frac{1.5708}{P_n}$
Circular Pitch, p	$p = \frac{\pi D}{N}$ or $\frac{\pi d}{n}$ or $\frac{\pi}{P}$	$p_n = \frac{\pi}{P_n}$
Pitch Diameter Pinion, d	$\frac{n}{P}$	$\frac{n}{P_n \cos\psi}$
Gear, D	$\frac{N}{P}$	$\frac{N}{P_n \cos\psi}$
Outside Diameter Pinion, d_o	$\frac{n+2}{P}$	$\frac{1}{P_n}\left(\frac{n}{\cos\psi} + 2\right)$
Gear, D_o	$\frac{N+2}{P}$	$\frac{1}{P_n}\left(\frac{N}{\cos\psi} + 2\right)$
Center Distance, C	$\frac{N+n}{2P}$	$\frac{N+n}{2P_n \cos\psi}$

All dimensions are in inches.
P = Transverse Diametral Pitch
P_n = Normal Diametral Pitch
t_n = Normal Tooth Thickness at Pitch Diameter
p_n = Normal Circular Pitch
ψ = Helix Angle
n = Number of pinion teeth
N = Number of gear teeth

American National Standard Tooth Proportions for Fine-Pitch Involute Spur and Helical Gears.—The proportions of spur gears in this Standard (ANSI B6.7-1977) follow closely ANSI B6.1-1968, R1974, "Tooth Proportions for Coarse-Pitch Involute Spur Gears." The main difference between fine-pitch and coarse-pitch gears is the greater clearance specified for fine-pitch gears. The increased clearance provides for any foreign material that may tend to accumulate at the bottoms of the teeth and also the relatively larger fillet radius resulting from proportionately greater wear on the tips of fine-pitch cutting tools.

Pressure Angle: The standard pressure angle for fine-pitch gears is 20 degrees and is recommended for most applications. For helical gears this pressure angle applies in the *nor-*

mal plane. In certain cases, notably sintered or molded gears, or in gearing where greatest strength and wear resistance are desired, a 25-degree pressure angle may be required. However, pressure angles greater than 20 degrees tend to require use of generating tools having very narrow point widths, and higher pressure angles require closer control of center distance when backlash requirements are critical.

In those cases where consideration of angular position or backlash is critical and both pinion and gear contain relatively large numbers of teeth, a 14½-degree pressure angle may be desirable. In general, pressure angles less than 20 degrees require greater amounts of tooth modification to avoid undercutting problems and are limited to larger total numbers of teeth in pinion and gear when operating at a standard center distance. Information Sheet B in the Standard provides tooth proportions for both 14½- and 25-degree pressure angle fine-pitch gears. Table 4 provides tooth proportions for fine-pitch spur and helical gears with 14½-, 20-, and 25-degree pressure angles, and Table 5 provides tooth parts.

Diametral Pitches: Diametral pitches preferred are: 20, 24, 32, 40, 48, 64, 72, 80, 96, and 120.

Table 5. American National Standard Fine Pitch Standard Gear Tooth Parts— 14½-, 20-, and 25-Degree Pressure Angles

Diametral Pitch	Circular Pitch	Circular Thickness	Standard Addend.	Standard Dedend.	Special Dedend.[a]
P	*p*	*t*	*a*	*b*	*b*
20	0.1571	0.0785	0.0500	0.0620	0.0695
24	0.1309	0.0654	0.0417	0.0520	0.0582
32	0.0982	0.0491	0.0313	0.0395	0.0442
40	0.0785	0.0393	0.0250	0.0320	0.0358
48	0.0654	0.0327	0.0208	0.0270	0.0301
64	0.0491	0.0245	0.0156	0.0208	0.0231
72	0.0436	0.0218	0.0139	0.0187	0.0208
80	0.0393	0.0196	0.0125	0.0170	0.0189
96	0.0327	0.0164	0.0104	0.0145	0.0161
120	0.0262	0.0131	0.0083	0.0120	0.0132

[a] Based upon clearance for shaved or ground teeth.

The working depth is equal to twice the addendum. The whole depth is equal to the addendum plus the dedendum. For minimum number of teeth see page 2027.

Other American Spur Gear Standards.—An appended information sheet in the American National Standard ANSI B6.1-1968, R1974 provides tooth proportion information for three spur gear forms with the notice that they are "not recommended for new designs." These forms are therefore considered to be obsolescent but the information is given on their proportions because they have been used widely in the past. These forms are the 14½-degree full depth form, the 20-degree stub involute form and the 14½-degree composite form which were covered in the former American Standard (ASA B6.1-1932). The basic rack for the 14½-degree full depth form is shown on page 2005; basic formulas for these proportions are given in Table 6.

Table 6. Formulas for Tooth Parts—Former American Standard Spur Gear Tooth Forms *ASA B6.1-1932*

To Find	Diametral Pitch, *P* Known	Circular Pitch, *p* Known
14½-Degree Involute Full-depth Teeth		
Addendum	$a = 1.000 \div P$	$a = 0.3183 \times p$
Minimum Dedendum	$b = 1.157 \div P$	$b = 0.3683 \times p$
Working Depth	$h_k = 2.000 \div P$	$h_k = 0.6366 \times p$
Minimum Whole Depth	$h_t = 2.157 \div P$	$h_t = 0.6866 \times p$
Basic Tooth Thickness on Pitch Line	$t = 1.5708 \div P$	$t = 0.500 \times p$
Minimum Clearance	$c = 0.157 \div P$	$c = 0.050 \times p$
20-Degree Involute Stub Teeth		
Addendum	$a = 0.800 \div P$	$a = 0.2546 \times p$
Minimum Dedendum	$b = 1.000 \div P$	$b = 0.3183 \times p$
Working Depth	$h_k = 1.600 \div P$	$h_k = 0.5092 \times p$
Minimum Whole Depth	$h_t = 1.800 \div P$	$h_t = 0.5729 \times p$
Basic Tooth Thickness on Pitch Line	$t = 1.5708 \div P$	$t = 0.500 \times p$
Minimum Clearance	$c = 0.200 \div P$	$c = 0.0637 \times p$

Note: Radius of fillet equals 1⅓ × clearance for 14½-degree full-depth teeth and 1½ × clearance for 20-degree full-depth teeth.

Note: A suitable working tolerance should be considered in connection with all minimum recommendations.

Fellows Stub Tooth.—The system of stub gear teeth introduced by the Fellows Gear Shaper Co. is based upon the use of two diametral pitches. One diametral pitch, say, 8, is used as the basis for obtaining the dimensions for the addendum and dedendum, while another diametral pitch, say, 6, is used for obtaining the dimensions of the thickness of the tooth, the number of teeth, and the pitch diameter. Teeth made according to this system are designated as 6⁄8 pitch, 12⁄14 pitch, etc., the numerator in this fraction indicating the pitch determining the thickness of the tooth and the number of teeth, and the denominator, the pitch determining the depth of the tooth. The clearance is made greater than in the ordinary gear-tooth system and equals 0.25 ÷ denominator of the diametral pitch. The pressure angle is 20 degrees.

This type of stub gear tooth is now used infrequently. For information as to the tooth part dimensions see 18th and earlier editions of Machinery's Handbook.

Basic Gear Dimensions.—The basic dimensions for all involute spur gears may be obtained using the formulas shown in Table 1. This table is used in conjunction with Table 3 to obtain dimensions for coarse pitch gears and Table 5 to obtain dimensions for fine pitch standard spur gears. To obtain the dimensions of gears that are specified at a standard circular pitch, the equivalent diametral pitch is first calculated by using the formula in Table 1. If the required number of teeth in the pinion (N_p) is less than the minimum specified in either Table 3 or Table 5, whichever is applicable, the gears must be proportioned by the long and short addendum method shown on page 2022.

Formulas for Outside and Root Diameters of Spur Gears that are Finish-hobbed, Shaped, or Pre-shaved

Notation

D = Pitch Diameter	a = Standard Addendum
D_O = Outside Diameter	b = Standard Minimum Dedendum
D_R = Root Diameter	b_s = Standard Dedendum
P = Diametral Pitch	b_{ps} = Dedendum for Pre-shaving

14½-, 20-, And 25-degree Involute Full-depth Teeth (19P and coarser)[a]

$$D_O = D + 2a = \frac{N}{P} + \left(2 \times \frac{1}{P}\right)$$

$$D_R = D - 2b = \frac{N}{P} - \left(2 \times \frac{1.157}{P}\right)$$ (Hobbed)[b]

$$D_R = D - 2b_s = \frac{N}{P} - \left(2 \times \frac{1.25}{P}\right)$$ (Shaped)[c]

$$D_R = D - 2b_{ps} = \frac{N}{P} - \left(2 \times \frac{1.35}{P}\right)$$ (Pre-shaved)[d]

$$D_R = D - 2b_{ps} = \frac{N}{P} - \left(2 \times \frac{1.40}{P}\right)$$ (Pre-shaved)[e]

20-degree Involute Fine-pitch Full-depth Teeth (20P and finer)

$$D_O = D + 2a = \frac{N}{P} + \left(2 \times \frac{1}{P}\right)$$

$$D_R = D - 2b = \frac{N}{P} - 2\left(\frac{1.2}{P} + 0.002\right)$$ (Hobbed or Shaped)[f]

$$D_R = D - 2b_{ps} = \frac{N}{P} - 2\left(\frac{1.35}{P} + 0.002\right)$$ (Pre-shaved)[g]

20-degree Involute Stub Teeth[a]

$$D_O = D + 2a = \frac{N}{P} + \left(2 \times \frac{0.8}{P}\right)$$

$$D_R = D - 2b = \frac{N}{P} - \left(2 \times \frac{1}{P}\right)$$ (Hobbed)

$$D_R = D - 2b_{ps} = \frac{N}{P} - \left(2 \times \frac{1.35}{P}\right)$$ (Pre-shaved)

[a] 14½-degree full-depth and 20-degree stub teeth are not recommended for new designs.

[b] According to ANSI B6.1-1968 a minimum clearance of 0.157/P may be used for the basic 20-degree and 25-degree pressure angle rack in the case of shallow root sections and the use of existing hobs and cutters.

[c] According to ANSI B6.1-1968 the preferred clearance is 0.250/P.

[d] According to ANSI B6.1-1968 the clearance for teeth which are shaved or ground is 0.350/P.

[e] When gears are preshave cut on a gear shaper the dedendum will usually need to be increased to 1.40/P to allow for the higher fillet trochoid produced by the shaper cutter; this is of particular importance on gears of few teeth or if the gear blank configuration requires the use of a small diameter shaper cutter, in which case the dedendum may need to be increased to as much as 1.45/P. This should be avoided on highly loaded gears where the consequently reduced J factor will increase gear tooth stress excessively.

[f] According to ANSI B6.7-1967 the standard clearance is 0.200/P + 0.002 (min.).

[g] According to ANSI B6.7-1967 the clearance for shaved or ground teeth is 0.350/P + 0.002 (min.).

Gears for Given Center Distance and Ratio.—When it is necessary to use a pair of gears of given ratio at a specified center distance C_1, it may be found that no gears of standard diametral pitch will satisfy the center distance requirement. Gears of standard diametral pitch P may need to be redesigned to operate at other than their standard pitch diameter D and standard pressure angle ϕ. The diametral pitch P_1 at which these gears will operate is

$$P_1 = \frac{N_P + N_G}{2C_1} \tag{1}$$

where N_p = number of teeth in pinion
N_G = number of teeth in gear

and their operating pressure angle ϕ_1 is

$$\phi_1 = \arccos\left(\frac{P_1}{P}\cos\phi\right) \tag{2}$$

Thus although the pair of gears are cut to a diametral pitch P and a pressure angle ϕ, they operate as standard gears of diametral pitch $P1$ and pressure angle ϕ_1. The pitch P and pressure angle ϕ should be chosen so that ϕ_1 lies between about 18 and 25 degrees.

The operating pitch diameters of the pinion D_{p1} and of the gear D_{G1} are

$$D_{P1} = \frac{N_P}{P_1} \quad \text{(3a)} \qquad \text{and} \qquad D_{G1} = \frac{N_G}{P_1} \quad \text{(3b)}$$

The base diameters of the pinion D_{PB1} and of the gear D_{GB1} are

$$D_{PB1} = D_{P1}\cos\phi_1 \quad \text{(4a)} \qquad \text{and} \qquad D_{GB1} = D_{G1}\cos\phi_1 \quad \text{(4b)}$$

The basic tooth thickness, t_1, at the operating pitch diameter for both pinion and gear is

$$t_1 = \frac{1.5708}{P_1} \tag{5}$$

The root diameters of the pinion D_{PR1} and gear D_{GR1} and the corresponding outside diameters D_{PO1} and D_{GO1} are not standard because each gear is to be cut with a cutter that is not standard for the operating pitch diameters D_{P1} and D_{G1}.

The root diameters are

$$D_{PR_1} = \frac{N_P}{P} - 2b_{P_1} \quad \text{(6a)} \qquad \text{and} \qquad D_{GR_1} = \frac{N_G}{P} - 2b_{G_1} \quad \text{(6b)}$$

where

$$b_{P_1} = b_c - \left(\frac{t_{P2} - 1.5708/P}{2\tan\phi}\right) \tag{7a}$$

and

$$b_{G1} = b_c - \frac{t_{G2} - 1.5708/P}{2\tan\phi} \tag{7b}$$

where b_c is the hob or cutter addendum for the pinion and gear.

The tooth thicknesses of the pinion t_{P2} and the gear t_{G2} are

$$t_{P2} = \frac{N_P}{P}\left(\frac{1.5708}{N_P} + \operatorname{inv}\phi_1 - \operatorname{inv}\phi\right) \tag{8a}$$

$$t_{G2} = \frac{N_G}{P}\left(\frac{1.5708}{N_G} + \mathrm{inv}\,\phi_1 - \mathrm{inv}\,\phi\right) \tag{8b}$$

The outside diameter of the pinion D_{PO} and the gear D_{GO} are

$$D_{PO} = 2 \times C_1 - D_{GR1} - 2(b_c - 1/P) \tag{9a}$$

and

$$D_{GO} = 2 \times C_1 - D_{PR1} - 2(b_c - 1/P) \tag{9b}$$

Example: Design gears of 8 diametral pitch, 20-degree pressure angle, and 28 and 88 teeth to operate at 7.50-inch center distance. The gears are to be cut with a hob of 0.169-inch addendum.

$$P_1 = \frac{28 + 88}{2 \times 7.50} = 7.7333 \tag{1}$$

$$\phi_1 = \arccos\left(\frac{7.7333}{8} \times 0.93969\right) = 24.719° \tag{2}$$

$$D_{P1} = \frac{28}{7.7333} = 3.6207 \text{ in.} \tag{3a}$$

and

$$D_{G1} = \frac{88}{7.7333} = 11.3794 \text{ in.} \tag{3b}$$

$$D_{PB1} = 3.6207 \times 0.90837 = 3.2889 \text{ in.} \tag{4a}$$

and

$$D_{GB1} = 11.3794 \times 0.90837 = 10.3367 \text{ in.} \tag{4b}$$

$$t_1 = \frac{1.5708}{7.7333} = 0.20312 \text{ in.} \tag{5}$$

$$D_{PR1} = \frac{28}{8} - 2 \times 0.1016 = 3.2968 \text{ in.} \tag{6a}$$

and

$$D_{GR1} = \frac{88}{8} - 2 \times (-0.0428) = 11.0856 \text{ in.} \tag{6b}$$

$$b_{P1} = 0.169 - \left(\frac{0.2454 - 1.5708/8}{2 \times 0.36397}\right) = 0.1016 \text{ in.} \tag{7a}$$

$$b_{G1} = 0.169 - \left(\frac{0.3505 - 1.5708/8}{2 \times 0.36397}\right) = -0.0428 \text{ in.} \tag{7b}$$

$$t_{P2} = \frac{28}{8}\left(\frac{1.5708}{28} + 0.028922 - 0.014904\right) = 0.2454 \text{ in.} \tag{8a}$$

$$t_{G2} = \frac{88}{8}\left(\frac{1.5708}{88} + 0.028922 - 0.014904\right) - 0.3505 \text{ in.} \tag{8b}$$

$$D_{PO1} = 2 \times 7.50 - 11.0856 - 2(0.169 - 1/8) = 3.8264 \text{ in.} \tag{9a}$$

$$D_{GO1} = 2 \times 7.50 - 3.2968 - 2(0.169 - 1/8) = 11.6152 \text{ in.} \tag{9b}$$

Tooth Thickness Allowance for Shaving.—Proper stock allowance is important for good results in shaving operations. If too much stock is left for shaving, the life of the shaving tool is reduced and, in addition, shaving time is increased. The following figures represent the amount of stock to be left on the teeth for removal by shaving under average conditions: For diametral pitches of 2 to 4, a thickness of 0.003 to 0.004 inch (one-half on each side of the tooth); for 5 to 6 diametral pitch, 0.0025 to 0.0035 inch; for 7 to 10 diametral pitch, 0.002 to 0.003 inch; for 11 to 14 diametral pitch, 0.0015 to 0.0020 inch; for 16 to 18 diametral pitch, 0.001 to 0.002 inch; for 20 to 48 diametral pitch, 0.0005 to 0.0015 inch; and for 52 to 72 diametral pitch, 0.0003 to 0.0007 inch.

The thickness of the gear teeth may be measured in several ways to determine the amount of stock left on the sides of the teeth to be removed by shaving. If it is necessary to measure the tooth thickness during the preshaving operation while the gear is in the gear shaper or hobbing machine, a gear tooth caliper or pins would be employed. Caliper methods of measuring gear teeth are explained in detail on page 2020 for measurements over single teeth, and on page 2109 for measurements over two or more teeth.

When the preshaved gear can be removed from the machine for checking, the center distance method may be employed. In this method, the preshaved gear is meshed without backlash with a gear of standard tooth thickness and the increase in center distance over standard is noted. The amount of total tooth thickness over standard that is left on the preshaved gear can then be determined by the formula: $t_2 = 2 \tan \phi \times d$, where t_2 = amount that the total thickness of the tooth exceeds the standard thickness, ϕ = pressure angle, and d = amount that the center distance between the two gears exceeds the standard center distance.

Circular Pitch for Given Center Distance and Ratio.—When it is necessary to use a pair of gears of given ratio at a specified center distance, it may be found that no gears of standard diametral pitch will satisfy the center distance requirement. Hence, circular pitch gears may be selected. To find the required circular pitch p, when the center distance C and total number of teeth N in both gears are known, use the following formula:

$$p = \frac{C \times 6.2832}{N}$$

Example: A pair of gears having a ratio of 3 is to be used at a center distance of 10.230 inches. If one gear has 60 teeth and the other 20, what must be their circular pitch?

$$p = \frac{10.230 \times 6.2832}{60 + 20} = 0.8035 \text{ inch}$$

Circular Thickness of Tooth when Outside Diameter is Standard.—For a full-depth or stub tooth gear of standard outside diameter, the tooth thickness on the pitch circle (circular thickness or arc thickness) is found by the following formula:

$$t = \frac{1.5708}{P}$$

where t = circular thickness and P = diametral pitch. In the case of Fellows stub tooth gears the diametral pitch used is the numerator of the pitch fraction (for example, 6 if the pitch is 6⁄8).

Example 1: Find the tooth thickness on the pitch circle of a 14½-degree full-depth tooth of 12 diametral pitch.

$$t = \frac{1.5708}{12} = 0.1309 \text{ inch}$$

Example 2: Find the tooth thickness on the tooth circle of a 20-degree full-depth involute tooth having a diametral pitch of 5.

$$t = \frac{1.5708}{5} = 0.31416\text{, say } 0.3142 \text{ inch}$$

The tooth thickness on the pitch circle can be determined very accurately by means of measurement over wires which are located in tooth spaces that are diametrically opposite or as nearly diametrically opposite as possible. Where measurement over wires is not feasible, the circular or arc tooth thickness can be used in determining the chordal thickness which is the dimension measured with a gear tooth caliper.

Circular Thickness of Tooth when Outside Diameter has been Enlarged.—When the outside diameter of a small pinion is not standard but is enlarged to avoid undercut and to improve tooth action, the teeth are located farther out radially relative to the standard pitch diameter and consequently the circular tooth thickness at the standard pitch diameter is increased. To find this increased arc thickness the following formula is used, where t = tooth thickness; e = amount outside diameter is increased over standard; ϕ = pressure angle; and p = circular pitch at the standard pitch diameter.

$$t = \frac{p}{2} + e\tan\phi$$

Example: The outside diameter of a pinion having 10 teeth of 5 diametral pitch and a pressure angle of 14½ degrees is to be increased by 0.2746 inch. The circular pitch equivalent to 5 diametral pitch is 0.6283 inch. Find the arc tooth thickness at the standard pitch diameter.

$$t = \frac{0.6283}{2} + (0.2746 \times \tan 14\tfrac{1}{2}°)$$

$$t = 0.3142 + (0.2746 \times 0.25862) = 0.3852 \text{ inch}$$

Circular Thickness of Tooth when Outside Diameter has been Reduced.—If the outside diameter of a gear is reduced, as is frequently done to maintain the standard center distance when the outside diameter of the mating pinion is increased, the circular thickness of the gear teeth at the standard pitch diameter will be reduced.This decreased circular thickness can be found by the following formula where t = circular thickness at the standard pitch diameter; e = amount outside diameter is reduced under standard; ϕ = pressure angle; and p = circular pitch.

$$t = \frac{p}{2} - e\tan\phi$$

Example: The outside diameter of a gear having a pressure angle of 14½ degrees is to be reduced by 0.2746 inch or an amount equal to the increase in diameter of its mating pinion. The circular pitch is 0.6283 inch. Determine the circular tooth thickness at the standard pitch diameter.

$$t = \frac{0.6283}{2} - (0.2746 \times \tan 14\tfrac{1}{2}°)$$

$$t = 0.3142 - (0.2746 \times 0.25862) = 0.2432 \text{ inch}$$

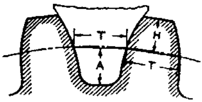
Chordal Thickness of Tooth when Outside Diameter is Standard.—To find the chordal or straight line thickness of a gear tooth the following formula can be used where t_c = chordal thickness; D = pitch diameter; and N = number of teeth.

$$t_c = D\sin\left(\frac{90°}{N}\right)$$

Example: A pinion has 15 teeth of 3 diametral pitch; the pitch diameter is equal to 15 ÷ 3 or 5 inches. Find the chordal thickness at the standard pitch diameter.

$$t_c = 5\sin\left(\frac{90°}{15}\right) = 5\sin 6° = 5 \times 0.10453 = 0.5226 \text{ inch}$$

Chordal Thicknesses and Chordal Addenda of Milled, Full-depth Gear Teeth and of Gear Milling Cutters



T = chordal thickness of gear tooth and cutter tooth at pitch line;
H = chordal addendum for full-depth gear tooth;
A = chordal addendum of cutter = (2.157 ÷ diametral pitch) − *H*
= (0.6866 × circular pitch) − *H*.

Diametral Pitch	Dimension	Number of Gear Cutter, and Corresponding Number of Teeth							
		No. 1 135 Teeth	No. 2 55 Teeth	No. 3 35 Teeth	No. 4 26 Teeth	No. 5 21 Teeth	No. 6 17 Teeth	No. 7 14 Teeth	No. 8 12 Teeth
1	T	1.5707	1.5706	1.5702	1.5698	1.5694	1.5686	1.5675	1.5663
	H	1.0047	1.0112	1.0176	1.0237	1.0294	1.0362	1.0440	1.0514
1½	T	1.0471	1.0470	1.0468	1.0465	1.0462	1.0457	1.0450	1.0442
	H	0.6698	0.6741	0.6784	0.6824	0.6862	0.6908	0.6960	0.7009
2	T	0.7853	0.7853	0.7851	0.7849	0.7847	0.7843	0.7837	0.7831
	H	0.5023	0.5056	0.5088	0.5118	0.5147	0.5181	0.5220	0.5257
2½	T	0.6283	0.6282	0.6281	0.6279	0.6277	0.6274	0.6270	0.6265
	H	0.4018	0.4044	0.4070	0.4094	0.4117	0.4144	0.4176	0.4205
3	T	0.5235	0.5235	0.5234	0.5232	0.5231	0.5228	0.5225	0.5221
	H	0.3349	0.3370	0.3392	0.3412	0.3431	0.3454	0.3480	0.3504
3½	T	0.4487	0.4487	0.4486	0.4485	0.4484	0.4481	0.4478	0.4475
	H	0.2870	0.2889	0.2907	0.2919	0.2935	0.2954	0.2977	0.3004
4	T	0.3926	0.3926	0.3926	0.3924	0.3923	0.3921	0.3919	0.3915
	H	0.2511	0.2528	0.2544	0.2559	0.2573	0.2590	0.2610	0.2628
5	T	0.3141	0.3141	0.3140	0.3139	0.3138	0.3137	0.3135	0.3132
	H	0.2009	0.2022	0.2035	0.2047	0.2058	0.2072	0.2088	0.2102
6	T	0.2618	0.2617	0.2617	0.2616	0.2615	0.2614	0.2612	0.2610
	H	0.1674	0.1685	0.1696	0.1706	0.1715	0.1727	0.1740	0.1752
7	T	0.2244	0.2243	0.2243	0.2242	0.2242	0.2240	0.2239	0.2237
	H	0.1435	0.1444	0.1453	0.1462	0.1470	0.1480	0.1491	0.1502
8	T	0.1963	0.1963	0.1962	0.1962	0.1961	0.1960	0.1959	0.1958
	H	0.1255	0.1264	0.1272	0.1279	0.1286	0.1295	0.1305	0.1314
9	T	0.1745	0.1745	0.1744	0.1744	0.1743	0.1743	0.1741	0.1740
	H	0.1116	0.1123	0.1130	0.1137	0.1143	0.1151	0.1160	0.1168
10	T	0.1570	0.1570	0.1570	0.1569	0.1569	0.1568	0.1567	0.1566
	H	0.1004	0.1011	0.1017	0.1023	0.1029	0.1036	0.1044	0.1051
11	T	0.1428	0.1428	0.1427	0.1427	0.1426	0.1426	0.1425	0.1424
	H	0.0913	0.0919	0.0925	0.0930	0.0935	0.0942	0.0949	0.0955
12	T	0.1309	0.1309	0.1308	0.1308	0.1308	0.1307	0.1306	0.1305
	H	0.0837	0.0842	0.0848	0.0853	0.0857	0.0863	0.0870	0.0876
14	T	0.1122	0.1122	0.1121	0.1121	0.1121	0.1120	0.1119	0.1118
	H	0.0717	0.0722	0.0726	0.0731	0.0735	0.0740	0.0745	0.0751
16	T	0.0981	0.0981	0.0981	0.0981	0.0980	0.0980	0.0979	0.0979
	H	0.0628	0.0632	0.0636	0.0639	0.0643	0.0647	0.0652	0.0657
18	T	0.0872	0.0872	0.0872	0.0872	0.0872	0.0871	0.0870	0.0870
	H	0.0558	0.0561	0.0565	0.0568	0.0571	0.0575	0.0580	0.0584
20	T	0.0785	0.0785	0.0785	0.0785	0.0784	0.0784	0.0783	0.0783
	H	0.0502	0.0505	0.0508	0.0511	0.0514	0.0518	0.0522	0.0525

Chordal Thicknesses and Chordal Addenda of Milled, Full-depth Gear Teeth and of Gear Milling Cutters

Circular Pitch	Dimension	Number of Gear Cutter, and Corresponding Number of Teeth							
		No. 1 135 Teeth	No. 2 55 Teeth	No. 3 35 Teeth	No. 4 26 Teeth	No. 5 21 Teeth	No. 6 17 Teeth	No. 7 14 Teeth	No. 8 12 Teeth
1/4	T	0.1250	0.1250	0.1249	0.1249	0.1249	0.1248	0.1247	0.1246
	H	0.0799	0.0804	0.0809	0.0814	0.0819	0.0824	0.0830	0.0836
5/16	T	0.1562	0.1562	0.1562	0.1561	0.1561	0.1560	0.1559	0.1558
	H	0.0999	0.1006	0.1012	0.1018	0.1023	0.1030	0.1038	0.1045
8/8	T	0.1875	0.1875	0.1874	0.1873	0.1873	0.1872	0.1871	0.1870
	H	0.1199	0.1207	0.1214	0.1221	0.1228	0.1236	0.1245	0.1254
7/16	T	0.2187	0.2187	0.2186	0.2186	0.2185	0.2184	0.2183	0.2181
	H	0.1399	0.1408	0.1416	0.1425	0.1433	0.1443	0.1453	0.1464
1/2	T	0.2500	0.2500	0.2499	0.2498	0.2498	0.2496	0.2495	0.2493
	H	0.1599	0.1609	0.1619	0.1629	0.1638	0.1649	0.1661	0.1673
9/16	T	0.2812	0.2812	0.2811	0.2810	0.2810	0.2808	0.2806	0.2804
	H	0.1799	0.1810	0.1821	0.1832	0.1842	0.1855	0.1868	0.1882
5/8	T	0.3125	0.3125	0.3123	0.3123	0.3122	0.3120	0.3118	0.3116
	H	0.1998	0.2012	0.2023	0.2036	0.2047	0.2061	0.2076	0.2091
11/16	T	0.3437	0.3437	0.3436	0.3435	0.3434	0.3432	0.3430	0.3427
	H	0.2198	0.2213	0.2226	0.2239	0.2252	0.2267	0.2283	0.2300
3/4	T	0.3750	0.3750	0.3748	0.3747	0.3747	0.3744	0.3742	0.3740
	H	0.2398	0.2414	0.2428	0.2443	0.2457	0.2473	0.2491	0.2509
13/16	T	0.4062	0.4062	0.4060	0.4059	0.4059	0.4056	0.4054	0.4050
	H	0.2598	0.2615	0.2631	0.2647	0.2661	0.2679	0.2699	0.2718
7/8	T	0.4375	0.4375	0.4373	0.4372	0.4371	0.4368	0.4366	0.4362
	H	0.2798	0.2816	0.2833	0.2850	0.2866	0.2885	0.2906	0.2927
15/16	T	0.4687	0.4687	0.4685	0.4684	0.4683	0.4680	0.4678	0.4674
	H	0.2998	0.3018	0.3035	0.3054	0.3071	0.3092	0.3114	0.3137
1	T	0.5000	0.5000	0.4998	0.4997	0.4996	0.4993	0.4990	0.4986
	H	0.3198	0.3219	0.3238	0.3258	0.3276	0.3298	0.3322	0.3346
1 1/8	T	0.5625	0.5625	0.5623	0.5621	0.5620	0.5617	0.5613	0.5610
	H	0.3597	0.3621	0.3642	0.3665	0.3685	0.3710	0.3737	0.3764
1 1/4	T	0.6250	0.6250	0.6247	0.6246	0.6245	0.6241	0.6237	0.6232
	H	0.3997	0.4023	0.4047	0.4072	0.4095	0.4122	0.4152	0.4182
1 8/8	T	0.6875	0.6875	0.6872	0.6870	0.6869	0.6865	0.6861	0.6856
	H	0.4397	0.4426	0.4452	0.4479	0.4504	0.4534	0.4567	0.4600
1 1/2	T	0.7500	0.7500	0.7497	0.7495	0.7494	0.7489	0.7485	0.7480
	H	0.4797	0.4828	0.4857	0.4887	0.4914	0.4947	0.4983	0.5019
1 3/4	T	0.8750	0.8750	0.8746	0.8744	0.8743	0.8737	0.8732	0.8726
	H	0.5596	0.5633	0.5666	0.5701	0.5733	0.5771	0.5813	0.5855
2	T	1.0000	1.0000	0.9996	0.9994	0.9992	0.9986	0.9980	0.9972
	H	0.6396	0.6438	0.6476	0.6516	0.6552	0.6596	0.6644	0.6692
2 1/4	T	1.1250	1.1250	1.1246	1.1242	1.1240	1.1234	1.1226	1.1220
	H	0.7195	0.7242	0.7285	0.7330	0.7371	0.7420	0.7474	0.7528
2 1/2	T	1.2500	1.2500	1.2494	1.2492	1.2490	1.2482	1.2474	1.2464
	H	0.7995	0.8047	0.8095	0.8145	0.8190	0.8245	0.8305	0.8365
3	T	1.5000	1.5000	1.4994	1.4990	1.4990	1.4978	1.4970	1.4960
	H	0.9594	0.9657	0.9714	0.9774	0.9828	0.9894	0.9966	1.0038

Chordal Thickness of Tooth when Outside Diameter is Special.—When the outside diameter is larger or smaller than standard the chordal thickness at the standard pitch diameter is found by the following formula where t_c = chordal thickness at the standard pitch diameter D; t = circular thickness at the standard pitch diameter of the enlarged pinion or reduced gear being measured.

$$t_c = t - \frac{t^3}{6 \times D^2}$$

Example 1: The outside diameter of a pinion having 10 teeth of 5 diametral pitch has been *enlarged* by 0.2746 inch. This enlargement has increased the circular tooth thickness at the standard pitch diameter (as determined by the formula previously given) to 0.3852 inch. Find the equivalent chordal thickness.

$$t_c = 0.3852 - \frac{0.385^3}{6 \times 2^2} = 0.3852 - 0.0024 = 0.3828 \text{ inch}$$

(The error introduced by rounding the circular thickness to three significant figures before cubing it only affects the fifth decimal place in the result.)

Example 2: A gear having 30 teeth is to mesh with the pinion in Example 1 and is *reduced* so that the circular tooth thickness at the standard pitch diameter is 0.2432 inch. Find the equivalent chordal thickness.

$$t_c = 0.2432 - \frac{0.243^3}{6 \times 6^2} = 0.2432 - 0.00007 = 0.2431 \text{ inch}$$

Chordal Addendum.—In measuring the chordal thickness, the vertical scale of a gear tooth caliper is set to the chordal or “corrected” addendum to locate the caliper jaws at the pitch line (see *Method of setting a gear tooth caliper* on page 2021). The simplified formula which follows may be used in determining the chordal addendum either when the addendum is standard for full-depth or stub teeth or when the addendum is either longer or shorter than standard as in case of an enlarged pinion or a gear which is to mesh with an enlarged pinion and has a reduced addendum to maintain the standard center distance. If a_c = chordal addendum; a = addendum; and t = circular thickness of tooth at pitch diameter D; then,

$$a_c = a + \frac{t^2}{4D}$$

Example 1: The outside diameter of an 8 diametral pitch 14-tooth pinion with 20-degree full-depth teeth is to be increased by using an enlarged addendum of 1.234 ÷ 8 = 0.1542 inch (see Table 1 on page 2019). The basic tooth thickness of the enlarged pinion is 1.741 ÷ 8 = 0.2176 inch. What is the chordal addendum?

$$\text{Chordal addendum} = 0.1542 + \frac{0.2176^2}{4 \times (14 \div 8)} = 0.1610 \text{ inch}$$

Example 2: The outside diameter of a 14½-degree pinion having 12 teeth of 2 diametral pitch is to be enlarged 0.624 inch to avoid undercut (see Table 2 on page 2019), thus increasing the addendum from 0.5000 to 0.8120 inch and the arc thickness at the pitch line from 0.7854 to 0.9467 inch. Then,

$$\text{Chordal addendum of pinion} = 0.8120 + \frac{0.9467^2}{4 \times (12 \div 2)} = 0.8493 \text{ inch}$$

Table 1. Addendums and Tooth Thicknesses for Coarse-Pitch Long-Addendum Pinions and their Mating Short-Addendum Gears—20- and 25-degree Pressure Angles *ANSI B6.1-1968 (R1974)*

Number of Teeth in Pinion	Addendum		Basic Tooth Thickness		Number of Teeth in Gear
	Pinion	Gear	Pinion	Gear	
N_P	a_P	a_G	t_P	t_G	N_G (min)
20-Degree Involute Full Depth Tooth Form (Less than 20 Diametral Pitch)					
10	1.468	.532	1.912	1.230	25
11	1.409	.591	1.868	1.273	24
12	1.351	.649	1.826	1.315	23
13	1.292	.708	1.783	1.358	22
14	1.234	.766	1.741	1.400	21
15	1.175	.825	1.698	1.443	20
16	1.117	.883	1.656	1.486	19
17	1.058	.942	1.613	1.529	18
25-Degree Involute Full Depth Tooth Form (Less than 20 Diametral Pitch)					
10	1.184	.816	1.742	1.399	15
11	1.095	.905	1.659	1.482	14

All values are for 1 diametral pitch. For any other sizes of teeth all linear dimensions should be divided by the diametral pitch. Basic tooth thicknesses do not include an allowance for backlash.

Table 2. Enlarged Pinion and Reduced Gear Dimensions to Avoid Interference Coarse Pitch 14½-degree Involute Full Depth Teeth

Number of Pinion Teeth	Changes in Pinion and Gear Diameters	Circular Tooth Thickness		Min. No. of Teeth in Mating Gear	
		Pinion	Mating Gear	To Avoid Undercut	For Full Involute Action
10	1.3731	1.9259	1.2157	54	27
11	1.3104	1.9097	1.2319	53	27
12	1.2477	1.8935	1.2481	52	28
13	1.1850	1.8773	1.2643	51	28
14	1.1223	1.8611	1.2805	50	28
15	1.0597	1.8449	1.2967	49	28
16	0.9970	1.8286	1.3130	48	28
17	0.9343	1.8124	1.3292	47	28
18	0.8716	1.7962	1.3454	46	28
19	0.8089	1.7800	1.3616	45	28
20	0.7462	1.7638	1.3778	44	28
21	0.6835	1.7476	1.3940	43	28
22	0.6208	1.7314	1.4102	42	27
23	0.5581	1.7151	1.4265	41	27
24	0.4954	1.6989	1.4427	40	27
25	0.4328	1.6827	1.4589	39	26
26	0.3701	1.6665	1.4751	38	26
27	0.3074	1.6503	1.4913	37	26
28	0.2447	1.6341	1.5075	36	25
29	0.1820	1.6179	1.5237	35	25
30	0.1193	1.6017	1.5399	34	24
31	0.0566	1.5854	1.5562	33	24

All dimensions are given in inches and are for 1 diametral pitch. For other pitches divide tabular values by desired diametral pitch.

Add to the standard outside diameter of the pinion the amount given in the second column of the table divided by the desired diametral pitch, and (to maintain standard center distance) subtract the same amount from the outside diameter of the mating gear. Long addendum pinions will mesh with standard gears, but the center distance will be greater than standard.

Example 3: The outside diameter of the mating gear for the pinion in Example 3 is to be reduced 0.624 inch. The gear has 60 teeth and the addendum is reduced from 0.5000 to 0.1881 inch (to maintain the standard center distance), thus reducing the arc thickness to 0.6240 inch. Then,

$$\text{Chordal addendum of gear} = 0.1881 + \frac{0.6240^2}{4 \times (60 \div 2)} = 0.1913 \text{ inch}$$

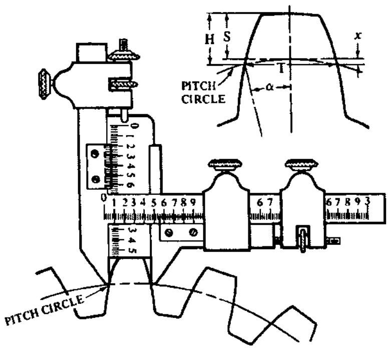
When a gear addendum is reduced as much as the mating pinion addendum is enlarged, the minimum number of gear teeth required to prevent undercutting depends upon the enlargement of the mating pinion. To illustrate, if a 14½-degree pinion with 13 teeth is enlarged 1.185 inches, then the reduced mating gear should have a minimum of 51 teeth to avoid undercut (see Table 2 on page 2019).

Tables for Chordal Thicknesses and Chordal Addenda of Milled, Full-depth Teeth.—Two convenient tables for checking gears with milled, full-depth teeth are given on pages 2016 and 2017. The first shows chordal thicknesses and chordal addenda for the lowest number of teeth cut by gear cutters Nos. 1 through 8, and for the commonly used diametral pitches. The second gives similar data for commonly used circular pitches. In each case the data shown are accurate for the number of gear teeth indicated, but are approximate for other numbers of teeth within the range of the cutter under which they appear in the table. For the higher diametral pitches and lower circular pitches, the error introduced by using the data for any tooth number within the range of the cuuter under which it appears is comparatively small. The chordal thicknesses and chordal addenda for gear cutters Nos. 1 through 8 of the more commonly used diametral and circular pitches can be obtained from the table and formulas on pages 2016 and 2017.

Caliper Measurement of Gear Tooth.—In cutting gear teeth, the general practice is to adjust the cutter or hob until it grazes the outside diameter of the blank; the cutter is then sunk to the total depth of the tooth space plus whatever slight additional amount may be required to provide the necessary play or backlash between the teeth. (For recommendations concerning backlash and excess depth of cut required, see *Backlash* starting on page 2036.) If the outside diameter of the gear blank is correct, the tooth thickness should also be correct after the cutter has been sunk to the depth required for a given pitch and backlash. However, it is advisable to check the tooth thickness by measuring it, and the vernier gear-tooth caliper (see following illustration) is commonly used in measuring the thickness.

The vertical scale of this caliper is set so that when it rests upon the top of the tooth as shown, the lower ends of the caliper jaws will be at the height of the pitch circle; the horizontal scale then shows the chordal thickness of the tooth at this point. If the gear is being cut on a milling machine or with the type of gear-cutting machine employing a formed milling cutter, the tooth thickness is checked by first taking a trial cut for a short distance at one side of the blank; then the gear blank is indexed for the next space and another cut is taken far enough to mill the full outline of the tooth. The tooth thickness is then measured.

Before the gear-tooth caliper can be used, it is necessary to determine the correct chordal thickness and also the chordal addendum (or “corrected addendum” as it is sometimes called). The vertical scale is set to the chordal addendum, thus locating the ends of the jaws at the height of the pitch circle. The rules or formulas to use in determining the chordal thickness and chordal addendum will depend upon the outside diameter of the gear; for example, if the outside diameter of a small pinion is enlarged to avoid undercut and improve the tooth action, this must be taken into account in figuring the chordal thickness and chordal addendum as shown by the accompanying rules. The detail of a gear tooth included with the gear-tooth caliper illustration, represents the chordal thickness T, the addendum S, and the chordal addendum H. For the caliper measurements over two or more teeth see *Checking Spur Gear Size by Chordal Measurement Over Two or More Teeth* starting on page 2109.



Method of setting a gear tooth caliper

Selection of Involute Gear Milling Cutter for a Given Diametral Pitch and Number of Teeth.—When gear teeth are cut by using formed milling cutters, the cutter must be selected to suit both the pitch and the number of teeth, because the shapes of the tooth spaces vary according to the number of teeth. For instance, the tooth spaces of a small pinion are not of the same shape as the spaces of a large gear of equal pitch. Theoretically, there should be a different formed cutter for every tooth number, but such refinement is unnecessary in practice. The involute formed cutters commonly used are made in series of eight cutters for each diametral pitch (see *Series of Involute, Finishing Gear Milling Cutters for Each Pitch*). The shape of each cutter in this series is correct for a certain number of teeth only, but it can be used for other numbers within the limits given. For instance, a No. 6 cutter may be used for gears having from 17 to 20 teeth, but the tooth outline is correct only for 17 teeth or the lowest number in the range, which is also true of the other cutters listed. When this cutter is used for a gear having, say, 19 teeth, too much material is removed from the upper surfaces of the teeth, although the gear meets ordinary requirements. When greater accuracy of tooth shape is desired to ensure smoother or quieter operation, an intermediate series of cutters having half-numbers may be used provided the number of gear teeth is between the number listed for the regular cutters (see *Series of Involute, Finishing Gear Milling Cutters for Each Pitch*).

Involute gear milling cutters are designed to cut a composite tooth form, the center portion being a true involute while the top and bottom portions are cycloidal. This composite form is necessary to prevent tooth interference when milled mating gears are meshed with each other. Because of their composite form, milled gears will not mate satisfactorily enough for high grade work with those of generated, full-involute form. Composite form hobs are available, however, which will produce generated gears that mesh with those cut by gear milling cutters.

Metric Module Gear Cutters: The accompanying table for selecting the cutter number to be used to cut a given number of teeth may be used also to select metric module gear cutters except that the numbers are designated in reverse order. For example, cutter No. 1, in the metric module system, is used for 12–13 teeth, cutter No. 2 for 14–16 teeth, etc.

Circular Pitch in Gears—Pitch Diameters, Outside Diameters, and Root Diameters

For any particular circular pitch and number of teeth, use the table as shown in the example to find the pitch diameter, outside diameter, and root diameter. *Example:* Pitch diameter for 57 teeth of 6-inch circular pitch = 10 × pitch diameter given under factor for 5 teeth plus pitch diameter given under factor for 7 teeth. (10 × 9.5493) + 13.3690 = 108.862 inches.

Outside diameter of gear equals pitch diameter plus outside diameter factor from next-to-last column in table = 108.862 + 3.8197 = 112.682 inches.

Root diameter of gear equals pitch diameter minus root diameter factor from last column in table = 108.862 – 4.4194 = 104.443 inches.

Circular Pitch in Inches	Factor for Number of Teeth									Outside Dia. Factor	Root Diameter Factor
	1	2	3	4	5	6	7	8	9		
	Pitch Diameter Corresponding to Factor for Number of Teeth										
6	1.9099	3.8197	5.7296	7.6394	9.5493	11.4591	13.3690	15.2788	17.1887	3.8197	4.4194
5½	1.7507	3.5014	5.2521	7.0028	8.7535	10.5042	12.2549	14.0056	15.7563	3.5014	4.0511
5	1.5915	3.1831	4.7746	6.3662	7.9577	9.5493	11.1408	12.7324	14.3239	3.1831	3.6828
4½	1.4324	2.8648	4.2972	5.7296	7.1620	8.5943	10.0267	11.4591	12.8915	2.8648	3.3146
4	1.2732	2.5465	3.8197	5.0929	6.3662	7.6394	8.9127	10.1859	11.4591	2.5465	2.9463
3½	1.1141	2.2282	3.3422	4.4563	5.5704	6.6845	7.7986	8.9127	10.0267	2.2282	2.5780
3	0.9549	1.9099	2.8648	3.8197	4.7746	5.7296	6.6845	7.6394	8.5943	1.9099	2.2097
2½	0.7958	1.5915	2.3873	3.1831	3.9789	4.7746	5.5704	6.3662	7.1620	1.5915	1.8414
2	0.6366	1.2732	1.9099	2.5465	3.1831	3.8197	4.4563	5.0929	5.7296	1.2732	1.4731
1⅞	0.5968	1.1937	1.7905	2.3873	2.9841	3.5810	4.1778	4.7746	5.3715	1.1937	1.3811
1¾	0.5570	1.1141	1.6711	2.2282	2.7852	3.3422	3.8993	4.4563	5.0134	1.1141	1.2890
1⅝	0.5173	1.0345	1.5518	2.0690	2.5863	3.1035	3.6208	4.1380	4.6553	1.0345	1.1969
1½	0.4775	0.9549	1.4324	1.9099	2.3873	2.8648	3.3422	3.8197	4.2972	0.9549	1.1049
1 7/16	0.4576	0.9151	1.3727	1.8303	2.2878	2.7454	3.2030	3.6606	4.1181	0.9151	1.0588
1⅜	0.4377	0.8754	1.3130	1.7507	2.1884	2.6261	3.0637	3.5014	3.9391	0.8754	1.0128
1 5/16	0.4178	0.8356	1.2533	1.6711	2.0889	2.5067	2.9245	3.3422	3.7600	0.8356	0.9667
1¼	0.3979	0.7958	1.1937	1.5915	1.9894	2.3873	2.7852	3.1831	3.5810	0.7958	0.9207
1 3/16	0.3780	0.7560	1.1340	1.5120	1.8900	2.2680	2.6459	3.0239	3.4019	0.7560	0.8747
1⅛	0.3581	0.7162	1.0743	1.4324	1.7905	2.1486	2.5067	2.8648	3.2229	0.7162	0.8286
1 1/16	0.3382	0.6764	1.0146	1.3528	1.6910	2.0292	2.3674	2.7056	3.0438	0.6764	0.7826
1	0.3183	0.6366	0.9549	1.2732	1.5915	1.9099	2.2282	2.5465	2.8648	0.6366	0.7366
15/16	0.2984	0.5968	0.8952	1.1937	1.4921	1.7905	2.0889	2.3873	2.6857	0.5968	0.6905
⅞	0.2785	0.5570	0.8356	1.1141	1.3926	1.6711	1.9496	2.2282	2.5067	0.5570	O.6445
13/16	0.2586	0.5173	0.7759	1.0345	1.2931	1.5518	1.8104	2.0690	2.3276	0.5173	0.5985
¾	0.2387	0.4475	0.7162	0.9549	1.1937	1.4324	1.6711	1.9099	2.1486	0.4775	0.5524
11/16	0.2188	0.4377	0.6565	0.8754	1.0942	1.3130	1.5319	1.7507	1.9695	0.4377	0.5064
⅔	0.2122	0.4244	0.6366	0.8488	1.0610	1.2732	1.4854	1.6977	1.9099	0.4244	0.4910
⅝	0.1989	0.3979	0.5968	0.7958	0.9947	1.1937	1.3926	1.5915	1.7905	0.3979	0.4604
9/16	0.1790	0.3581	0.5371	0.7162	0.8952	1.0743	1.2533	1.4324	1.6114	0.3581	0.4143
½	0.1592	0.3183	0.4775	0.6366	0.7958	0.9549	1.1141	1.2732	1.4324	0.3183	0.3683
7/16	0.1393	0.2785	0.4178	0.5570	0.6963	0.8356	0.9748	1.1141	1.2533	0.2785	0.3222
⅜	0.1194	0.2387	0.3581	0.4775	0.5968	0.7162	0.8356	0.9549	1.0743	0.2387	0.2762
⅓	0.1061	0.2122	0.3183	0.4244	0.5305	0.6366	0.7427	0.8488	0.9549	0.2122	0.2455
5/16	0.0995	0.1989	0.2984	0.3979	0.4974	0.5968	0.6963	0.7958	0.8952	0.1989	0.2302
¼	0.0796	0.1592	0.2387	0.3183	0.3979	0.4775	0.5570	0.6366	0.7162	0.1592	0.1841
3/16	0.0597	0.1194	0.1790	0.2387	0.2984	0.3581	0.4178	0.4775	0.5371	0.1194	0.1381
⅛	0.0398	0.0796	0.1194	0.1592	0.1989	0.2387	0.2785	0.3183	0.3581	0.0796	0.0921
1/16	0.0199	0.0398	0.0597	0.0796	0.0995	0.1194	0.1393	0.1592	0.1790	0.0398	0.0460

Increasing Pinion Diameter to Avoid Undercut or Interference.—On coarse-pitch pinions with small numbers of teeth (10 to 17 for 20-degree and 10 and 11 for 25-degree pressure angle involute tooth forms) undercutting of the tooth profile or fillet interference with the tip of the mating gear can be avoided by making certain changes from the standard tooth proportions that are specified in Table on page 2007. These changes consist essentially in increasing the addendum and hence the outside diameter of the pinion and decreasing the addendum and hence the outside diameter of the mating gear. These changes in outside diameters of pinion and gear do not change the velocity ratio or the procedures in cutting the teeth on a hobbing machine or generating type of shaper or planer.

Data in Table 1 on page 2019 are taken from ANSI Standard B6.1-1968, reaffirmed 1974, and show for 20-degree and 25-degree full-depth standard tooth forms, respectively, the addendums and tooth thicknesses for long addendum pinions and their mating short addendum gears when the number of teeth in the pinion is as given. Similar data for former standard 14½-degree full-depth teeth (20 diametral pitch and coarser) are given in Table 2 on page 2019.

Example: A 14-tooth, 20-degree pressure angle pinion of 6 diametral pitch is to be enlarged. What will be the outside diameters of the pinion and a 60-tooth mating gear? If the mating gear is to have the minimum number of teeth to avoid undercut, what will be its outside diameter?

$$D_o(\text{ pinion}) = \frac{N_P}{P} + 2a = \frac{14}{6} + 2\left(\frac{1.234}{6}\right) = 2.745 \text{ inches}$$

$$D_o(\text{ gear}) = \frac{N_G}{P} + 2a = \frac{60}{6} + 2\left(\frac{0.766}{6}\right) = 10.255 \text{ inches}$$

For a mating gear with minimum number of teeth to avoid undercut:

$$D_o(\text{ gear}) = \frac{N_G}{P} + 2a = \frac{21}{6} + 2\left(\frac{0.766}{6}\right) = 3.755 \text{ inches}$$

Series of Involute, Finishing Gear Milling Cutters for Each Pitch

Number of Cutter	Will cut Gears from	Number of Cutter	Will cut Gears from
1	135 teeth to a rack	5	21 to 25 teeth
2	55 to 134 teeth	6	17 to 20 teeth
3	35 to 54 teeth	7	14 to 16 teeth
4	26 to 34 teeth	8	12 to 13 teeth
The regular cutters listed above are used ordinarily. The cutters listed below (an intermediate series having half numbers) may be used when greater accuracy of tooth shape is essential in cases where the number of teeth is between the numbers for which the regular cutters are intended.			
Number of Cutter	Will cut Gears from	Number of Cutter	Will cut Gears from
1½	80 to 134 teeth	5½	19 to 20 teeth
2½	42 to 54 teeth	6½	15 to 16 teeth
3½	30 to 34 teeth	7½	13 teeth
4½	23 to 25 teeth	…	…

Roughing cutters are made with No. 1 form only. Dimensions of roughing and finishing cutters are given on page 791. Dimensions of cutters for bevel gears are given on page 792.

Enlarged Fine-Pitch Pinions: American Standard ANSI B6.7–1977, Information Sheet A provides a different system for 20-degree pressure angle pinion enlargement than is used for coarse-pitch gears. Pinions with 11 through 23 teeth (9 through 14 teeth for 25-degree pressure angle) are enlarged so that a standard tooth thickness rack with addendum $1.05/P$ will start contact 5° of roll above the base circle radius. The use of $1.05/P$ for the addendum allows for center distance variation and eccentricity of the mating gear outside diameter; the 5° roll angle avoids the fabrication of the involute in the troublesome area near the base circle.

Pinions with less than 11 teeth (9 teeth for 25-degree pressure angle) are enlarged to the extent that the highest point of undercut coincides with the start of contact with the standard rack described previously. The height of undercut considered is that produced by a sharp-cornered 120 pitch hob. Pinions with less than 13 teeth (11 teeth for 25-degree pressure angle) are truncated to provide a top land of $0.275/P$. Data for enlarged pinions may be found in Tables 3a, 3b, 3c, and 3d.

Table 3a. Increase in Dedendum, Δ for 20 -, and 25 -Degree Pressure Angle Fine-Pitch Enlarged Pinions and Reduced Gears *ANSI B6. 7-1977*

Diametral Pitch, *P*	Δ	Diametral Pitch, *P*	Δ	Diametral Pitch, *P*	Δ	Diametral Pitch, *P*	Δ	Diametral Pitch, *P*	Δ
20	0.0000	32	0.0007	48	0.0012	72	0.0015	96	0.0016
24	0.0004	40	0.0010	64	0.0015	80	0.0015	120	0.0017

Δ = increase in standard dedendum to provide increased clearance. See footnote to Table 3d.

Table 3b. Dimensions Required when Using Enlarged, Fine-pitch, 14½-Degree Pressure Angle Pinions *ANSI B6.7-1977*, Information Sheet B

Enlarged Pinion			Standard Center-distance System (Long and Short Addendum)				Enlarged Center-distance System		
			Reduced Mating Gear				Increase over St'd. Center Distance		
No. of Teeth *n*	Outside Diameter	Cir. Tooth Thickness at Standard Pitch Dia.	Decrease in Standard Outside Dia.[b]	Cir. Tooth Thickness at Standard Pitch Dia.	Recommended Minimum No. of Teeth *N*	Contact Ratio, *n* Mating with *N*	Enlarged Pinion Mating with St'd. Gear	Two Equal Enlarged Mating Pinions[a]	Contact Ratio of Two Equal Enlarged Mating Pinions
10	13.3731	1.9259	1.3731	1.2157	54	1.831	0.6866	1.3732	1.053
11	14.3104	1.9097	1.3104	1.2319	53	1.847	0.6552	1.3104	1.088
12	15.2477	1.8935	1.2477	1.2481	52	1.860	0.6239	1.2477	1.121
13	16.1850	1.8773	1.1850	1.2643	51	1.873	0.5925	1.1850	1.154
14	17.1223	1.8611	1.1223	1.2805	50	1.885	0.5612	1.2223	1.186
15	18.0597	1.8448	1.0597	1.2967	49	1.896	0.5299	1.0597	1.217
16	18.9970	1.8286	0.9970	1.3130	48	1.906	0.4985	0.9970	1.248
17	19.9343	1.8124	0.9343	1.3292	47	1.914	0.4672	0.9343	1.278
18	20.8716	1.7962	0.8716	1.3454	46	1.922	0.4358	0.8716	1.307
19	21.8089	1.7800	0.8089	1.3616	45	1.929	0.4045	0.8089	1.336
20	22.7462	1.7638	0.7462	1.3778	44	1.936	0.3731	0.7462	1.364
21	23.6835	1.7476	0.6835	1.3940	43	1.942	0.3418	0.6835	1.392
22	24.6208	1.7314	0.6208	1.4102	42	1.948	0.3104	0.6208	1.419
23	25.5581	1.7151	0.5581	1.4265	41	1.952	0.2791	0.5581	1.446
24	26.4954	1.6989	0.4954	1.4427	40	1.956	0.2477	0.4954	1.472
25	27.4328	1.6827	0.4328	1.4589	39	1.960	0.2164	0.4328	1.498
26	28.3701	1.6665	0.3701	1.4751	38	1.963	0.1851	0.3701	1.524
27	29.3074	1.6503	0.3074	1.4913	37	1.965	0.1537	0.3074	1.549
28	30.2447	1.6341	0.2448	1.5075	36	1.967	0.1224	0.2448	1.573
29	31.1820	1.6179	0.1820	1.5237	35	1.969	0.0910	0.1820	1.598
30	32.1193	1.6017	0.1193	1.5399	34	1.970	0.0597	0.1193	1.622
31	33.0566	1.5854	0.0566	1.5562	33	1.971	0.0283	0.0566	1.646

[a] If enlarged mating pinions are of unequal size, the center distance is increased by an amount equal to one-half the sum of their increase over standard outside diameters. Data in this column are not given in the standard.

[b] To maintain standard center distance when using an enlarged pinion, the mating gear diameter must be decreased by the amount of the pinion enlargement.

All dimensions are given in inches and are for 1 diametral pitch. For other pitches divide tabulated dimensions by the diametrical pitch.

Table 3c. Tooth Proportions Recommended for Enlarging Fine-Pitch Pinions of 20-Degree Pressure Angle—20 Diametral Pitch and Finer

ANSI B6.7-1977

Enlarged Pinion Dimensions					Enlarged C.D. System Pinion Mating with Standard Gear		Standard Center Distance (Long and Short Addendums) Reduced Gear Dimensions				
Number of Teeth,[a] n	Outside Diameter, D_{oP}	Addendum, a_P	Basic Tooth Thickness, t_P	Dedendum Based on 20 Pitch,[b] b_P	Contact Ratio Two Equal Pinions	Contact Ratio with a 24-Tooth Gear	Addendum, a_G	Basic Tooth Thickness, t_G	Dedendum Based on 20 Pitch,[b] b_G	Recommended Minimum No. of Teeth, N	Contact Ratio n Mating with N
7	10.0102	1.5051	2.14114	0.4565	0.697	1.003	0.2165	1.00045	2.0235	42	1.079
8	11.0250	1.5125	2.09854	0.5150	0.792	1.075	0.2750	1.04305	1.9650	40	1.162
9	12.0305	1.5152	2.05594	0.5735	0.893	1.152	0.3335	1.08565	1.9065	39	1.251
10	13.0279	1.5140	2.01355	0.6321	0.982	1.211	0.3921	1.12824	1.8479	38	1.312
11	14.0304	1.5152	1.97937	0.6787	1.068	1.268	0.4387	1.16222	1.8013	37	1.371
12	15.0296	1.5148	1.94703	0.7232	1.151	1.322	0.4832	1.19456	1.7568	36	1.427
13	15.9448	1.4724	1.91469	0.7676	1.193	1.353	0.5276	1.22690	1.7124	35	1.457
14	16.8560	1.4280	1.88235	0.8120	1.232	1.381	0.5720	1.25924	1.6680	34	1.483
15	17.7671	1.3836	1.85001	0.8564	1.270	1.408	0.6164	1.29158	1.6236	33	1.507
16	18.6782	1.3391	1.81766	0.9009	1.323	1.434	0.6609	1.32393	1.5791	32	1.528
17	19.5894	1.2947	1.78532	0.9453	1.347	1.458	0.7053	1.35627	1.5347	31	1.546
18	20.5006	1.2503	1.75298	0.9897	1.385	1.482	0.7497	1.38861	1.4903	30	1.561
19	21.4116	1.2058	1.72064	1.0342	1.423	1.505	0.7942	1.42095	1.4458	29	1.574
20	22.3228	1.1614	1.68839	1.0786	1.461	1.527	0.8386	1.45320	1.4014	28	1.584
21	23.2340	1.1170	1.65595	1.1230	1.498	1.548	0.8830	1.48564	1.3570	27	1.592
22	24.1450	1.0725	1.62361	1.1675	1.536	1.568	0.9275	1.51798	1.3125	26	1.598
23	25.0561	1.0281	1.59127	1.2119	1.574	1.588	0.9719	1.55032	1.2681	25	1.601
24	26.0000	1.0000	1.57080	1.2400	1.602	1.602	1.0000	1.57080	1.2400	24	1.602

[a] Caution should be exercised in the use of pinions above the horizontal lines. They should be checked for suitability, particularly in the areas of contact ratio (less than 1.2 is not recommended), center distance, clearance, and tooth strength.

[b] The actual dedendum is calculated by dividing the values in this column by the desired diametral pitch and then adding to the result an amount Δ found in Table 3a. As an example, a 20-degree pressure angle 7-tooth pinion meshing with a 42-tooth gear would have, for 24 diametral pitch, a dedendum of $0.4565 \div 24 + 0.0004 = 0.0194$. The 42-tooth gear would have a dedendum of $2.0235 \div 24 + 0.004 = 0.0847$ inch.

All dimensions are given in inches.

Table 3d. Tooth Proportions Recommended for Enlarging Fine-Pitch Pinions of 20-Degree Pressure Angle—20 Diametral Pitch and Finer

ANSI B6.7-1977

Enlarged Pinion Dimensions					Enlarged C.D. System Pinion Mating with Standard Gear		Standard Center Distance (Long and Short Addendums) Reduced Gear Dimensions				
Number of Teeth,[a] n	Outside Diameter, D_{oP}	Addendum, a_P	Basic Tooth Thickness, t_P	Dedendum Based on 20 Pitch, Dedendum Based on 20 Pitch, [b] b_P	Contact Ratio Two Equal Pinions	Contact Ratio with a 15-Tooth Gear	Addendum, a_G	Basic Tooth Thickness, t_G	Dedendum Based on 20 Pitch, [b] b_G	Recommended Minimum No. of Teeth, N	Contact Ratio n Mating with N
6	8.7645	1.3822	2.18362	0.5829	0.696	0.954	0.3429	0.95797	1.8971	24	1.030
7	9.7253	1.3626	2.10029	0.6722	0.800	1.026	0.4322	1.04130	1.8078	23	1.108
8	10.6735	1.3368	2.01701	0.7616	0.904	1.094	0.5216	1.12459	1.7184	22	1.177
9	11.6203	1.3102	1.94110	0.8427	1.003	1.156	0.6029	1.20048	1.6371	20	1.234
10	12.5691	1.2846	1.87345	0.9155	1.095	1.211	0.6755	1.26814	1.5645	19	1.282
11	13.5039	1.2520	1.80579	0.9880	1.183	1.261	0.7480	1.33581	1.4920	18	1.322
12	14.3588	1.1794	1.73813	1.0606	1.231	1.290	0.8206	1.40346	1.4194	17	1.337
13	15.2138	1.1069	1.67047	1.1331	1.279	1.317	0.8931	1.47112	1.3469	16	1.347
14	16.0686	1.0343	1.60281	1.2057	1.328	1.343	0.9657	1.53878	1.2743	15	1.352
15	17.0000	1.0000	1.57030	1.2400	1.358	1.358	1.0000	1.57080	1.2400	15	1.358

[a] Caution should be exercised in the use of pinions above the horizontal lines. They should be checked for suitability, particularly in the areas of contact ratio (less than 1.2 is not recommended), center distance, clearance, and tooth strength.

[b] The actual dedendum is calculated by dividing the values in this column by the desired diametral pitch and then adding to the result an amount Δ found in Table 3a. As an example, a 20-degree pressure angle 7-tooth pinion meshing with a 42-tooth gear would have, for 24 diametral pitch, a dedendum of $0.4565 \div 24 + 0.0004 = 0.0194$. The 42-tooth gear would have a dedendum of $2.0235 \div 24 + 0.004 = 0.0847$ inch.

All dimensions are given in inches.

All values are for 1 diametral pitch. For any other sizes of teeth, all linear dimensions should be divided by the diametral pitch.

Note: The tables in the ANSI B6.7-1977 standard also specify Form Diameter, Roll Angle to Form Diameter, and Top Land. These are not shown here. The top land is in no case less than $0.275/P$. The form diameters and the roll angles to form diameter shown in the Standard are the values which should be met with a standard hob when generating the tooth thicknesses shown in the tables. These form diameters provides more than enough length of involute profile for any mating gear smaller than a rack. However,since these form diameters are based on gear tooth generation using standard hobs, they should impose little or no hardship on manufacture except in cases of the most critical quality levels. In such cases, form diameter specifications and master gear design should be based upon actual mating conditions.

Minimum Number of Teeth to Avoid Undercutting by Hob.—The data in the above tables give tooth proportions for low numbers of teeth to avoid interference between the gear tooth tip and the pinion tooth flank. Consideration must also be given to possible undercutting of the pinion tooth flank by the hob used to cut the pinion. The minimum number of teeth N_{min} of standard proportion that may be cut without undercut is:

$N_{min} = 2P \csc^2 \phi \, [a_H - r_t(1 - \sin\phi)]$ where: a_H = cutter addendum; r_t = radius at cutter tip or corners; ϕ = cutter pressure angle; and P = diametral pitch.

Gear to Mesh with Enlarged Pinion.—Data in the fifth column of Table 2 show minimum number of teeth in a mating gear which can be cut with hob or rack type cutter without undercut, when outside diameter of gear has been reduced an amount equal to the pinion enlargement to retain the standard center distance. To calculate N for the gear, insert addendum a of enlarged mating pinion in the formula $N = 2a \times \csc^2\phi$.

Example: A gear is to mesh with a 24-tooth pinion of 1 diametral pitch which has been enlarged 0.4954 inch, as shown by the table. The pressure angle is 14½ degrees. Find minimum number of teeth N for reduced gear.

$$\text{Pinion addendum} = 1 + (0.4954 \div 2) = 1.2477$$

$$\text{Hence,} \quad N = 2 \times 1.2477 \times 15.95 = 39.8 \text{ (use 40)}$$

In the case of fine pitch gears with reduced outside diameters, the recommended minimum numbers of teeth given in Tables 3b, 3c, and 3d, are somewhat more than the minimum numbers required to prevent undercutting and are based upon studies made by the *American Gear Manufacturers Association*.

Standard Center-distance System for Enlarged Pinions.—In this system, sometimes referred to as “long and short addendums,” the center distance is made standard for the numbers of teeth in pinion and gear. The outside diameter of the gear is decreased by the same amount that the outside of the pinion is enlarged.

The advantages of this system are: 1) No change in center distance or ratio is required; 2) The operating pressure angle remains standard; and 3) A slightly greater contact ratio is obtained than when the center distance is increased.

The disadvantages are 1) The gears as well as the pinion must be changed from standard dimensions; 2) Pinions having fewer than the minimum number of teeth to avoid undercut cannot be satisfactorily meshed together; and 3) In most cases where gear trains include idler gears, the standard center-distance system cannot be used.

Enlarged Center-distance System for Enlarged Pinions.—If an enlarged pinion is meshed with another enlarged pinion or with a gear of standard outside diameter, the center distance must be increased. For fine-pitch gears, it is usually satisfactory to increase the center distance by an amount equal to one-half of the enlargements (see eighth column of Table 3b). This is an approximation as theoretically there is a slight increase in backlash.

The advantages of this system are: 1) Only the pinions need be changed from the standard dimensions; 2) Pinions having fewer than 18 teeth may engage other pinions in this range; 3) The pinion tooth, which is the weaker member, is made stronger by the enlargement; and 4) The tooth contact stress, which controls gear durability, is lowered by being moved away from the pinion base circle.

The disadvantages are: 1) Center distances must be enlarged over the standard; 2) The operating pressure angle increases slightly with different combinations of pinions and gears, which is usually not important; and 3) The contact ratio is slightly smaller than that obtained with the standard center-distance system.

This consideration is of minor importance as in the worst case the loss is approximately only 6 per cent.

Enlarged Pinions Meshing without Backlash: When two enlarged pinions are to mesh without backlash, their center distance will be greater than the standard and less than that for the enlarged center-distance system. This center distance may be calculated by the formulas given in the following section.

Center Distance at Which Modified Mating Spur Gears Will Mesh with No Backlash.—When the tooth thickness of one or both of a pair of mating spur gears has been increased or decreased from the standard value ($\pi \div 2P$), the center distance at which they will mesh tightly (without backlash) may be calculated from the following formulas:

$$\operatorname{inv}\phi_1 = \operatorname{inv}\phi_1 + \frac{P(t+T)-\pi}{n+N}$$

$$C = \frac{n+N}{2P}$$

$$C_1 = \frac{\cos\phi}{\cos\phi_1} \times C$$

In these formulas, P = diametral pitch; n = number of teeth in pinion; N = number of teeth in gear; t and T are the actual tooth thicknesses of the pinion and gear, respectively, on their standard pitch circles; inv ϕ = involute function of standard pressure angle of gears; C = standard center distance for the gears; C_1 = center distance at which the gears mesh without backlash; and inv ϕ_1 = involute function of operating pressure angle when gears are meshed tightly at center distance C_1.

Example: Calculate the center distance for no backlash when an enlarged 10-tooth pinion of 100 diametral pitch and 20-degree pressure angle is meshed with a standard 30-tooth gear, the circular tooth thickness of the pinion and gear, respectively, being 0.01873 and 0.015708 inch.

$$\operatorname{inv}\phi_1 = \operatorname{inv}20° + \frac{100(0.01873 + 0.015708) - \pi}{(10+30)}$$

From the table of involute functions, inv 20-degrees = 0.014904. Therefore,

$$\operatorname{inv}\phi_1 = 0.014904 + \frac{0.34438 - 0.31416}{4} = 0.022459$$

$$\phi_1 = 22°49' \text{ from page 99}$$

$$C = \frac{n+N}{2P} = \frac{10+30}{2 \times 100} = 0.2000 \text{ inch}$$

$$C_1 = \frac{\cos 20°}{\cos 22°49'} \times 0.2000 = \frac{0.93969}{0.92175} \times 0.2000 = 0.2039 \text{ inch}$$

Contact Diameter.—For two meshing gears it is important to know the contact diameter of each. A first gear with number of teeth, n, and outside diameter, d_0, meshes at a standard center distance with a second gear with number of teeth, N, and outside diameter, D_0; both gears have a diametral pitch, P, and pressure angle, ϕ, a, A, b, and B are unnamed angles used only in the calculations. The contact diameter, d_c, is found by a three-step calculation that can be done by hand using a trigonometric table and a logarithmic table or a desk calculator. Slide rule calculation is not recommended because it is not accurate enough to give good results. The three-step formulas to find the contact diameter, d_c, of the first gear are:

$$\cos A = \frac{N\cos\phi}{D_o \times P} \qquad (1)$$

$$\tan b = \tan\phi - \frac{N}{n}(\tan A - \tan\phi) \qquad (2)$$

$$d_c = \frac{n\cos\phi}{P\cos b} \tag{3}$$

Similarly the three-step formulas to find the contact diameter, D_c, of the second gear are:

$$\cos a = \frac{n\cos\phi}{d_o \times P} \tag{4}$$

$$\tan B = \tan\phi - \frac{n}{N}(\tan a - \tan\phi) \tag{5}$$

$$D_c = \frac{N\cos\phi}{P\cos B} \tag{6}$$

Contact Ratio.—The contact ratio of a pair of mating spur gears must be well over 1.0 to assure a smooth transfer of load from one pair of teeth to the next pair as the two gears rotate under load. Because of a reduction in contact ratio due to such factors as tooth deflection, tooth spacing errors, tooth tip breakage, and outside diameter and center distance tolerances, the contact ratio of gears for power transmission as a general rule should not be less than about 1.4. A contact ratio of as low as 1.15 may be used in extreme cases, provided the tolerance effects mentioned above are accounted for in the calculation. The formula for determining the contact ratio, m_f, using the nomenclature in the previous section is:

$$m_f = \frac{N}{6.28318}(\tan A - \tan B) \tag{7a}$$

or

$$m_f = \frac{N}{6.28318}(\tan a - \tan b) \tag{7b}$$

or

$$m_f = \frac{\sqrt{R_0^2 - R_B^2} + \sqrt{r_0^2 - r_B^2} - C\sin\theta}{P\cos\theta} \tag{7c}$$

where R_0 = outside radius of first gear; R_B = = base radius of first gear ; r_0 = outside radius of second gear; r_B = base radius of second gear; C = center distance; I = pressure angle; and, p = circular pitch.

Both formulas Equations (7a) and Equations (7b) should give the same answer. It is good practice to use both formulas as a check on the previous calculations.

Lowest Point of Single Tooth Contact.—This diameter on the pinion (sometimes referred to as LPSTQ is used to find the maximum contact compressive stress (sometimes called the Hertz Stress) of a pair of mating spur gears. The two-step formulas for determining this pinion diameter, d_L, using the same nomenclature as in the previous sections with c and C as unnamed angles used only in the calculations are:

$$\tan c = \tan a - \frac{6.28318}{n} \tag{8}$$

$$d_L = \frac{n\cos\phi}{P\cos c} \tag{9}$$

In some cases it is necessary to have a plot of the compressive stress over the whole cycle of contact; in this case the LPSTC for the gear is required also. The similar two-step formulas for this gear diameter are:

$$\tan C = \tan A - \frac{6.28318}{N} \tag{10}$$

$$D_L = \frac{N \cos\phi}{P \cos C} \tag{11}$$

Maximum Hob Tip Radius.—The standard gear tooth proportions given by the formulas in Table 2 on page 2004 provide a specified size for the rack fillet radius in the general form of (a constant) × (pitch). For any given standard this constant may vary up to a maximum which it is geometrically impossible to exceed; this maximum constant, r_c (max), is found by the formula:

$$r_c\ (\max) = \frac{0.785398 \cos\phi - b \sin\phi}{1 - \sin\phi} \tag{12}$$

where b is the similar constant in the specified formula for the gear dedendum. The hob tip radius of any standard hob to finish cut any standard gear may vary from zero up to this limiting value.

Undercut Limit for Hobbed Involute Gears.—It is well to avoid designing and specifying gears that will have a hobbed trochoidal fillet that undercuts the involute gear tooth profile. This should be avoided because it may cause the involute profile to be cut away up to a point above the required contact diameter with the mating gear so that involute action is lost and the contact ratio reduced to a level that may be too low for proper conjugate action. An undercut fillet will also weaken the beam strength and thus raise the fillet tensile stress of the gear tooth. To assure that the hobbed gear tooth will not have an undercut fillet, the following formula must be satisfied:

$$\frac{b - r_c}{\sin\phi} + r_c \leq 0.5 n \sin\phi \tag{13}$$

where b is the dedendum constant; r_c is the hob or rack tip radius constant; n is the number of teeth in the gear; and ϕ is the gear and hob pressure angle. If the gear is not standard or the hob does not roll at the gear pitch diameter, this formula can not be applied and the determination of the expected existence of undercut becomes a considerably more complicated procedure.

Highest Point of Single Tooth Contact.—This diameter is used to place the maximum operating load for the determination of the gear tooth fillet stress. The two-step formulas for determining this diameter, d_H, of the pinion using the same nomenclature as in the previous sections with d and D as unnamed angles used only in the calculations are:

$$\tan d = \tan b + \frac{6.28318}{n} \tag{14}$$

$$d_H = \frac{n \cos\phi}{P \cos d} \tag{15}$$

Similarly for the gear:

$$\tan D = \tan B + \frac{6.28318}{N} \tag{16}$$

$$D_H = \frac{N \cos\phi}{P \cos D} \tag{17}$$

True Involute Form Diameter.—The point on the gear tooth at which the fillet and the involute profile are tangent to each other should be determined to assure that it lies at a smaller diameter than the required contact diameter with the mating gear. If the TIF diameter is larger than the contact diameter, then fillet interference will occur with severe damage to the gear tooth profile and rough action of the gear set. This two-step calculation is made by using the following two formulas with e and E as unnamed angles used only in the calculations:

$$\tan e = \tan\phi - \frac{4}{n}\left(\frac{b - r_c}{\sin 2\phi} + \frac{r_c}{2\cos\phi}\right) \tag{18}$$

$$d_{TIF} = \frac{n\cos\phi}{P\cos e} \tag{19}$$

As in the previous sections, ϕ is the pressure angle of the gear; n is the number of teeth in the pinion; b is the dedendum constant, r_c is the rack or hob tip radius constant, P is the gear diametral pitch and d_{TIF} is the true involute form diameter.

Similarly, for the mating gear:

$$\tan E = \tan\phi - \frac{4}{N}\left(\frac{b - r_c}{\sin 2\phi} + \frac{r_c}{2\cos\phi}\right) \tag{20}$$

$$D_{TIF} = \frac{N\cos\phi}{P\cos E} \tag{21}$$

where N is the number of teeth in this mating gear and D_{TIF} is the true involute form diameter.

Profile Checker Settings.—The actual tooth profile tolerance will need to be determined on high performance gears that operate either at high unit loads or at high pitch-line velocity. This is done on an involute checker, a machine which requires two settings, the gear base radius and the roll angle in degrees to significant points on the involute. From the smallest diameter outward these significant points are: TIF, Contact Diameter, LPSTC, Pitch Diameter, HPSTC, and Outside Diameter.

The base radius is:

$$R_b = \frac{N\cos\phi}{2P} \tag{22}$$

The roll angle, in degrees, at any point is equal to the tangent of the pressure angle at that point multiplied by 57.2958. The following table shows the tangents to be used at each significant diameter.

Significant Point on Tooth Profile	Pinion	Gear	For Computation
TIF	tan e	tan E	(See Formulas (18) & (20))
Contact Dia.	tan b	tan B	(See Formulas (2) & (5))
LPSTC	tan c	tan C	(See Formulas (8) & (10))
Pitch Dia.	tan ϕ	tan ϕ	(ϕ = Pressure angle)
HPSTC	tan d	tan D	(See Formulas (14) & (16))
Outside Dia.	tan a	tan A	(See Formulas (4) & (1))

Example: Find the significant diameters, contact ratio and hob tip radius for a 10-diametral pitch, 23-tooth, 20-degree pressure angle pinion of 2.5-inch outside diameter if it is to mesh with a 31-tooth gear of 3.3-inch outside diameter.

Thus: $n = 23$

$d_O = 2.5$

$P = 10$

$N = 31$

$D_O = 3.3$

$\phi = 20°$

1) Pinion contact diameter, d_c

$$\cos A = \frac{31 \times 0.93969}{3.3 \times 10} \quad (1)$$
$$= 0.88274 \qquad A = 28°1'30''$$

$$\tan b = 0.36397 - \frac{31}{23}(0.53227 - 0.36397) \quad (2)$$
$$= 0.13713 \qquad b = 7°48'26''$$

$$d_c = \frac{23 \times 0.93969}{10 \times 0.99073} \quad (3)$$
$$= 2.1815 \text{ inches}$$

2) Gear contact diameter, D_c

$$\cos a = \frac{23 \times 0.93963}{2.5 \times 10} \quad (4)$$
$$= 0.86452 \qquad a = 30°10'20''$$

$$\tan B = 0.36397 - \frac{23}{31}(0.58136 - 0.36937) \quad (5)$$
$$= 0.20267 \qquad B = 11°27'26''$$

$$D_c = \frac{31 \times 0.93969}{10 \times 0.98000} \quad (6)$$
$$= 2.9725 \text{ inches}$$

3) Contact ratio, m_f

$$m_f = \frac{31}{6.28318}(0.53227 - 0.20267) \quad (7a)$$
$$= 1.626$$

$$m_f = \frac{23}{6.28318}(0.58136 - 0.13713) \quad (7b)$$
$$= 1.626$$

4) Pinion LPSTC, d_L

$$\tan c = 0.58136 - \frac{6.28318}{23} \quad (8)$$
$$= 0.30818 \qquad c = 17°7'41''$$

$$d_L = \frac{23 \times 0.93969}{10 \times 0.95565} \quad (9)$$
$$= 2.2616 \text{ inches}$$

5) Gear LPSTC, D_L

$$\tan C = 0.53227 - \frac{6.28318}{31} \quad (10)$$
$$= 0.32959 \qquad C = 18°14'30''$$

$$D_L = \frac{31 \times 0.93969}{10 \times 0.94974} \quad (11)$$
$$= 3.0672 \text{ inches}$$

6) Maximum permissible hob tip radius, r_c (max). The dedendum factor is 1.25.

$$r_c\ (\max) = \frac{0.785398 \times 0.93969 - 1.25 \times 0.34202}{1 - 0.34202} = 0.4719 \text{ inch} \quad (12)$$

7) If the hob tip radius r_c is 0.30, determine if the pinion involute is undercut.

$$\frac{1.25 - 0.30}{0.34202} + 0.30 \le 0.5 \times 23 \times 0.34202 \qquad 3.0776 < 3.9332 \quad (13)$$

8) therefore there is no involute undercut.

9) Pinion HPSTC, D_H

$$\tan d = 0.13713 + \frac{6.28318}{23} = 0.41031 \qquad d = 22°18'32'' \quad (14)$$

$$d_H = \frac{23 \times 0.93969}{10 \times 0.92515} = 2.3362 \text{ inches} \quad (15)$$

10) Gear HPSTC, D_H

$$\tan D = 0.20267 + \frac{6.28318}{31} = 0.40535 \qquad D = 22°3'55'' \quad (16)$$

$$D_H = \frac{31 \times 0.93969}{10 \times 0.92676} = 3.1433 \text{ inches} \quad (17)$$

11) Pinion TIF diameter, d_{TIF}

$$\tan e = 0.36397 - \frac{4}{23}\left(\frac{1.25 - 0.30}{0.64279} + \frac{0.30}{2 \times 0.93969}\right) = 0.07917 \qquad e = 4°31'36'' \quad (18)$$

$$d_{TIF} = \frac{23 \times 0.93969}{10 \times 0.99688} = 2.1681 \text{ inches} \quad (19)$$

12) Gear TIF diameter, D_{TIF}

$$\tan E = 0.36397 - \frac{4}{31}\left(\frac{1.25 - 0.30}{0.64279} + \frac{0.30}{2 \times 0.93969}\right) = 0.15267 \qquad E = 8°40'50'' \quad (20)$$

$$D_{TIF} = \frac{31 \times 0.93969}{10 \times 0.98855} = 2.9468 \text{ inches} \quad (21)$$

Gear Blanks for Fine-pitch Gears.—The accuracy to which gears can be produced is considerably affected by the design of the gear blank and the accuracy to which the various surfaces of the blank are machined. The following recommendations should not be regarded as inflexible rules, but rather as minimum average requirements for gear-blank quality compatible with the expected quality class of the finished gear.

Design of Gear Blanks: The accuracy to which gears can be produced is affected by the design of the blank, so the following points of design should be noted: 1) Gears designed with a hole should have the hole large enough that the blank can be adequately supported during machining of the teeth and yet not so large as to cause distortion; 2) Face widths should be wide enough, in proportion to outside diameters, to avoid springing and to permit obtaining flatness in important surfaces; 3) Short bore lengths should be avoided wherever possible. It is feasible, however, to machine relatively thin blanks in stacks, provided the surfaces are flat and parallel to each other; 4) Where gear blanks with hubs are to be designed, attention should be given to the wall sections of the hubs. Too thin a section will not permit proper clamping of the blank during machining operations and may also affect proper mounting of the gear; and 5) Where pinions or gears integral with their shafts are to be designed, deflection of the shaft can be minimized by having the shaft length and shaft diameter well proportioned to the gear or pinion diameter. The foregoing general principles may also be useful when applied to blanks for coarser pitch gears.

Specifying Spur and Helical Gear Data on Drawings.—The data that may be shown on drawings of spur and helical gears falls into three groups: The first group consists of data basic to the design of the gear; the second group consists of data used in manufacturing and inspection; and the third group consists of engineering reference data. The accompanying table may be used as a checklist for the various data which may be placed on gear drawings and the sequence in which they should appear.

Explanation of Terms Used in Gear Specifications: 1) Number of teeth is the number of teeth in 360 deg of gear circumference. In a sector gear, both the actual number of teeth in the sector and the theoretical number of teeth in 360 deg should be given.

2) Diametral pitch is the ratio of the number of teeth in the gear to the number of inches in the standard pitch diameter. It is used in this standard as a nominal specification of tooth size.

a) Normal diametral pitch is the diametral pitch in the normal plane.

b) Transverse diametral pitch is the diametral pitch in the transverse plane.

c) Module is the ratio of the number of teeth in the gear to the number of mm in the standard pitch diameter.

d) Normal module is the module measured in the normal plane.

e) Transverse module is the module measured in the transverse plane.

3) Pressure angle is the angle between the gear tooth profile and a radial line at the pitch point. It is used in this standard to specify the pressure angle of the basic rack used in defining the gear tooth profile.

a) Normal pressure angle is the pressure angle in the normal plane.

b) Transverse pressure angle is the pressure angle in the transverse plane.

4) Helix angle is the angle between the pitch helix and an element of the pitch cylinder, unless otherwise specified.

a) Hand of helix is the direction in which the teeth twist as they recede from an observer along the axis. A right hand helix twists clockwise and a left hand helix twists counterclockwise.

5) Standard pitch diameter is the diameter of the pitch circle. It equals the number of teeth divided by the transverse diametral pitch.

6) Tooth form may be specified as standard addendum, long addendum, short addendum, modified involute or special. If a modified involute or special tooth form is required, a detailed view should be shown on the drawing. If a special tooth form is specified, roll angles must be supplied (see page 2031).

7) Addendum is the radial distance between the standard pitch circle and the outside circle. The actual value depends on the specification of outside diameter.

8) Whole depth is the total radial depth of the tooth space. The actual value is dependent on the specification of outside diameter and root diameter.

9) Maximum calculated circular thickness on the standard pitch circle is the tooth thickness which will provide the desired minimum backlash when the gear is assembled in mesh with its mate on minimum center distance. Control may best be exerted by testing in tight mesh with a master which integrates all errors in the several teeth in mesh through the arc of action as explained on page 2042. This value is independent of the effect of runout.

a) Maximum calculated *normal* circular thickness is the circular tooth thickness in the normal plane which satisfies requirements explained in (9).

10) Gear testing radius is the distance from its axis of rotation to the standard pitch line of a standard master when in intimate contact under recommended pressure on a variable-center-distance running gage. Maximum testing radius should be calculated to provide the maximum circular tooth thickness specified in (9) when checked as explained on page 2042. This value is affected by the runout of the gear. Tolerance on testing radius must be equal to or greater than the total composite error permitted by the quality class specified in (11).

11) Quality class is specified for convenience when talking or writing about the accuracy of the gear.

12) Maximum total composite error, and (13). Maximum tooth-to-tooth composite error. Actual tolerance values (12 and 13) permitted by the quality class (11) are specified in inches to provide machine operator or inspector with tolerances required to inspect the gear.

13) Testing pressure recommendations are given on page 2042. Incorrect testing pressure will result in incorrect measurement of testing radius.

14) Master specifications by tool or code number may be required to call for the use of a special master gear when tooth thickness deviates excessively from standard.

15) Measurement over two 0.xxxx diameter pins may be specified to assist the manufacturing department in determining size at machine for setup only.

16) Outside diameter is usually shown on the drawing of the gear together with other blank dimensions so that it will not be necessary for machine operators to search gear tooth data for this dimension. Since outside diameter is also frequently used in the manufacture and inspection of the teeth, it may be included in the data block with other tooth specifications if preferred. To permit use of topping hobs for cutting gears on which the tooth thickness has been modified from standard, the outside diameter should be related to the specified gear testing radius (10).

17) Maximum root diameter is specified to assure adequate clearance for the outside diameter of the mating gear. This dimension is usually considered acceptable if the gear is checked with a master and meets specifications (10) through (13).

18) Active profile diameter of a gear is the smallest diameter at which the mating gear tooth profile can make contact. Because of difficulties involved in checking, this specification is not recommended for gears finer than 48 pitch.

19) Surface roughness on active profile surfaces may be specified in microinches to be checked by instrument up to about 32 pitch, or by visual comparison in the finer pitch ranges. It is difficult to determine accurately the surface roughness of fine pitch gears. For many commercial applications surface roughness may be considered acceptable on gears which meet the maximum tooth-to-tooth-error specification (13).

20) Mating gear part number may be shown as a convenient reference. If the gear is used in several applications, all mating gears may be listed but usual practice is to record this information in a reference file.

21) Number of teeth in mating gear, and (23). Minimum operating center distance. This information is often specified to eliminate the necessity of getting prints of the mating gear and assemblies for checking the design specifications, interference, backlash, determination of master gear specification, and acceptance or rejection of gears made out of tolerance.

Data for Spur and Helical Gear Drawings

Type of Data	Min. Spur Gear Data	Min. Helical Gear-Data	Add'l Optional Data	Item Number[a]	Data[a]
Basic Specifications	X	X		1	Number of teeth
	X			2	Diametral pitch or module
		X		2a	Normal diametral pitch or module
			X	2b	Transverse diametral pitch or module
	X			3	Pressure angle
		X		3a	Normal pressure angle
			X	3b	Transverse pressure angle
		X		4	Helix angle
		X		4a	Hand of helix
	X	X		5	Standard pitch diameter
	X	X		6	Tooth form
			X	7	Addendum
			X	8	Whole depth
	X			9	Max. calc. circular thickness on std. pitch circle
		X		9a	Max. calc. normal circular thickness on std.pitch circle
Manufacturing and Inspection			X	10	Roll angles
	X	X		11	A.G.M.A. quality class
	X	X		12	Max. total composite error
	X	X		13	Max. tooth-to-tooth composite error
			X	14	Testing pressure (Ounces)
	X	X		15	Master specification
			X	16	Meas. over two .xxxx dia. pins (For setup only)
	X	X		17	Outside diameter (Preferably shown on drawing of gear)
			X	18	Max. root diameter
			X	19	Active profile diameter
			X	20	Surface roughness of active profile
Engineering Reference			X	21	Mating gear part number
			X	22	Number of teeth in mating gear
			X	23	Minimum operating center distance

[a] An item-by-item explanation of the terms used in this table is given beginning on page 2034.

Backlash

In general, backlash in gears is play between mating teeth. For purposes of measurement and calculation, backlash is defined as the amount by which a tooth space exceeds the thickness of an engaging tooth. It does not include the effect of center-distance changes of the mountings and variations in bearings. When not otherwise specified, numerical values of backlash are understood to be given on the pitch circles. The general purpose of backlash is to prevent gears from jamming together and making contact on both sides of their teeth simultaneously. Lack of backlash may cause noise, overloading, overheating of the-gears and bearings, and even seizing and failure.

Excessive backlash is objectionable, particularly if the drive is frequently reversing, or if there is an overrunning load as in cam drives. On the other hand, specification of an unnecessarily small amount of backlash allowance will increase the cost of gears, because errors in runout, pitch, profile, and mounting must be held correspondingly smaller. Backlash does not affect involute action and usually is not detrimental to proper gear action.

Determining Proper Amount of Backlash.—In specifying proper backlash and tolerances for a pair of gears, the most important factor is probably the maximum permissible amount of runout in both gear and pinion (or worm). Next are the allowable errors in profile, pitch, tooth thickness, and helix angle. Backlash between a pair of gears will vary as successive teeth make contact because of the effect of composite tooth errors, particularly runout, and errors in the gear center distances and bearings.

Other important considerations are speed and space for lubricant film. Slow-moving gears, in general, require the least backlash. Fast-moving fine-pitch gears are usually lubricated with relatively light oil, but if there is insufficient clearance for an oil film, and particularly if oil trapped at the root of the teeth cannot escape, heat and excessive tooth loading will occur.

Heat is a factor because gears may operate warmer, and, therefore, expand more, than the housings. The heat may result from oil churning or from frictional losses between the teeth, at bearings or oil seals, or from external causes. Moreover, for the same temperature rise, the material of the gears—for example, bronze and aluminum—may expand more than the material of the housings, usually steel or cast iron.

The higher the helix angle or spiral angle, the more transverse backlash is required for a given normal backlash. The transverse backlash is equal to the normal backlash divided by the cosine of the helix angle.

In designs employing normal pressure angles higher than 20 degrees, special consideration must be given to backlash, because more backlash is required on the pitch circles to obtain a given amount of backlash in a direction normal to the tooth profiles.

Errors in boring the gear housings, both in center distance and alignment, are of extreme importance in determining allowance to obtain the backlash desired. The same is true in the mounting of the gears, which is affected by the type and adjustment of bearings, and similar factors. Other influences in backlash specification are heat treatment subsequent to cutting the teeth, lapping operations, need for recutting, and reduction of tooth thickness through normal wear.

Minimum backlash is necessary for timing, indexing, gun-sighting, and certain instrument gear trains. If the operating speed is very low and the necessary precautions are taken in the manufacture of such gear trains, the backlash may be held to extremely small limits. However, the specification of “zero backlash,” so commonly stipulated for gears of this nature, usually involves special and expensive techniques, and is difficult to obtain.

Table 1. AGMA Recommended Backlash Range for Coarse-Pitch Spur, Helical and Herringbone Gearing

Center Distance (Inches)	Normal Diametral Pitches				
	0.5–1.99	2–3.49	3.5–5.99	6–9.99	10–19.99
	Backlash, Normal Plane, Inches[a]				
Up to 5					.005–.015
Over 5 to 10				.010–.020	.010–.020
Over 10 to 20			.020–.030	.015–.025	.010–.020
Over 20 to 30		.030–.040	.025–.030	.020–.030	
Over 30 to 40	.040–.060	.035–.045	.030–.040	.025–.035	
Over 40 to 50	.050–.070	.040–.055	.035–.050	.030–.040	
Over 50 to 80	.060–.080	.045–.065	.040–.060		
Over 80 to 100	.070–.095	.050–.080			
Over 100 to 120	.080–.110				

[a] Suggested backlash, on nominal centers, measured after rotating to the point of closest engagement. For helical and herringbone gears, divide above values by the cosine of the helix angle to obtain the transverse backlash.

The above backlash tolerances contain allowance for gear expansion due to differential in the operating temperature of the gearing and their supporting structure. The values may be used where the operating temperatures are up to 70 deg F higher than the ambient temperature.

For most gearing applications the recommended backlash ranges will provide proper running clearance between engaging teeth of mating gears. Deviation below the minimum or above the maximum values shown, which do not affect operational use of the gearing, should not be cause for rejection.

Definite backlash tolerances on coarse-pitch gearing are to be considered binding on the gear manufacturer only when agreed upon in writing.

Some applications may require less backlash than shown in the above table. In such cases the amount and tolerance should be by agreement between manufacturer and purchaser.

Recommended Backlash: In the following tables American Gear Manufacturers Association recommendations for backlash ranges for various kinds of gears are given.* For purposes of measurement and calculation, backlash is defined as the amount by which a tooth space exceeds the thickness of an engaging tooth. When not otherwise specified, numerical values of backlash are understood to be measured at the tightest point of mesh on the pitch circle in a direction normal to the tooth surface when the gears are mounted in their specified position.

Coarse-Pitch Gears: Table 1 gives the recommended backlash range for coarse-pitch spur, helical and herringbone gearing. Because backlash for helical and herringbone gears is more conveniently measured in the normal plane, Table 1 has been prepared to show backlash in the normal plane for coarse-pitch helical and herringbone gearing and in the transverse plane for spur gears. To obtain backlash in the transverse plane for helical and herringbone gears, divide the normal plane backlash in Table 1 by the cosine of the helix angle.

Table 2. AGMA Recommended Backlash Range for Bevel and Hypoid Gears

Diametral Pitch	Normal Backlash, Inch		Diametral Pitch	Normal Backlash, Inch	
	Quality Numbers 7 through 13	Quality Numbers 3 through 6		Quality Numbers 7 through 13	Quality Numbers 3 through 6
1.00 to 1.25	0.020–0.030	0.045–0.065	5.00 to 6.00	0.005–0.007	0.006–0.013
1.25 to 1.50	0.018–0.026	0.035–0.055	6.00 to 8.00	0.004–0.006	0.005–0.010
1.50 to 1.75	0.016–0.022	0.025–0.045	8.00 to 10.00	0.003–0.005	0.004–0.008
1.75 to 2.00	0.014–0.018	0.020–0.040	10.00 to 16.00	0.002–0.004	0.003–0.005
2.00 to 2.50	0.012–0.016	0.020–0.030	16.00 to 20.00	0.001–0.003	0.002–0.004
2.50 to 3.00	0.010–0.013	0.015–0.025	20 to 50	0.000–0.002	0.000–0.002
3.00 to 3.50	0.008–0.011	0.012–0.022	50 to 80	0.000–0.001	0.000–0.001
3.50 to 4.00	0.007–0.009	0.010–0.020	80 and finer	0.000–0.0007	0.000–0.0007
4.00 to 5.00	0.006–0.008	0.008–0.016	…	…	…

Measured at tightest point of mesh

The backlash tolerances given in this table contain allowances for gear expansion due to a differential in the operating temperature of the gearing and their supporting structure. The values may be used where the operating temperature is up to 70 degrees F. higher than the ambient temperature. These backlash values will provide proper running clearances for most gear applications.

The following important factors must be considered in establishing backlash tolerances:

V) Center distance tolerance; W) Parallelism of gear axes; X) Side runout or wobble;

Y) Tooth thickness tolerance; Z) Pitch line runout tolerance; AA) Profile tolerance;

AB) Pitch tolerance; AC) Lead tolerance; AD) Types of bearings and subsequent wear;

AE) Deflection under load; AF) Gear tooth wear; AG) Pitch line velocity; AH) Lubrication requirements; and AI) Thermal expansion of gears and housing.

A tight mesh may result in objectionable gear sound, increased power losses, overheating, rupture of the lubricant film, overloaded bearings and premature gear failure. However, it is recognized that there are some gearing applications where a tight mesh (zero backlash) may be required.

Specifying unnecessarily close backlash tolerances will increase the cost of the gearing. It is obvious from the above summary that the desired amount of backlash is difficult to evaluate. It is, therefore, recommended that when a designer, user or purchaser includes a reference to backlash in a gearing specification and drawing, consultation be arranged with the manufacturer.

* Extracted from Gear Classification Manual, AGMA 390.03 with permission of the publisher, the American Gear Manufacturers Association, 1500 King St., Alexandria, VA 22314.

Bevel and Hypoid Gears: Table 2 gives similar backlash range values for bevel and hypoid gears. These are values based upon average conditions for general purpose gearing, but may require modification to meet specific needs.

Backlash on bevel and hypoid gears can be controlled to some extent by axial adjustment of the gears during assembly. However, due to the fact that actual adjustment of a bevel or hypoid gear in its mounting will alter the amount of backlash, it is imperative that the amount of backlash cut into the gears during manufacture is not excessive. Bevel and hypoid gears must always be capable of operation without interference when adjusted for zero backlash. This requirement is imposed by the fact that a failure of the axial thrust bearing might permit the gears to operate under this condition. Therefore, bevel and hypoid gears should never be designed to operate with normal backlash in excess of $0.080/P$ where P is diametral pitch.

Fine-Pitch Gears: Table 3 gives similar backlash range values for fine-pitch spur, helical and herringbone gearing.

Providing Backlash.—In order to obtain the amount of backlash desired, it is necessary to decrease tooth thicknesses. However, because of manufacturing and assembling inaccuracies not only in the gears but also in other parts, the allowances made on tooth thickness almost always must exceed the desired amount of backlash. Since the amounts of these allowances depend on the closeness of control exercised on all manufacturing operations, no general recommendations for them can be given.

It is customary to make half of the allowance for backlash on the tooth thickness of each gear of a pair, although there are exceptions. For example, on pinions having very low numbers of teeth it is desirable to provide all of the allowance on the mating gear, so as not to weaken the pinion teeth. In worm gearing, ordinary practice is to provide all of the allowance on the worm which is usually made of a material stronger than that of the worm gear.

In some instances the backlash allowance is provided in the cutter, and the cutter is then operated at the standard tooth depth. In still other cases, backlash is obtained by setting the distance between two tools for cutting the two sides of the teeth, as in straight bevel gears, or by taking side cuts, or by changing the center distance between the gears in their mountings. In spur and helical gearing, backlash allowance is usually obtained by sinking the cutter deeper into the blank than the standard depth. The accompanying table gives the excess depth of cut for various pressure angles.

Excess Depth of Cut *E* to Provide Backlash Allowance

Distribution of Backlash	Pressure Angle ϕ, Degrees				
	14½	17½	20	25	30
Excess Depth of Cut *E* to Obtain Circular Backlash *B*[a]					
All on One Gear	1.93B	1.59B	1.37B	1.07B	0.87B
One-half on Each Gear	0.97B	0.79B	0.69B	0.54B	0.43B
Excess Depth of Cut *E* to Obtain Backlash B_b Normal to Tooth Profile[b]					
All on One Gear	2.00B_b	1.66B_b	1.46B_b	1.18B_b	1.99B_b
One-half on Each Gear	1.00B_b	0.83B_b	0.73B_b	0.59B_b	0.50B_b

[a] Circular backlash is the amount by which the width of a tooth space is greater than the thickness of the engaging tooth on the pitch circles. As described in pages 2036 and 2040 this is what is meant by backlash unless otherwise specified.

[b] Backlash measured normal to the tooth profile by inserting a feeler gage between meshing teeth; to convert to circular backlash, $B = B_b \div \cos\phi$.

Control of Backlash Allowances in Production.—Measurement of the tooth thickness of gears is perhaps the simplest way of controlling backlash allowances in production.

There are several ways in which this may be done including: 1) chordal thickness measurements as described on page 2018; 2) caliper measurements over two or more teeth as described on page 2109; and 3) measurements over wires.

In this last method, first the theoretical measurement over wires when the backlash allowance is zero is determined by the method described on page 2094; then the amount this measurement must be reduced to obtain a desired backlash allowance is taken from the table on page 2108.

It should be understood, as explained in the section *Measurement of Backlash* that merely making tooth thickness allowances will not guarantee the amount of backlash in the ready-to-run assembly of two or more gears. Manufacturing limitations will introduce such gear errors as runout, pitch error, profile error, and lead error, and gear-housing errors in both center distance and alignment. All of these make the backlash of the assembled gears different from that indicated by tooth thickness measurements on the individual gears.

Measurement of Backlash.—Backlash is commonly measured by holding one gear of a pair stationary and rocking the other back and forth. The movement is registered by a dial indicator having its pointer or finger in a plane of rotation at or near the pitch diameter and in a direction parallel to a tangent to the pitch circle of the moving gear. If the direction of measurement is normal to the teeth, or other than as specified above, it is recommended that readings be converted to the plane of rotation and in a tangent direction at or near the pitch diameter, for purposes of standardization and comparison.

In spur gears, parallel helical gears, and bevel gears, it is immaterial whether the pinion or gear is held stationary for the test. In crossed helical and hypoid gears, readings may vary according to which member is stationary; hence, it is customary to hold the pinion stationary and measure on the gear.

In some instances, backlash is measured by thickness gages or feelers. A similar method utilizes a soft lead wire inserted between the teeth as they pass through mesh. In both methods, it is likewise recommended that readings be converted to the plane of rotation and in a tangent direction at or near the pitch diameter, taking into account the normal pressure angle, and the helix angle or spiral angle of the teeth.

Sometimes backlash in parallel helical or herringbone gears is checked by holding the gear stationary, and moving the pinion axially back and forth, readings being taken on the face or shaft of the pinion, and converted to the plane of rotation by calculation. Another method consists of meshing a pair of gears tightly together on centers and observing the variation from the specified center distance. Such readings should also be converted to the plane of rotation and in a tangent direction at or near the pitch diameter for the reasons previously given.

Measurements of backlash may vary in the same pair of gears, depending on accuracy of manufacturing and assembling. Incorrect tooth profiles will cause a change of backlash at different phases of the tooth action. Eccentricity may cause a substantial difference between maximum and minimum backlash at different positions around the gears. In stating amounts of backlash, it should always be remembered that merely making allowances on tooth thickness does not guarantee the minimum amount of backlash that will exist in assembled gears.

Fine-Pitch Gears: The measurement of backlash of fine-pitch gears, when assembled, cannot be made in the same manner and by the same techniques employed for gears of coarser pitches. In the very fine pitches, it is virtually impossible to use indicating devices for measuring backlash. Sometimes a toolmaker's microscope is used for this purpose to good advantage on very small mechanisms.

Another means of measuring backlash in fine-pitch gears is to attach a beam to one of the shafts and measure the angular displacement in inches when one member is held stationary. The ratio of the length of the beam to the nominal pitch radius of the gear or pinion to which the beam is attached gives the approximate ratio of indicator reading to circular backlash. Because of the limited means of measuring backlash between a pair of fine-pitch gears, gear centers and tooth thickness of the gears when cut must be held to very close limits. Tooth thickness of fine-pitch spur and helical gears can best be checked on a variable-center-distance fixture using a master gear. When checked in this manner, tooth thickness change = 2 × center distance change × tangent of transverse pressure angle, approximately.

Control of Backlash in Assemblies.—Provision is often made for adjusting one gear relative to the other, thereby affording complete control over backlash at initial assembly and throughout the life of the gears. Such practice is most common in bevel gearing. It is fairly common in spur and helical gearing when the application permits slight changes between shaft centers. It is practical in worm gearing only for single thread worms with low lead angles. Otherwise faulty contact results.

Another method of controlling backlash quite common in bevel gears and less common in spur and helical gears is to match the high and low spots of the runout gears of one to one ratio and mark the engaging teeth at the point where the runout of one gear cancels the runout of the mating gear.

Table 3. AGMA Backlash Allowance and Tolerance for Fine-Pitch Spur, Helical and Herringbone Gearing

Backlash Designation	Normal Diametral Pitch Range	Tooth Thinning to Obtain Backlash[a] Allowance, per Gear, Inch	Tooth Thinning to Obtain Backlash[a] Tolerance, per Gear, Inch	Resulting Approximate Backlash (per Mesh) Normal Plane[b] Inch
A	20 thru 45	.002	0 to .002	.004 to .008
	46 thru 70	.0015	0 to .002	.003 to .007
	71 thru 90	.001	0 to .00175	.002 to .0055
	91 thru 200	.00075	0 to .00075	.0015 to .003
B	20 thru 60	.001	0 to .001	.002 to .004
	61 thru 120	.00075	0 to .00075	.0015 to .003
	121 thru 200	.0005	0 to .0005	.001 to .002
C	20 thru 60	.0005	0 to .0005	.001 to .002
	61 thru 120	.00035	0 to .0004	.0007 to .0015
	121 thru 200	.0002	0 to .0 to .0003	.0004 to .001
D	20 thru 60	.0025	0 to .00025	.0005 to .001
	61 thru 120	.0002	0 to .0002	.0004 to .0008
	121 thru 200	.0001	0 to .0001	.0002 to .0004
E	20 thru 60		0 to .00025	0 to .0005
	61 thru 120		0 to .0002	0 to .0004
	121 thru 200	Zero[c]	0 to .0001	0 to .0002

[a] These dimensions are shown primarily for the benefit of the gear manufacturer and represent the amount that the thickness of teeth should be reduced in the pinion and gear below the standard calculated value, to provide for backlash in the mesh. In some cases, particularly with pinions involving small numbers of teeth, it may be desirable to provide for total backlash by thinning the teeth in the gear member only by twice the allowance value shown in column (3). In this case both members will have the tolerance shown in column (4). In some cases, particularly in meshes with a small number of teeth, backlash may be achieved by an increase in basic center at distance. In such cases, neither member is reduced by the allowance shown in column (3).

[b] These dimensions indicate the approximate backlash that will occur in a mesh in which each of the mating pairs of gears have the teeth thinned by the amount referred to in Note 1, and are meshed on theoretical centers.

[c] Backlash in gear sets can also be achieved by increasing the center distance above nominal and using the teeth at standard tooth thickness. Class E backlash designation infers gear sets operating under these conditions.

Backlash in gears is the play between mating tooth surfaces. For purposes of measurement and calculation, backlash is defined as the amount by which a tooth space exceeds the thickness of an engaging tooth. When not otherwise specified, numerical values of backlash are understood to be measured at the tightest point of mesh on the pitch circle in a direction normal to the tooth surface when the gears are mounted in their specified position.

Allowance is the basic amount that a tooth is thinned from basic calculated circular tooth thickness to obtain the required backlash class.

Tolerance is the total permissible variation in the circular thickness of the teeth.

Angular Backlash in Gears.—When the backlash on the pitch circles of a meshing pair of gears is known, the angular backlash or angular play corresponding to this backlash may be computed from the following formulas.

$$\theta_D = \frac{6875B}{D} \text{ minutes} \qquad \theta_d = \frac{6875B}{d} \text{ minutes}$$

In these formulas, B = backlash between gears, in inches; D = pitch diameter of larger gear, in inches; d = pitch diameter of smaller gear, in inches; θ_D = angular backlash or angular movement of larger gear in minutes when smaller gear is held fixed and larger gear rocked back and forth; and θ_d = angular backlash or angular movement of smaller gear, in minutes, when the larger gear is held fixed and the smaller gear rocked back and forth.

Inspection of Gears.—Perhaps the most widely used method of determining relative accuracy in a gear is to rotate the gear through at least one complete revolution in intimate contact with a master gear of known accuracy. The gear to be tested and the master gear are mounted on a variable-center-distance fixture and the resulting radial displacements or changes in center distance during rotation of the gear are measured by a suitable device. Except for the effect of backlash, this so-called “composite check” approximates the action of the gear under operating conditions and gives the combined effect of the following errors: runout; pitch error; tooth-thickness variation; profile error; and lateral runout (sometimes called wobble).

Tooth-to-Tooth Composite Error, illustrated below, is the error that shows up as flicker on the indicator of a variable-center-distance fixture as the gear being tested is rotated from tooth to tooth in intimate contact with the master gear. Such flicker shows the combined or composite effect of circular pitch error, tooth-thickness variation, and profile error.

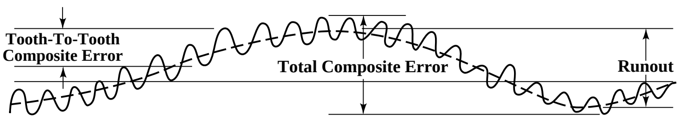


Diagram Showing Nature of Composite Errors

Total Composite Error, shown above, is made up of runout, wobble, and the tooth-to-tooth composite error; it is the total center-distance displacement read on the indicating device of the testing fixture, as shown in the accompanying diagram.

Pressure for Composite Checking of Fine-Pitch Gears.—In using a variable-center-distance fixture, excessive pressure on fine-pitch gears of narrow face width will result in incorrect readings due to deflection of the teeth. Based on tests, the following checking pressures are recommended for gears of 0.100-inch face width: 20 to 29 diametral pitch, 28 ounces; 30 to 39 pitch, 24 ounces; 40 to 49 pitch, 20 ounces; 50 to 59 pitch, 16 ounces; 60 to 79 pitch, 12 ounces; 80 to 99 pitch, 8 ounces, 100 to 149 pitch, 4 ounces; and 150 and finer pitches, 2 ounces, minimum. These recommended checking pressures are based on the use of antifriction mountings for the movable head of the checking fixture and include the pressure of the indicating device. For face widths less than 0.100 inch, the recommended pressures should be reduced proportionately; for larger widths, no increase is necessary although the force may be increased safely in the proper proportion.

British Standard for Spur and Helical Gears

British Standard For Spur And Helical Gears.—BS 436: Part 1: 1967: Spur and Helical Gears, Basic Rack Form, Pitches and Accuracy for Diametral Pitch Series, now has sections concerned with basic requirements for general tooth form, standard pitches, accuracy and accuracy testing procedures, and the showing of this information on engineering drawings to make sure that the gear manufacturer receives the required data. The latest form of the standard complies with ISO agreements. The standard pitches are in accordance with ISO R54, and the basic rack form and its modifications are in accordance with the ISO R53 "Basic Rack of Cylindrical Gears for General Engineering and for Heavy Engineering Standard".

Five grades of gear accuracy in previous versions are replaced by grades 3 to 12 of the draft ISO Standard. Grades 1 and 2 cover master gears that are not dealt with here. BS 436: Part 1: 1967 is a companion to the following British Standards:

BS 235 "Gears for Traction"
BS 545 "Bevel Gears (Machine Cut)"
BS 721 "Worm Gearing"
BS 821 "Iron Castings for Gears and Gear Blanks (Ordinary, Medium and High Grade)"
BS 978 "Fine Pitch Gears"Part 1, "Involute, Spur and Helical Gears"; Part 2, "Cycloidal Gears" (with addendum 1, PD 3376: "Double Circular Arc Type Gears."; Part 3, "Bevel Gears"
BS 1807 "Gears for Turbines and Similar Drives" Part 1, "Accuracy" Part 2, "Tooth Form and Pitches"
BS 2519 "Glossary of Terms for Toothed Gearing"
BS 3027 "Dimensions for Worm Gear Units"
BS 3696 "Master Gears"

Part 1 of BS 436 applies to external and internal involute spur and helical gears on parallel shafts and having normal diametral pitch of 20 or coarser. The basic rack and tooth form are specified, also first and second preference standard pitches and fundamental tolerances that determine the grades of gear accuracy, and requirements for terminology and notation. These requirements include: center distance a; reference circle diameter d, for pinion d_1 and wheel d_2; tip diameter d_a for pinion d_{a1} and wheel d_{a2}; center distance modification coefficient γ; face width b for pinion b_1 and wheel b_2; addendum modification coefficient x; for pinion x_1 and wheel x_2; length of arc l; diametral pitch P_t; normal diametral pitch p_n; transverse pitch p_t; number of teeth z, for pinion z_1 and wheel z_2; helix angle at reference cylinder $ß$; pressure angle at reference cylinder α; normal pressure angle at reference cylinder α_n; transverse pressure angle at reference cylinder α_t; and transverse pressure angle, working, α_{tw}.

The basic rack tooth profile has a pressure angle of 20°.

The Standard permits the total tooth depth to be varied within 2.25 to 2.40, so that the root clearance can be increased within the limits of 0.25 to 0.040 to allow for variations in manufacturing processes; and the root radius can be varied within the limits of 0.25 to 0.39. Tip relief can be varied within the limits shown at the right in the illustration.

Standard normal diametral pitches P_n, BS 436 Part 1:1967, are in accordance with ISO R54. The preferred series, rather than the second choice, should be used where possible.

Preferred normal diametral pitches for spur and helical gears (second choices in parentheses) are: 20 (18), 16 (14), 12 (11), 10 (9), 8 (7), 6 (5.5), 5 (4.5), 4 (3.5), 3 (2.75), 2.5 (2.25), 2 (1.75), 1.5, 1.25, and 1.

Information to be Given on Drawings: British Standard BS 308, "Engineering Drawing Practice", specifies data to be included on drawings of spur and helical gears. For all gears the data should include: number of teeth, normal diametral pitch, basic rack tooth form, axial pitch, tooth profile modifications, blank diameter, reference circle diameter, and helix angle at reference cylinder (0° for straight spur gears), tooth thickness at reference cylinder, grade of gear, drawing number of mating gear, working center distance, and backlash.

For single helical gears, the above data should be supplemented with hand and lead of the tooth helix; and for double helical gears, with the hand in relation to a specific part of the face width and the lead of tooth helix.

Inspection instructions also should be included, care being taken to avoid conflicting requirements for accuracy of individual elements, and single- and dual-flank testing. Supplementary data covering specific design, manufacturing and inspection requirements or limitations may also be needed, together with other dimensions and their tolerances, material, heat treatment, hardness, case depth, surface texture, protective finishes, and drawing scale.

Addendum Modification to Involute Spur and Helical Gears.—The British Standards Institute guide PD 6457:1970 contains certain design recommendations aimed at making it possible to use standard cutting tools for some sizes of gears. Essentially, the guide covers addendum modification and includes formulas for both English and metric units.

Addendum Modification is an enlargement or reduction of gear tooth dimensions that results from displacement of the reference plane of the generating rack from its normal position. The displacement is represented by the coefficient X, X1 , or X2 , where X is the equivalent dimension for gears of unit module or diametral pitch. The addendum modification establishes a datum tooth thickness at the reference circle of the gear but does not necessarily establish the height of either the reference addendum or the working addendum. In any pair of gears, the datum tooth thicknesses are those that always give zero backlash at the meshing center distance. Normal practice requires allowances for backlash for all unmodified gears.

Taking full advantage of the adaptability of the involute system allows various tooth design features to be obtained. Addendum modification has the following applications: avoiding undercut tooth profiles; achieving optimum tooth proportions and control of the proportion of receding to approaching contact; adapting a gear pair to a predetermined center distance without recourse to non-standard pitches; and permitting use of a range of working pressure angles using standard geometry tools.

BS 436, Part 3:1986 "Spur and Helical Gears".—This part provides methods for calculating contact and root bending stresses for metal involute gears, and is somewhat similar to the ANSI/AGMA Standard for calculating stresses in pairs of involute spur or helical gears. Stress factors covered in the British Standard include the following:

Tangential Force is the nominal force for contact and bending stresses.

Zone Factor accounts for the influence of tooth flank curvature at the pitch point on Hertzian stress.

Contact Ratio Factor takes account of the load-sharing influence of the transverse contact ratio and the overlap ratio on the specific loading.

Elasticity Factor takes into account the influence of the modulus of elasticity of the material and of Poisson's ratio on the Hertzian stress.

Basic Endurance Limit for contact makes allowance for the surface hardness.

Material Quality covers the quality of the material used.

Lubricant Influence, Roughness, and Speed The lubricant viscosity, surface roughness and pitch line speed affect the lubricant film thickness, which in turn, affects the Hertzian stresses.

Work Hardening Factor accounts for the increase in surface durability due to the meshing action.

Size Factor covers the possible influences of size on the material quality and its response to manufacturing processes.

Life Factor accounts for the increase in permissible stresses when the number of stress cycles is less than the endurance life.

Application Factor allows for load fluctuations from the mean load or loads in the load histogram caused by sources external to the gearing.

Dynamic Factor allows for load fluctuations arising from contact conditions at the gear mesh.

Load Distribution accounts for the increase in local load due to maldistribution of load across the face of the gear tooth caused by deflections, alignment tolerances and helix modifications.

Minimum Demanded and Actual Safety Factor The minimum demanded safety factor is agreed between the supplier and the purchaser. The actual safety factor is calculated.

Geometry Factors allow for the influence of the tooth form, the effect of the fillet and the helix angle on the nominal bending stress for the application of load at the highest point of single pair tooth contact.

Sensitivity Factor allows for the sensitivity of the gear material to the presence of notches such as the root fillet.

Surface Condition Factor accounts for reduction of the endurance limit due to flaws in the material and the surface roughness of the tooth root fillets.

ISO TC/600.—The ISO TC/600 Standard is similar to BS 436, Part 3:1986, but is far more comprehensive. For general gear design, the ISO Standard provides a complicated method of arriving at a conclusion similar to that reached by the less complex British Standard. Factors additional to the above that are included in the ISO Standard include the following

Application Factor takes account of dynamic overloads from sources external to the gearing.

Dynamic Factor allows for internally generated dynamic loads caused by vibrations of the pinion and wheel against each other.

Load Distribution makes allowance for the effects of non-uniform distribution of load across the face width, depending on the mesh alignment error of the loaded gear pair and the mesh stiffness.

Transverse Load Distribution Factor takes into account the effect of the load distribution on gear tooth contact stresses.

Gear Tooth Stiffness Constants are defined as the load needed to deform one or several meshing gear teeth having 1 mm face width, by an amount of 1 μm (0.00004 in).

Allowable Contact Stress is the permissible Hertzian pressure on the gear tooth face.

Minimum demanded and Calculated Safety Factors The minimum demanded safety factor is agreed between the supplier and the customer. The calculated safety factor is the actual safety factor of the gear pair.

Zone Factor accounts for the influence on the Hertzian pressure of the tooth flank curvature at the pitch point.

Elasticity Factor takes account of the influence of the material properties such as the modulus of elasticity and Poisson's ratio.

Contact Ratio Factor accounts for the influence of the transverse contact ratio and the overlap ratio on the specific surface load of the gears.

Helix Angle Factor makes allowance for the influence of the helix angle on the surface durability.

Endurance Limit is the limit of repeated Hertzian stresses that can be permanently endured by a given material

Life Factor takes account of a higher permissible Hertzian stress if only limited durability is demanded.

Lubrication Film Factor The film of lubricant between the tooth flanks influences the surface load capacity. Factors include the oil viscosity, pitch line velocity and roughness of the tooth flanks.

Work Hardening Factor takes account of the increase in surface durability due to meshing a steel wheel with a hardened pinion having smooth tooth surfaces.

Coefficient of Friction The mean value of the local coefficient of friction depends on the lubricant, surface roughness, the lay of surface irregularities, material properties of the tooth flanks, and the force and size of tangential velocities.

Bulk Temperature Thermal Flash Factor is dependent on the moduli of elasticity and thermal contact coefficients of pinion and wheel materials and the geometry of the line of action.

Welding Factor Accounts for different tooth materials and heat treatments.

Geometrical Factor is defined as a function of the gear ratio and the dimensionless parameter on the line of action.

Integral Temperature Criterion The integral temperature of the gears depends on the lubricant viscosity and tendency toward cuffing and scoring of the gear materials.

Examination of the above factors shows the similarity in the approach of the British and the ISO Standards to that of the ANSI/AGMA Standards. Slight variations in the methods used to calculate the factors will result in different allowable stress figures. Experimental work using some of the stressing formulas has shown wide variations and designers must continue to rely on experience to arrive at satisfactory results.

Standards Nomenclature

All standards are referenced and identified throughout this book by an alphanumeric prefix which designates the organization that administered the development work on the standard, and followed by a standards number.

All standards are reviewed by the relevant committees at regular time intervals, as specified by the overseeing standards organization, to determine whether the standard should be confirmed (reissued without changes other than correction of typographical errors), updated, or removed from service.

The following is for example use only. ANSI B18.8.2-1984, R1994 is a standard for Taper, Dowel, Straight, Grooved, and Spring Pins. ANSI refers to the American National Standards Institute that is responsible for overseeing the development or approval of the standard, and B18.8.2 is the number of the standard. The first date, 1984, indicates the year in which the standard was issued, and the sequence R1994 indicates that this standard was reviewed and reaffirmed in that 1994. The current designation of the standard, ANSI/ASME B18.8.2-1995, indicates that it was revised in 1995; it is ANSI approved; and, ASME (American Society of Mechanical Engineers) was the standards body responsible for development of the standard. This standard is sometimes also designated ASME B18.8.2-1995.

ISO (International Organization for Standardization) standards use a slightly different format, for example, ISO 5127-1:1983. The entire ISO reference number consists of a prefix ISO, a serial number, and the year of publication.

Aside from the content, ISO standards differ from American National standards in that they often smaller focused documents, which in turn reference other standards or other parts of the same standard. Unlike the numbering scheme used by ANSI, ISO standards related to a particular topic often do not carry sequential numbers nor are they in consecutive series.

British Standards Institute standards use the following format: BS 1361: 1971 (1986). The first part is the organization prefix BS, followed by the reference number and the date of issue. The number in parenthesis is the date that the standard was most recently reconfirmed. British Standards may also be designated *withdrawn* (no longer to be used) and *obsolescent* (going out of use, but may be used for servicing older equipment).

Organization	Web Address	Organization	Web Address
ISO (International Organization for Standardization)	www.iso.ch	JIS (Japanese Industrial Standards)	www.jisc.org
IEC (International Electrotechnical Commission)	www.iec.ch	ASME (American Society of Mechanical Engineers)	www.asme.org
ANSI (American National Standards Institute)	www.ansi.org	SAE (Society of Automotive Engineers)	www.sae.org
BSI (British Standards Institute)	www.bsi-inc.org	SME (Society of Manufacturing Engineers)	www.sme.org

INTERNAL GEARING

Internal Spur Gears.—An internal gear may be proportioned like a standard spur gear turned "outside in" or with addendum and dedendum in reverse positions; however, to avoid interference or improve the tooth form and action, the internal diameter of the gear should be increased and the outside diameter of the mating pinion is also made larger than the size based upon standard or conventional tooth proportions. The extent of these enlargements will be illustrated by means of examples given following table, *Rules for Internal Gears—20-degree Full-Depth Teeth*. The 20-degree involute full-depth tooth form is recommended for internal gears; the 20-degree stub tooth and the 14½-degree full-depth tooth are also used.

Methods of Cutting Internal Gears.—Internal spur gears are cut by methods similar in principle to those employed for external spur gears.

They may be cut by one of the following methods: 1) By a generating process, as when using a Fellows gear shaper; 2) by using a formed cutter and milling the teeth; 3) by planing, using a machine of the template or form-copying type (especially applicable to gears of large pitch); and 4) by using a formed tool that reproduces its shape and is given a planing action either on a slotting or a planing type of machine.

Internal gears frequently have a web at one side that limits the amount of clearance space at the ends of the teeth. Such gears may be cut readily on a gear shaper. The most practical method of cutting very large internal gears is on a planer of the form-copying type. A regular spur gear planer is equipped with a special tool holder for locating the tool in the position required for cutting internal teeth.

Formed Cutters for Internal Gears.—When formed cutters are used, a special cutter usually is desirable, because the tooth spaces of an internal gear are not the same shape as the tooth spaces of external gearing having the same pitch and number of teeth. This difference is because an internal gear is a spur gear "turned outside in." According to one rule, the standard No. 1 cutter for external gearing may be used for internal gears of 4 diametral pitch and finer, when there are 60 or more teeth. This No. 1 cutter, as applied to external gearing, is intended for all gears having from 135 teeth to a rack. The finer the pitch and the larger the number of teeth, the better the results obtained with a No. 1 cutter. The standard No. 1 cutter is considered satisfactory for jobbing work, and usually when the number of gears to be cut does not warrant obtaining a special cutter, although the use of the No. 1 cutter is not practicable when the number of teeth in the pinion is large in proportion to the number of teeth in the internal gear.

Arc Thickness of Internal Gear Tooth.—*Rule:* If internal diameter of an internal gear is enlarged as determined by Rules 1 and 2 for Internal Diameters (see *Rules for Internal Gears—20-degree Full-Depth Teeth*), the arc tooth thickness at the pitch circle equals 1.3888 divided by the diametral pitch, assuming a pressure angle of 20 degrees.

Arc Thickness of Pinion Tooth.—*Rule:* If the pinion for an internal gear is larger than conventional size (see Outside Diameter of Pinion for Internal Gear, under *Rules for Internal Gears—20-degree Full-Depth Teeth*), then the arc tooth thickness on the pitch circle equals 1.7528 divided by the diametral pitch, assuming a pressure angle of 20 degrees.

Note: For chordal thickness and chordal addendum, see rules and formulas for spur gears.

Relative Sizes of Internal Gear and Pinion.—If a pinion is too large or too near the size of its mating internal gear, serious interference or modification of the tooth shape may occur.

Rule: For internal gears having a 20-degree pressure angle and full-depth teeth, the difference between the numbers of teeth in gear and pinion should not be less than 12. For teeth of stub form, the smallest difference should be 7 or 8 teeth. For a pressure angle of 14½ degrees, the difference in tooth numbers should not be less than 15.

Rules for Internal Gears—20-degree Full-Depth Teeth

To Find	Rule
Pitch Diameter	*Rule:* To find the pitch diameter of an internal gear, divide the number of internal gear teeth by the diametral pitch. The pitch diameter of the mating pinion also equals the number of pinion teeth divided by the diametral pitch, the same as for external spur gears.
Internal Diameter (Enlarged to Avoid Interference)	*Rule* 1: For internal gears to mesh with pinions having 16 teeth or more, subtract 1.2 from the number of teeth and divide the remainder by the diametral pitch. *Example:* An internal gear has 72 teeth of 6 diametral pitch and the mating pinion has 18 teeth; then $\text{Internal diameter} = \frac{72 - 1.2}{6} = 11.8 \text{ inches}$ *Rule* 2: If circular pitch is used, subtract 1.2 from the number of internal gear teeth, multiply the remainder by the circular pitch, and divide the product by 3.1416.
Internal Diameter (Based upon Spur Gear Reversed)	*Rule:* If the internal gear is to be designed to conform to a spur gear turned outside in, subtract 2 from the number of teeth and divide the remainder by the diametral pitch to find the internal diameter. *Example:* (Same as Example above.) $\text{Internal diameter} = \frac{72 - 2}{6} = 11.666 \text{ inches}$
Outside Diameter of Pinion for Internal Gear	*Note:* If the internal gearing is to be proportioned like standard spur gearing, use the rule or formula previously given for spur gears in determining the outside diameter. The rule and formula following apply to a pinion that is enlarged and intended to mesh with an internal gear enlarged as determined by the preceding Rules 1 and 2 above. *Rule:* For pinions having 16 teeth or more, add 2.5 to the number of pinion teeth and divide by the diametral pitch. *Example* 1: A pinion for driving an internal gear is to have 18 teeth (full depth) of 6 diametral pitch; then $\text{Outside diameter} = \frac{18 + 2.5}{6} = 3.416 \text{ inches}$ By using the rule for external spur gears, the outside diameter = 3.333 inches.
Center Distance	*Rule:* Subtract the number of pinion teeth from the number of internal gear teeth and divide the remainder by two times the diametral pitch.
Tooth Thickness	See paragraphs, *Arc Thickness of Internal Gear Tooth* and *Effect of Diameter of Cutting on Profile and Pressure Angle of Worms*, on previous page.

Hypoid Gears

Hypoid gears are offset and in effect, are spiral gears whose axes do not intersect but are staggered by an amount decided by the application. Due to the offset, contact between the teeth of the two gears does not occur along a surface line of the cones as it does with spiral bevels having intersecting axes, but along a curve in space inclined to the surface line. The basic solids of the hypoid gear members are not cones, as in spiral bevels, but are hyperboloids of revolution which cannot be projected into the common plane of ordinary flat gears, thus the name hypoid. The visualization of hypoid gears is based on an imaginary flat gear which is a substitute for the theoretically correct helical surface. If certain rules are observed during the calculations to fix the gear dimensions, the errors that result from the use of an imaginary flat gear as an approximation are negligible.

The staggered axes result in meshing conditions that are beneficial to the strength and running properties of the gear teeth. A uniform sliding action takes place between the teeth, not only in the direction of the tooth profile but also longitudinally, producing ideal conditions for movement of lubricants. With spiral gears, great differences in sliding motion arise over various portions of the tooth surface, creating vibration and noise. Hypoid gears are almost free from the problems of differences in these sliding motions and the teeth also have larger curvature radii in the direction of the profile. Surface pressures are thus reduced so that there is less wear and quieter operation.

The teeth of hypoid gears are 1.5 to 2 times stronger than those of spiral bevel gears of the same dimensions, made from the same material. Certain limits must be imposed on the dimensions of hypoid gear teeth so that their proportions can be calculated in the same way as they are for spiral bevel gears. The offset must not be larger than 1/7th of the ring gear outer diameter, and the tooth ratio must not be much less than 4 to 1. Within these limits, the tooth proportions can be calculated in the same way as for spiral bevel gears and the radius of lengthwise curvature can be assumed in such a way that the normal module is a maximum at the center of the tooth face width to produce stabilized tooth bearings.

If the offset is larger or the ratio is smaller than specified above, a tooth form must be selected that is better adapted to the modified meshing conditions. In particular, the curvature of the tooth length curve must be determined with other points in view. The limits are only guidelines since it is impossible to account for all other factors involved, including the pitch line speed of the gears, lubrication, loads, design of shafts and bearings, and the general conditions of operation.1

Of the three different designs of hypoid bevel gears now available, the most widely used, especially in the automobile industry, is the Gleason system. Two other hypoid gear systems have been introduced by Oerlikon (Swiss) and Klingelnberg (German). All three methods use the involute gear form, but they have teeth with differing curvatures, produced by the cutting method. Teeth in the Gleason system are arc shaped and their depth tapers. Both the European systems are designed to combine rolling with the sideways motion of the teeth and use a constant tooth depth. Oerlikon uses an epicycloidal tooth form and Klingelnberg uses a true involute form.

With their circular arcuate tooth face curves, Gleason hypoid gears are produced with multi-bladed face milling cutters. The gear blank is rolled relative to the rotating cutter to make one inter-tooth groove, then the cutter is withdrawn and returned to its starting position while the blank is indexed into the position for cutting the next tooth. Both roughing and finishing cutters are kept parallel to the tooth root lines, which are at an angle to the gear pitch line. Depending on this angularity, plus the spiral angle, a correction factor must be calculated for both the leading and trailing faces of the gear tooth.

In operation, the convex faces of the teeth on one gear always bear on the concave faces of the teeth on the mating gear. For correct meshing between the pinion and gear wheel, the spiral angles should not vary over the full face width. The tooth form generated is a logarithmic spiral and, as a compromise, the cutter radius is made equal to the mean radius of a corresponding logarithmic spiral.

The involute tooth face curves of the Klingelnberg system gears have constant-pitch teeth cut by (usually) a single-start taper hob. The machine is set up to rotate both the cutter and the gear blank at the correct relative speeds. The surface of the hob is set tangential to a circle radius, which is the gear base circle, from which all the parallel involute curves are struck. To keep the hob size within reasonable dimensions, the cone must lie a minimum distance within the teeth and this requirement governs the size of the module.

Both the module and the tooth depth are constant over the full face width and the spiral angle varies. The cutting speed variations, especially with regard to crown wheels, over the cone surface of the hob, make it difficult to produce a uniform surface finish on the teeth, so a finishing cut is usually made with a truncated hob which is tilted to produce the required amount of crowning automatically, for correct tooth marking and finishing. The dependence of the module, spiral angle and other features on the base circle radius, and the need for suitable hob proportions restrict the gear dimensions and the system cannot be used for gears with a low or zero angle. However, gears can be cut with a large root radius giving teeth of high strength. The favorable geometry of the tooth form gives quieter running and tolerance of inaccuracies in assembly.

Teeth of gears made by the Oerlikon system have elongated epicycloidal form, produced with a face-type rotating cutter. Both the cutter and the gear blank rotate continuously, with no indexing. The cutter head has separate groups of cutters for roughing, outside cutting and inside cutting so that tooth roots and flanks are cut simultaneously, but the feed is divided into two stages. As stresses are released during cutting, there is some distortion of the blank and this distortion will usually be worse for a hollow crown wheel than for a solid pinion.

All the heavy cuts are taken during the first stages of machining with the Oerlikon system and the second stage is used to finish the tooth profile accurately, so distortion effects are minimized. As with the Klingelnberg process, the Oerlikon system produces a variation in spiral angle and module over the width of the face, but unlike the Klingelnberg method, the tooth length curve is cycloidal. It is claimed that, under load, the tilting force in an Oerlikon gear set acts at a point 0.4 times the distance from the small diameter end of the gear and not in the mid-tooth position as in other gear systems, so that the radius is obviously smaller and the tilting moment is reduced, resulting in lower loading of the bearings.

Gears cut by the Oerlikon system have tooth markings of different shape than gears cut by other systems, showing that more of the face width of the Oerlikon tooth is involved in the load-bearing pattern. Thus, the surface loading is spread over a greater area and becomes lighter at the points of contact.

Bevel Gearing

Types of Bevel Gears.—Bevel gears are conical gears, that is, gears in the shape of cones, and are used to connect shafts having intersecting axes. Hypoid gears are similar in general form to bevel gears, but operate on axes that are offset. With few exceptions, most bevel gears may be classified as being either of the straight-tooth type or of the curved-tooth type. The latter type includes spiral bevels, Zerol bevels, and hypoid gears. The following is a brief description of the distinguishing characteristics of the different types of bevel gears.

Straight Bevel Gears: The teeth of this most commonly used type of bevel gear are straight but their sides are tapered so that they would intersect the axis at a common point called the pitch cone apex if extended inward. The face cone elements of most straight bevel gears, however, are now made parallel to the root cone elements of the mating gear to obtain uniform clearance along the length of the teeth. The face cone elements of such gears, therefore, would intersect the axis at a point inside the pitch cone. Straight bevel gears are the easiest to calculate and are economical to produce.

Straight bevel gear teeth may be generated for full-length contact or for localized contact. The latter are slightly convex in a lengthwise direction so that some adjustment of the gears during assembly is possible and small displacements due to load deflections can occur without undesirable load concentration on the ends of the teeth. This slight lengthwise rounding of the tooth sides need not be computed in the design but is taken care of automatically in the cutting operation on the newer types of bevel gear generators.

Zerol Bevel Gears: The teeth of Zerol bevel gears are curved but lie in the same general direction as the teeth of straight bevel gears. They may be thought of as spiral bevel gears of zero spiral angle and are manufactured on the same machines as spiral bevel gears. The face cone elements of Zerol bevel gears do not pass through the pitch cone apex but instead are approximately parallel to the root cone elements of the mating gear to provide uniform tooth clearance. The root cone elements also do not pass through the pitch cone apex because of the manner in which these gears are cut. Zerol bevel gears are used in place of straight bevel gears when generating equipment of the spiral type but not the straight type is available, and may be used when hardened bevel gears of high accuracy (produced by grinding) are required.

Spiral Bevel Gears: Spiral bevel gears have curved oblique teeth on which contact begins gradually and continues smoothly from end to end. They mesh with a rolling contact similar to straight bevel gears. As a result of their overlapping tooth action, however, spiral bevel gears will transmit motion more smoothly than straight bevel or Zerol bevel gears, reducing noise and vibration that become especially noticeable at high speeds.

One of the advantages associated with spiral bevel gears is the complete control of the localized tooth contact. By making a slight change in the radii of curvature of the mating tooth surfaces, the amount of surface over which tooth contact takes place can be changed to suit the specific requirements of each job. Localized tooth contact promotes smooth, quiet running spiral bevel gears, and permits some mounting deflections without concentrating the load dangerously near either end of the tooth. Permissible deflections established by experience are given under the heading *Mountings for Bevel Gears*.

Because their tooth surfaces can be ground, spiral bevel gears have a definite advantage in applications requiring hardened gears of high accuracy. The bottoms of the tooth spaces and the tooth profiles may be ground simultaneously, resulting in a smooth blending of the tooth profile, the tooth fillet, and the bottom of the tooth space. This feature is important from a strength standpoint because it eliminates cutter marks and other surface interruptions that frequently result in stress concentrations.

Hypoid Gears: In general appearance, hypoid gears resemble spiral bevel gears, except that the axis of the pinion is offset relative to the gear axis. If there is sufficient offset, the shafts may pass one another thus permitting the use of a compact straddle mounting on the gear and pinion. Whereas a spiral bevel pinion has equal pressure angles and symmetrical profile curvatures on both sides of the teeth, a hypoid pinion properly conjugate to a mating gear having equal pressure angles on both sides of the teeth must have nonsymmetrical profile curvatures for proper tooth action. In addition, to obtain equal arcs of motion for both sides of the teeth, it is necessary to use unequal pressure angles on hypoid pinions. Hypoid gears are usually designed so that the pinion has a larger spiral angle than the gear. The advantage of such a design is that the pinion diameter is increased and is stronger than a corresponding spiral bevel pinion. This diameter increment permits the use of comparatively high ratios without the pinion becoming too small to allow a bore or shank of adequate size. The sliding action along the lengthwise direction of their teeth in hypoid gears is a function of the difference in the spiral angles on the gear and pinion. This sliding effect makes such gears even smoother running than spiral bevel gears. Grinding of hypoid gears can be accomplished on the same machines used for grinding spiral bevel and Zerol bevel gears.

Applications of Bevel and Hypoid Gears.—Bevel and hypoid gears may be used to transmit power between shafts at practically any angle and speed. The particular type of gearing best suited for a specific job, however, depends on the mountings and the operating conditions.

Straight and Zerol Bevel Gears: For peripheral speeds up to 1000 feet per minute, where maximum smoothness and quietness are not the primary consideration, straight and Zerol bevel gears are recommended. For such applications, plain bearings may be used for radial and axial loads, although the use of antifriction bearings is always preferable. Plain bearings permit a more compact and less expensive design, which is one reason why straight and Zerol bevel gears are much used in differentials. This type of bevel gearing is the simplest to calculate and set up for cutting, and is ideal for small lots where fixed charges must be kept to a minimum.

Zerol bevel gears are recommended in place of straight bevel gears where hardened gears of high accuracy are required, because Zerol gears may be ground; and when only spiral-type equipment is available for cutting bevel gears.

Spiral Bevel and Hypoid Gears: Spiral bevel and hypoid gears are recommended for applications where peripheral speeds exceed 1000 feet per minute or 1000 revolutions per minute. In many instances, they may be used to advantage at lower speeds, particularly where extreme smoothness and quietness are desired. For peripheral speeds above 8000 feet per minute, ground gears should be used.

For large reduction ratios the use of spiral and hypoid gears will reduce the overall size of the installation because the continuous pitch line contact of these gears makes it practical to obtain smooth performance with a smaller number of teeth in the pinion than is possible with straight or Zerol bevel gears.

Hypoid gears are recommended for industrial applications: when maximum smoothness of operation is desired; for high reduction ratios where compactness of design, smoothness of operation, and maximum pinion strength are important; and for nonintersecting shafts.

Bevel and hypoid gears may be used for both speed-reducing and speed-increasing drives. In speed-increasing drives, however, the ratio should be kept as low as possible and the pinion mounted on antifriction bearings; otherwise bearing friction will cause the drive to lock.

Notes on the Design of Bevel Gear Blanks.—The quality of any finished gear is dependent, to a large degree, on the design and accuracy of the gear blank. A number of factors that affect manufacturing economy as well as performance must be considered.

A gear blank should be designed to avoid localized stresses and serious deflections within itself. Sufficient thickness of metal should be provided under the roots of gear teeth to give them proper support. As a general rule, the amount of metal under the root should equal the whole depth of the tooth; this metal depth should be maintained under the small ends of the teeth as well as under the middle. On webless-type ring gears, the minimum stock between the root line and the bottom of tap drill holes should be one-third the tooth depth. For heavily loaded gears, a preliminary analysis of the direction and magnitude of the forces is helpful in the design of both the gear and its mounting. Rigidity is also necessary for proper chucking when cutting the teeth. For this reason, bores, hubs, and other locating surfaces must be in proper proportion to the diameter and pitch of the gear. Small bores, thin webs, or any condition that necessitates excessive overhang in cutting should be avoided.

Other factors to be considered are the ease of machining and, in gears that are to be hardened, proper design to ensure the best hardening conditions. It is desirable to provide a locating surface of generous size on the backs of gears. This surface should be machined or ground square with the bore and is used both for locating the gear axially in assembly and for holding it when the teeth are cut. The front clamping surface must, of course, be flat and parallel to the back surface. In connection with cutting the teeth on Zerol bevel, spiral bevel, and hypoid gears, clearance must be provided for face-mill type cutters; front and rear hubs should not intersect the extended root line of the gear or they will interfere with the path of the cutter. In addition, there must be enough room in the front of the gear for the clamp nut that holds the gear on the arbor, or in the chuck, while cutting the teeth. The same considerations must be given to straight bevel gears that are to be generated using a circular-type cutter instead of reciprocating tools.

Mountings for Bevel Gears.—Rigid mountings should be provided for bevel gears to keep the displacements of the gears under operating loads within recommended limits. To align gears properly, care should be taken to ensure accurately machined mountings, properly fitted keys, and couplings that run true and square.

As a result of deflection tests on gears and their mountings, and having observed these same units in service, the *Gleason Works* recommends that the following allowable deflections be used for gears from 6 to 15 inches in diameter: neither the pinion nor the gear should lift or depress more than 0.003 inch at the center of the face width; the pinion should not yield axially more than 0.003 inch in either direction; and the gear should not yield axially more than 0.003 inch in either direction on 1 to 1 ratio gears (miter gears), or near miters, or more than 0.010 inch away from the pinion on higher ratios.

When deflections exceed these limits, additional problems are involved in obtaining satisfactory gears. It becomes necessary to narrow and shorten the tooth contacts to suit the more flexible mounting. These changes decrease the bearing area, raise the unit tooth pressure, and reduce the number of teeth in contact, resulting in increased noise and the danger of surface failure as well as tooth breakage.

Spiral bevel and hypoid gears in general should be mounted on antifriction bearings in an oil-tight case. Designs for a given set of conditions may use plain bearings for radial and thrust loads, maintaining gears in satisfactory alignment is usually more easily accomplished with ball or roller bearings.

Bearing Spacing and Shaft Stiffness: Bearing spacing and shaft stiffness are extremely important if gear deflections are to be minimized. For both straddle mounted and overhung mounted gears the spread between bearings should never be less than 70 per cent of the pitch diameter of the gear. On overhung mounted gears the spread should be at least 2½ times the overhang and, in addition, the shaft diameter should be equal to or preferably greater than the overhang to provide sufficient shaft stiffness. When two spiral bevel or hypoid gears are mounted on the same shaft, the axial thrust should be taken at one place only and near the gear where the greater thrust is developed. Provision should be made for adjusting both the gear and pinion axially in assembly. Details on how this may be accomplished are given in the *Gleason Works* booklet, "Assembling Bevel Gears."

Cutting Bevel Gear Teeth.—A correctly formed bevel gear tooth has the same sectional shape throughout its length, but on a uniformly diminishing scale from the large to the small end. The only way to obtain this correct form is by using a generating type of bevel gear cutting machine. This accounts, in part, for the extensive use of generating type gear cutting equipment in the production of bevel gears.

Bevel gears too large to be cut by generating equipment (100 inches or over in diameter) may be produced by a form-copying type of gear planer. With this method, a template or former is used to mechanically guide a single cutting tool in the proper path to cut the profile of the teeth. Since the tooth profile produced by this method is dependent on the contour of the template used, it is possible to produce tooth profiles to suit a variety of requirements.

Although generating methods are to be preferred, there are still some cases where straight bevel gears are produced by milling. Milled gears cannot be produced with the accuracy of generated gears and generally are not suitable for use in high-speed applications or where angular motion must be transmitted with a high degree of accuracy. Milled gears are used chiefly as replacement gears in certain applications, and gears which are subsequently to be finished on generating type equipment are sometimes roughed out by milling. Formulas and methods used for the cutting of bevel gears are given in the latter part of this section.

In producing gears by generating methods, the tooth curvature is generated from a straight-sided cutter or tool having an angle equal to the required pressure angle. This tool represents the side of a crown gear tooth. The teeth of a true involute crown gear, however, have sides which are very slightly curved. If the curvature of the cutting tool conforms to that of the involute crown gear, an involute form of bevel gear tooth will be obtained. The use of a straight-sided tool is more practical and results in a very slight change of tooth shape to what is known as the “octoid” form. Both the octoid and involute forms of bevel gear tooth give theoretically correct action.

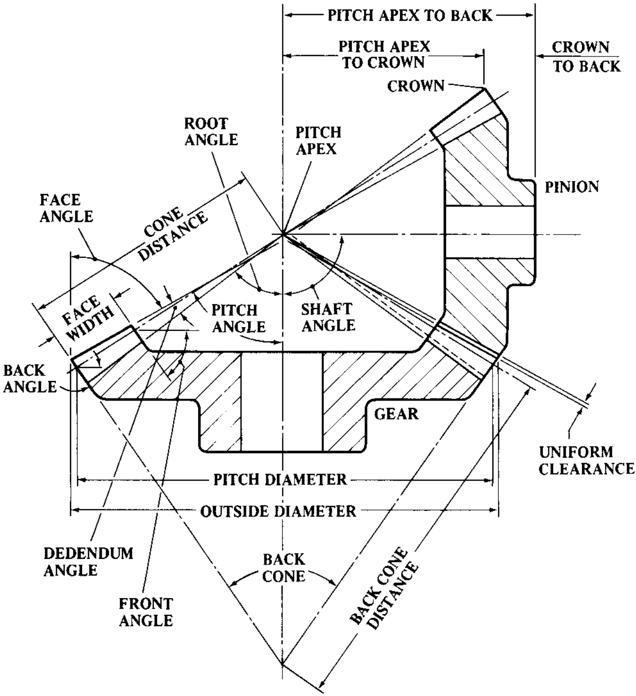
Bevel gear teeth, like those for spur gears, differ as to pressure angle and tooth proportions. The whole depth and the addendum at the large end of the tooth may be the same as for a spur gear of equal pitch. Most bevel gears, however, both of the straight tooth and spiral-bevel types, have lengthened pinion addendums and shortened gear addendums as in the case of some spur gears, the amount of departure from equal addendums varying with the ratio of gearing. Long addendums on the pinion are used principally to avoid undercut and to increase tooth strength. In addition, where long and short addendums are used, the tooth thickness of the gear is decreased and that of the pinion increased to provide a better balance of strength. See the Gleason Works System for straight and spiral bevel gears and also the British Standard.

Nomenclature for Bevel Gears.—The accompanying diagram, *Bevel Gear Nomenclature*, illustrates various angles and dimensions referred to in describing bevel gears. In connection with the face angles shown in the diagram, it should be noted that the face cones are made parallel to the root cones of the mating gears to provide uniform clearance along the length of the teeth.

American Standard for Bevel Gears.—American Standard ANSI/AGMA 2005-B88, Design Manual for Bevel Gears, replaces AGMA Standards 202.03, 208.03, 209.04, and 330.01, and provides standards for design of straight, zerol, and spiral bevel gears and hypoid gears with information on fabrication, inspection, and mounting. The information covers preliminary design, drawing formats, materials, rating, strength, inspection, lubrication, mountings, and assembly. Blanks for standard taper, uniform depth, duplex taper, and tilted root designs are included so that the material applies to users of Gleason, Klingelnberg, and Oerlikon gear cutting machines.

Formulas for Dimensions of Milled Bevel Gears.—As explained earlier, most bevel gears are produced by generating methods. Even so, there are applications for which it may be desired to cut a pair of mating bevel gears by using rotary formed milling cutters. Examples of such applications include replacement gears for certain types of equipment and gears for use in experimental developments.

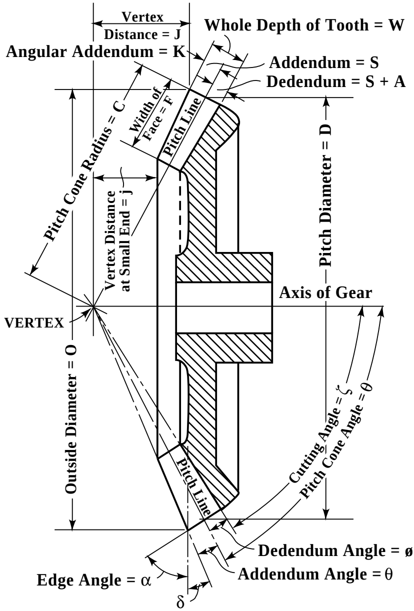
The tooth proportions of milled bevel gears differ in some respects from those of generated gears, the principal difference being that for milled bevel gears the tooth thicknesses of pinion and gear are made equal, and the addendum and dedendum of the pinion are respectively the same as those of the gear. The rules and formulas in the accompanying table may be used to calculate the dimensions of milled bevel gears with shafts at a right angle, an acute angle, and an obtuse angle.



Bevel Gear Nomenclature

In the accompanying diagram and list of notations, the various terms and symbols applied to milled bevel gears are as indicated.

N = number of teeth
P = diametral pitch
p = circular pitch
α = pitch cone angle and edge angle
Σ = angle between shafts
D = pitch diameter
S = addendum
$S+A$ = dedendum (A = clearance)



W = whole depth of tooth
T = thickness of tooth at pitch line
C = pitch cone radius
F = width of face
s = addendum at small end of tooth
t = thickness of tooth at pitch line at small end
θ = addendum angle
ϕ = dedendum angle
γ = face angle = pitch cone angle + addendum angle
δ = angle of compound rest
ζ = cutting angle
K = angular addendum
O = outside diameter
J = vertex distance
j = vertex distance at small end
N′ = number of teeth for which to select cutter

The formulas for milled bevel gears should be modified to make the clearance at the bottom of the teeth uniform instead of tapering toward the vertex. If this recommendation is followed, then the cutting angle (root angle) should be determined by subtracting the *addendum* angle from the pitch cone angle instead of subtracting the dedendum angle as in the formula given in the table.

Rules and Formulas for Calculating Dimensions of Milled Bevel Gears

To Find	Rule	Formula
Pitch Cone Angle of Pinion	Divide the sine of the shaft angle by the sum of the cosine of the shaft angle and the quotient obtained by dividing the number of teeth in the gear by the number of teeth in the pinion; this gives the tangent. *Note:* For shaft angles greater than 90° the cosine is negative.	$\tan\alpha P = \dfrac{\sin\Sigma}{\dfrac{N_G}{N_P} + \cos\Sigma}$ For 90° shaft angle, $\tan\alpha P = \dfrac{N_P}{N_G}$
Pitch Cone Angle of Gear	Subtract the pitch cone angle of the pinion from the shaft angle.	$\alpha G = \Sigma - \alpha P$
Pitch Diameter	Divide the number of teeth by the diametral pitch.	$D = N \div P$

Rules and Formulas for Calculating Dimensions of Milled Bevel Gears *(Continued)*

To Find		Rule	Formula
These dimensions are the same for both gear and pinion.	Addendum	Divide 1 by the diametral pitch.	$S = 1 \div P$
	Dedendum	Divide 1.157 by the diametral pitch.	$S + A = 1.157 \div P$
	Whole Depth of Tooth	Divide 2.157 by the diametral pitch.	$W = 2.157 \div P$
	Thickness of Tooth at Pitch Line	Divide 1.571 by the diametral pitch.	$T = 1.571 \div P$
	Pitch Cone Radius	Divide the pitch diameter by twice the sine of the pitch cone angle.	$C = \frac{D}{2 \times \sin\alpha}$
	Addendum of Small End of Tooth	Subtract the width of face from the pitch cone radius, divide the remainder by the pitch cone radius and multiply by the addendum.	$s = S \times \frac{C - F}{C}$
	Thickness of Tooth at Pitch Line at Small End	Subtract the width of face from the pitch cone radius, divide the remainder by the pitch cone radius and multiply by the thickness of the tooth at pitch line.	$t = T \times \frac{C - F}{C}$
	Addendum Angle	Divide the addendum by the pitch cone radius to get the tangent.	$\tan\theta = \frac{S}{C}$
	Dedendum Angle	Divide the dedendum by the pitch cone radius to get the tangent.	$\tan\phi = \frac{S + A}{C}$
	Face Width (Max.)	Divide the pitch cone radius by 3 or divide 8 by the diametral pitch, whichever gives the smaller value.	$F = \frac{C}{3}$ or $F = \frac{8}{P}$
	Circular Pitch	Divide 3.1416 by the diametral pitch.	$\rho = 3.1416 \div P$
Face Angle		Add the addendum angle to the pitch cone angle	$\gamma = \alpha + \theta$
Compound Rest Angle for Turning Blank		Subtract both the pitch cone angle and the addendum angle from 90 degrees.	$\delta = 90° - \alpha - \theta$
Cutting Angle		Subtract the dedendum angle from the pitch cone angle.	$\zeta = \alpha - \phi$
Angular Addendum		Multiply the addendum by the cosine of the pitch cone angle.	$K = S \times \cos\alpha$
Outside Diameter		Add twice the angular addendum to the pitch diameter.	$O = D + 2K$
Vertex or Apex Distance		Multiply one-half the outside diameter by the cotangent of the face angle.	$J = \frac{O}{2} \times \cot\gamma$
Vertex Distance at Small End of Tooth		Subtract the width of face from the pitch cone radius; divide the remainder by the pitch cone radius and multiply by the apex distance.	$j = J \times \frac{C - F}{C}$
Number of Teeth for which to Select Cutter		Divide the number of teeth by the cosine of the pitch cone angle.	$N' = \frac{N}{\cos\alpha}$

Numbers of Formed Cutters Used to Mill Teeth in Mating Bevel Gear and Pinion with Shafts at Right Angles

	Number of Teeth in Pinion																
Number of Teeth in Gear	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
12	7-7	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
13	6-7	6-6	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
14	5-7	6-6	6-6	…	…	…	…	…	…	…	…	…	…	…	…	…	…
15	5-7	5-6	5-6	5-5	…	…	…	…	…	…	…	…	…	…	…	…	…
16	4-7	5-7	5-6	5-6	5-5	…	…	…	…	…	…	…	…	…	…	…	…
17	4-7	4-7	4-6	5-6	5-5	5-5	…	…	…	…	…	…	…	…	…	…	…
18	4-7	4-7	4-6	4-6	4-5	4-5	5-5	…	…	…	…	…	…	…	…	…	…
19	3-7	4-7	4-6	4-6	4-6	4-5	4-5	4-4	…	…	…	…	…	…	…	…	…
20	3-7	3-7	4-6	4-6	4-6	4-5	4-5	4-4	4-4	…	…	…	…	…	…	…	…
21	3-8	3-7	3-7	3-6	4-6	4-5	4-5	4-5	4-4	4-4	…	…	…	…	…	…	…
22	3-8	3-7	3-7	3-6	3-6	3-5	4-5	4-5	4-4	4-4	4-4	…	…	…	…	…	…
23	3-8	3-7	3-7	3-6	3-6	3-5	3-5	3-5	3-4	4-4	4-4	4-4	…	…	…	…	…
24	3-8	3-7	3-7	3-6	3-6	3-6	3-5	3-5	3-4	3-4	3-4	4-4	4-4	…	…	…	…
25	2-8	2-7	3-7	3-6	3-6	3-6	3-5	3-5	3-5	3-4	3-4	3-4	4-4	3-3	…	…	…
26	2-8	2-7	3-7	3-6	3-6	3-6	3-5	3-5	3-5	3-4	3-4	3-4	3-4	3-3	3-3	…	…
27	2-8	2-7	2-7	2-6	3-6	3-6	3-5	3-5	3-5	3-4	3-4	3-4	3-4	3-4	3-3	3-3	…
28	2-8	2-7	2-7	2-6	2-6	3-6	3-5	3-5	3-5	3-4	3-4	3-4	3-4	3-4	3-3	3-3	3-3
29	2-8	2-7	2-7	2-7	2-6	2-6	3-5	3-5	3-5	3-4	3-4	3-4	3-4	3-4	3-3	3-3	3-3
30	2-8	2-7	2-7	2-7	2-6	2-6	2-5	2-5	3-5	3-5	3-4	3-4	3-4	3-4	3-4	3-3	3-3
31	2-8	2-7	2-7	2-7	2-6	2-6	2-6	2-5	2-5	2-5	3-4	3-4	3-4	3-4	3-4	3-3	3-3
32	2-8	2-7	2-7	2-7	2-6	2-6	2-6	2-5	2-5	2-5	2-4	2-4	3-4	3-4	3-4	3-3	3-3
33	2-8	2-8	2-7	2-7	2-6	2-6	2-6	2-5	2-5	2-5	2-4	2-4	2-4	3-4	3-4	3-4	3-3
34	2-8	2-8	2-7	2-7	2-6	2-6	2-6	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4	3-4	3-3
35	2-8	2-8	2-7	2-7	2-6	2-6	2-6	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4	2-4	2-3
36	2-8	2-8	2-7	2-7	2-6	2-6	2-6	2-5	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4	2-3
37	2-8	2-8	2-7	2-7	2-6	2-6	2-6	2-5	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4	2-3
38	2-8	2-8	2-7	2-7	2-6	2-6	2-6	2-5	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4	2-4
39	2-8	2-8	2-7	2-7	2-6	2-6	2-6	2-5	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4	2-4
40	1-8	2-8	2-7	2-7	2-6	2-6	2-6	2-5	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4	2-4
41	1-8	1-8	2-7	2-7	2-6	2-6	2-6	2-6	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4	2-4
42	1-8	1-8	2-7	2-7	2-6	2-6	2-6	2-6	2-5	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4
43	1-8	1-8	1-7	2-7	2-6	2-6	2-6	2-6	2-5	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4
44	1-8	1-8	1-7	1-7	2-6	2-6	2-6	2-6	2-5	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4
45	1-8	1-8	1-7	1-7	1-6	2-6	2-6	2-6	2-5	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4
46	1-8	1-8	1-7	1-7	1-7	2-6	2-6	2-6	2-5	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4
47	1-8	1-8	1-7	1-7	1-7	1-6	2-6	2-6	2-5	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4
48	1-8	1-8	1-7	1-7	1-7	1-6	1-6	2-6	2-5	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4
49	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	2-5	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4
50	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	2-5	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4
51	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-5	2-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4
52	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-5	1-5	2-5	2-5	2-4	2-4	2-4	2-4	2-4
53	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-5	1-5	1-5	2-5	2-4	2-4	2-4	2-4	2-4
54	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-5	1-5	1-5	1-5	2-4	2-4	2-4	2-4	2-4
55	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	2-4	2-4	2-4	2-4

Numbers of Formed Cutters Used to Mill Teeth in Mating Bevel Gear and Pinion with Shafts at Right Angles *(Continued)*

	Number of Teeth in Pinion																
Number of Teeth in Gear	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
56	1-8	1-8	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	2-4	2-4	2-4
57	1-8	1-8	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	2-4	2-4
58	1-8	1-8	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4	2-4
59	1-8	1-8	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
60	1-8	1-8	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
61	1-8	1-8	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
62	1-8	1-8	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
63	1-8	1-8	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
64	1-8	1-8	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
65	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
66	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
67	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
68	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
69	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
70	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
71	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
72	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
73	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
74	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
75	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
76	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
77	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
78	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
79	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
80	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
81	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
82	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
83	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
84	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
85	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
86	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
87	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
88	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
89	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
90	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
91	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
92	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
93	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
94	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
95	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
96	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
97	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
98	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
99	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4
100	1-8	1-8	1-7	1-7	1-7	1-6	1-6	1-6	1-6	1-5	1-5	1-5	1-5	1-4	1-4	1-4	1-4

Number of cutter for gear given first, followed by number for pinion. See text, page 2060

Selecting Formed Cutters for Milling Bevel Gears.—For milling 14½-degree pressure angle bevel gears, the standard cutter series furnished by manufacturers of formed milling cutters is commonly used. There are 8 cutters in the series for each diametral pitch to cover the full range from a 12-tooth pinion to a crown gear. The difference between formed cutters used for milling spur gears and those used for bevel gears is that bevel gear cutters are thinner because they must pass through the narrow tooth space at the small end of the bevel gear; otherwise the shape of the cutter and hence, the cutter number, are the same.

To select the proper number of cutter to be used when a bevel gear is to be milled, it is necessary, first, to compute what is called the "Number of Teeth, N' for which to Select Cutter." This number of teeth can then be used to select the proper number of bevel gear cutter from the spur gear milling cutter table on page 2023. The value of N' may be computed using the last formula in the table on page 2056.

Example 1: What numbers of cutters are required for a pair of bevel gears of 4 diametral pitch and 70 degree shaft angle if the gear has 50 teeth and the pinion 20 teeth?

The pitch cone angle of the pinion is determined by using the first formula in the table on page 2056:

$$\tan\alpha_P = \frac{\sin\Sigma}{\frac{N_G}{N_P} + \cos\Sigma} = \frac{\sin 70°}{\frac{50}{20} + \cos 70°} = 0.33064;\ \alpha_P = 18°18'$$

The pitch cone angle of the gear is determined from the second formula in the table on page 2056:

$$\alpha_G = \Sigma - \alpha_P = 70° - 18°18' = 51°42'$$

The numbers of teeth N′ for which to select the cutters for the gear and pinion may now be determined from the last formula in the table on page 2056:

$$N' \text{ for the pinion} = \frac{N_P}{\cos\alpha_P} = \frac{20}{\cos 18°18'} = 21.1 \approx 21 \text{ teeth}$$

$$N' \text{ for the gear} = \frac{N_G}{\cos\alpha_G} = \frac{50}{\cos 51°42'} = 80.7 \approx 81 \text{ teeth}$$

From the table on page 2023 the numbers of the cutters for pinion and gear are found to be, respectively, 5 and 2.

Example 2: Required the cutters for a pair of bevel gears where the gear has 24 teeth and the pinion 12 teeth. The shaft angle is 90 degrees. As in the first example, the formulas given in the table on page 2056 will be used.

$$\tan\alpha_P = N_P \div N_G = 12 \div 24 = 0.5000 \text{ and } \alpha_P = 26°34'$$

$$\alpha_G = \Sigma - \alpha_P = 90° - 26°34' = 63°26'$$

$$N' \text{ for pinion} = 12 \div \cos 26°34' = 13.4 \approx 13 \text{ teeth}$$

$$N' \text{ for gear} = 24 \div \cos 63°26' = 53.6 \approx 54 \text{ teeth}$$

And from the table on page 2023 the cutters for pinion and gear are found to be, respectively, 8 and 3.

Use of Table for Selecting Formed Cutters for Milling Bevel Gears.—The table beginning on page 2058 gives the numbers of cutters to use for milling various numbers of teeth in the gear and pinion. The table applies only to bevel gears with axes at right angles. Thus, in Example 2 given above, the numbers of the cutters could have been obtained directly by entering the table with the actual numbers of teeth in the gear, 24, and the pinion, 12.

Offset of Cutter for Milling Bevel Gears.—When milling bevel gears with a rotary formed cutter, it is necessary to take two cuts through each tooth space with the gear blank slightly off center, first on one side and then on the other, to obtain a tooth of approximately the correct form. The gear blank is also rotated proportionately to obtain the proper tooth thickness at the large and small ends. The amount that the gear blank or cutter should be offset from the central position can be determined quite accurately by the use of the table *Factors for Obtaining Offset for Milling Bevel Gears* in conjunction with the following rule: Find the factor in the table corresponding to the number of cutter used and to the ratio of the pitch cone radius to the face width; then divide this factor by the diametral pitch and subtract the result from half the thickness of the cutter at the pitch line.

Factors for Obtaining Offset for Milling Bevel Gears

No. of Cutter	Ratio of Pitch Cone Radius to Width of Face $\left(\frac{C}{F}\right)$												
	$\frac{3}{1}$	$\frac{3\frac{1}{4}}{1}$	$\frac{3\frac{1}{2}}{1}$	$\frac{3\frac{3}{4}}{1}$	$\frac{4}{1}$	$\frac{4\frac{1}{4}}{1}$	$\frac{4\frac{1}{2}}{1}$	$\frac{4\frac{3}{4}}{1}$	$\frac{5}{1}$	$\frac{5\frac{1}{2}}{1}$	$\frac{6}{1}$	$\frac{7}{1}$	$\frac{8}{1}$
1	0.254	0.254	0.255	0.256	0.257	0.257	0.257	0.258	0.258	0.259	0.260	0.262	0.264
2	0.266	0.268	0.271	0.272	0.273	0.274	0.274	0.275	0.277	0.279	0.280	0.283	0.284
3	0.266	0.268	0.271	0.273	0.275	0.278	0.280	0.282	0.283	0.286	0.287	0.290	0.292
4	0.275	0.280	0.285	0.287	0.291	0.293	0.296	0.298	0.298	0.302	0.305	0.308	0.311
5	0.280	0.285	0.290	0.293	0.295	0.296	0.298	0.300	0.302	0.307	0.309	0.313	0.315
6	0.311	0.318	0.323	0.328	0.330	0.334	0.337	0.340	0.343	0.348	0.352	0.356	0.362
7	0.289	0.298	0.308	0.316	0.324	0.329	0.334	0.338	0.343	0.350	0.360	0.370	0.376
8	0.275	0.286	0.296	0.309	0.319	0.331	0.338	0.344	0.352	0.361	0.368	0.380	0.386

Note.—For obtaining offset by above table, use formula:

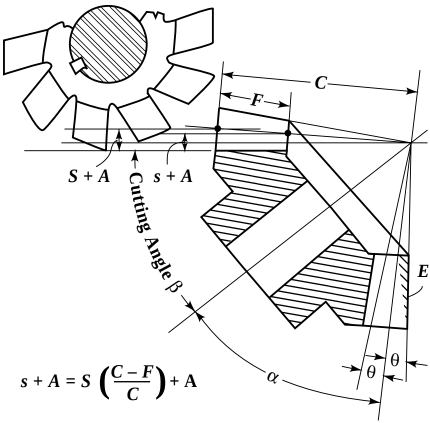
$$\text{Offset} = \frac{T}{2} - \frac{\text{factor from table}}{P}$$

P = diametral pitch of gear to be cut
T = thickness of cutter used, measured at pitch line

To illustrate, what would be the amount of offset for a bevel gear having 24 teeth, 6 diametral pitch, 30-degree pitch cone angle and $1\frac{1}{4}$-inch face or tooth length? In order to obtain a factor from the table, the ratio of the pitch cone radius to the face width must be determined. The pitch cone radius equals the pitch diameter divided by twice the sine of the pitch cone angle $= 4 \div (2 \times 0.5) = 4$ inches. As the face width is 1.25, the ratio is $4 \div 1.25$ or about $3\frac{1}{4}$ to 1. The factor in the table for this ratio is 0.280 with a No. 4 cutter, which would be the cutter number for this particular gear. The thickness of the cutter at the pitch line is measured by using a vernier gear tooth caliper. The depth $S + A$ (see Fig. 1; S = addendum; A = clearance) at which to take the measurement equals 1.157 divided by the diametral pitch; thus, $1.157 \div 6 = 0.1928$ inch. The cutter thickness at this depth will vary with different cutters and even with the same cutter as it is ground away, because formed bevel gear cutters are commonly provided with side relief. Assuming that the thickness is 0.1745 inch, and substituting the values in the formula given, we have:

$$\text{Offset} = \frac{0.1745}{2} - \frac{0.280}{6} = 0.0406 \text{ inch}$$

Adjusting the Gear Blank for Milling.—After the offset is determined, the blank is adjusted laterally by this amount, and the tooth spaces are milled around the blank. After having milled one side of each tooth to the proper dimensions, the blank is set over in the opposite direction the same amount from a position central with the cutter, and is rotated to line up the cutter with a tooth space at the small end. A trial cut is then taken, which will leave the tooth being milled a little too thick, provided the cutter is thin enough—as it should be—to pass through the small end of the tooth space of the finished gear. This trial tooth is made the proper thickness by rotating the blank toward the cutter. To test the amount of offset, measure the tooth thickness (with a vernier caliper) at the large and small ends. The caliper should be set so that the addendum at the small end is in proper proportion to the addendum at the large end; that is, in the ratio, $(C - F)/C$ (see Fig. 1).



$$s + A = S\left(\frac{C - F}{C}\right) + A$$

Fig. 1.

In taking these measurements, if the thicknesses at both ends (which should be in this same ratio) are too great, rotate the tooth toward the cutter and take trial cuts until the proper thickness at either the large or small end is obtained. If the large end of the tooth is the right thickness and the small end too thick, the blank was offset too much; inversely, if the small end is correct and the large end too thick, the blank was not set enough off center, and, either way, its position should be changed accordingly. The formula and table previously referred to will enable a properly turned blank to be set accurately enough for general work. The dividing head should be set to the cutting angle β (see Fig. 1), which is found by subtracting the addendum angle θ from the pitch cone angle α. After a bevel gear is cut by the method described, the sides of the teeth at the small end should be filed as indicated by the shade lines at *E*; that is, by filing off a triangular area from the point of the tooth at the large end to the point at the small end, thence down to the pitch line and back diagonally to a point at the large end.

Typical Steels Used for Bevel Gear Applications

SAE or AISI No.	Type of Steel	Purchase Specifications: Preliminary Heat Treatment	Purchase Specifications: Brinell Hard-ness Number	Purchase Specifications: ASTM Grain Size	Remarks
Carburizing Steels					
1024	Manganese	Normalize			Low Alloy — oil quench limited to thin sections
2512	Nickel Alloy	Normalize — Anneal	163–228	5–8	Aircraft quality
3310 3312X	Nickel-Chromium	Normalize, then heat to 1450°F, cool in furnace. Reheat to1170°F — cool in air	163–228	5–8	Used for maximum resistance to wear and fatigue
4028	Molybdenum	Normalize	163–217		Low Alloy
4615 4620	Nickel-Molybdenum	Normalize — 1700°F–1750°F	163–217	5–8	Good machining qualities. Well adapted to direct quench — gives tough core with minimum distortion
4815 4820	Nickel-Molybdenum	Normalize	163–241	5–8	For aircraft and heavily loaded service
5120	Chromium	Normalize	163–217	5–8	
8615 8620 8715 8720	Chromium-Nickel-Molybdenum	Normalize — cool at hammer	163–217	5–8	Used as an alternate for 4620
Oil Hardening and Flame Hardening Steels					
1141	Sulfurized free-cutting carbon steel	Normalize Heat-treated	179–228 255–269	5 or Coarser	Free-cutting steel used for unhardened gears, oil-treated gears, and for gears to be surface hardened where stresses are low
4140 4640	Chromium-Molybdenum Nickel-Molybdenum	For oil hardening, Normalize — Anneal For surface hardening, Normalize, reheat, quench, and draw	179–212 235–269 269–302 302–341		Used for heat-treated, oil-hardened, and surface-hardened gears. Machine qualities of 4640 are superior to 4140, and it is the preferred steel for flame hardening
6145	Chromium-Vanadium	Normalize— reheat, quench, and draw	235–269 269–302 302–341		Fair machining qualities. Used for surface hardened gears when 4640 is not available
8640 8739	Chromium-Nickel-Molybdenum	Same as for 4640			Used as an alternate for 4640
Nitriding Steels					
Nitralloy H & G	Special Alloy	Anneal	163–192		Normal hardness range for cutting is 20–28 Rockwell C

Other steels with qualities equivalent to those listed in the table may also be used.

Circular Thickness, Chordal Thickness, and Chordal Addendum of Milled Bevel Gear Teeth.—In the formulas that follow, T = circular tooth thickness on pitch circle at large end of tooth; t = circular thickness at small end; T_c and t_c = chordal thickness at large and small ends, respectively; S_c and s_c = chordal addendum at large and small ends, respectively; D = pitch diameter at large end; and $C, F, P, S, s,$ and α are as defined on page 2054.

$$T = \frac{1.5708}{P} \qquad T_c = T - \frac{T^3}{6D^2} \qquad S_c = S + \frac{T^2 \cos\alpha}{4D}$$

$$t = \frac{T(C-F)}{C} \qquad t_c = t - \frac{t^3}{6(D-2F\sin\alpha)^2} \qquad s_c = s + \frac{t^2\cos\alpha}{4(D-2F\sin\alpha)}$$

Worm Gearing

Worm Gearing.—Worm gearing may be divided into two general classes, fine-pitch worm gearing, and coarse-pitch worm gearing. Fine-pitch worm gearing is segregated from coarse-pitch worm gearing for the following reasons:

1) Fine-pitch worms and wormgears are used largely to transmit motion rather than power. Tooth strength except at the coarser end of the fine-pitch range is seldom an important factor; durability and accuracy, as they affect the transmission of uniform angular motion, are of greater importance.

2) Housing constructions and lubricating methods are, in general, quite different for fine-pitch worm gearing.

3) Because fine-pitch worms and wormgears are so small, profile deviations and tooth bearings cannot be measured with the same accuracy as can those of coarse pitches.

4) Equipment generally available for cutting fine-pitch wormgears has restrictions which limit the diameter, the lead range, the degree of accuracy attainable, and the kind of tooth bearing obtainable.

5) Special consideration must be given to top lands in fine-pitch hardened worms and wormgear-cutting tools.

6) Interchangeability and high production are important factors in fine-pitch worm gearing; individual matching of the worm to the gear, as often practiced with coarse-pitch precision worms, is impractical in the case of fine-pitch worm drives.

American Standard Design for Fine-pitch Worm Gearing (ANSI B6.9-1977).—This standard is intended as a design procedure for fine-pitch worms and wormgears having axes at right angles. It covers cylindrical worms with helical threads, and wormgears hobbed for fully conjugate tooth surfaces. It does not cover helical gears used as wormgears.

Hobs: The hob for producing the gear is a duplicate of the mating worm with regard to tooth profile, number of threads, and lead. The hob differs from the worm principally in that the outside diameter of the hob is larger to allow for resharpening and to provide bottom clearance in the wormgear.

Pitches: Eight standard axial pitches have been established to provide adequate coverage of the pitch range normally required: 0.030, 0.040, 0.050, 0.065, 0.080, 0.100, 0.130, and 0.160 inch.

Axial pitch is used as a basis for this design standard because: 1) Axial pitch establishes lead which is a basic dimension in the production and inspection of worms; 2) the axial pitch of the worm is equal to the circular pitch of the gear in the central plane; and 3) only one set of change gears or one master lead cam is required for a given lead, regardless of lead angle, on commonly-used worm-producing equipment.

Table 1. Formulas for Proportions of American Standard Fine-pitch Worms and Wormgears *ANSI B6.9-1977*

LETTER SYMBOLS

P = Circular pitch of wormgear

P = axial pitch of the worm, P_x, in the central plane

P_x = Axial pitch of worm

P_n = Normal circular pitch of worm and wormgear = $P_x \cos \lambda = P \cos \psi$

λ = Lead angle of worm

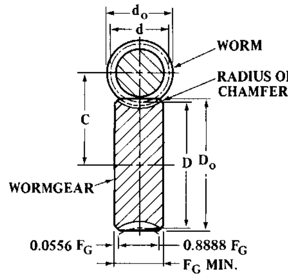
ψ = Helix angle of wormgear

n = Number of threads in worm

N = Number of teeth in wormgear

$N = nm_G$

m_G = Ratio of gearing = $N \div n$



Item	Formula	Item	Formula
WORM DIMENSIONS		WORMGEAR DIMENSIONS[a]	
Lead	$l = nP_x$	Pitch Diameter	$D = NP \div \pi = N\Pi_{\xi} \div \pi$
Pitch Diameter	$d = l \div (\pi \tan \lambda)$	Outside Diameter	$D_o = 2C - d + 2a$
Outside Diameter	$d_o = d + 2a$	Face Width	$F_{G\min} = 1.125 \times \sqrt{(d_o + 2c)^2 - (d_o - 4a)^2}$
Safe Minimum Length of Threaded Portion of Worm[b]	$F_W = \sqrt{D_o^2 - D^2}$		
DIMENSIONS FOR BOTH WORM AND WORMGEAR			
Addendum	$a = 0.3183P_n$	Tooth thickness	$t_n = 0.5P_n$
Whole Depth	$h_t = 0.7003P_n + 0.002$	Approximate normal pressure angle[c]	$\phi_n = 20$ degrees
Working Depth	$h_k = 0.6366P_n$		
Clearance	$c = h_t - h_k$	Center distance	$C = 0.5\,(d + D)$

[a] Current practice for fine-pitch worm gearing does not require the use of throated blanks. This results in the much simpler blank shown in the diagram which is quite similar to that for a spur or helical gear. The slight loss in contact resulting from the use of non-throated blanks has little effect on the load-carrying capacity of fine-pitch worm gears. It is sometimes desirable to use topping hobs for producing wormgears in which the size relation between the outside and pitch diameters must be closely controlled. In such cases the blank is made slightly larger than D_o by an amount (usually from 0.010 to 0.020) depending on the pitch. Topped wormgears will appear to have a small throat which is the result of the hobbing operation. For all intents and purposes, the throating is negligible and a blank so made is not to be considered as being a throated blank.

[b] This formula allows a sufficient length for fine-pitch worms.

[c] As stated in the text on page 2066, the actual pressure angle will be slightly greater due to the manufacturing process.

All dimensions in inches unless otherwise indicated.

Lead Angles: Fifteen standard lead angles have been established to provide adequate coverage: 0.5, 1, 1.5, 2, 3, 4, 5, 7, 9, 11, 14, 17, 21, 25, and 30 degrees.

This series of lead angles has been standardized to: 1) Minimize tooling; 2) permit obtaining geometric similarity between worms of different axial pitch by keeping the same lead angle; and 3) take into account the production distribution found in fine-pitch worm gearing applications.

For example, most fine-pitch worms have either one or two threads. This requires smaller increments at the low end of the lead angle series. For the less frequently used thread numbers, proportionately greater increments at the high end of the lead angle series are sufficient.

Pressure Angle of Worm: A pressure angle of 20 degrees has been selected as standard for cutters and grinding wheels used to produce worms within the scope of this Standard because it avoids objectionable undercutting regardless of lead angle.

Although the pressure angle of the cutter or grinding wheel used to produce the worm is 20 degrees, the normal pressure angle produced in the worm will actually be slightly greater, and will vary with the worm diameter, lead angle, and diameter of cutter or grinding wheel. A method for calculating the pressure angle change is given under the heading *Effect of Production Method on Worm Profile and Pressure Angle*.

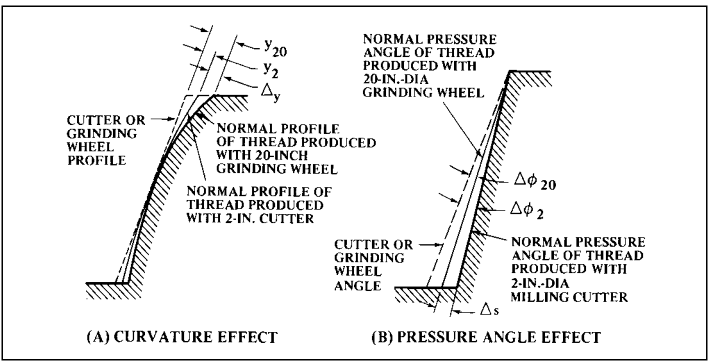
Pitch Diameter Range of Worms: The minimum recommended worm pitch diameter is 0.250 inch and the maximum is 2.000 inches.

Tooth Form of Worm and Wormgear: The shape of the worm thread in the normal plane is defined as that which is produced by a symmetrical double-conical cutter or grinding wheel having straight elements and an included angle of 40 degrees.

Because worms and wormgears are closely related to their method of manufacture, it is impossible to specify clearly the tooth form of the wormgear without referring to the mating worm. For this reason, worm specifications should include the method of manufacture and the diameter of cutter or grinding wheel used. Similarly, for determining the shape of the generating tool, information about the method of producing the worm threads must be given to the manufacturer if the tools are to be designed correctly.

The worm profile will be a curve that departs from a straight line by varying amounts, depending on the worm diameter, lead angle, and the cutter or grinding wheel diameter. A method for calculating this deviation is given in the Standard. The tooth form of the wormgear is understood to be made fully conjugate to the mating worm thread.

Effect of Diameter of Cutting on Profile and Pressure Angle of Worms



Effect of Production Method on Worm Profile and Pressure Angle.—In worm gearing, tooth bearing is usually used as the means of judging tooth profile accuracy since direct profile measurements on fine-pitch worms or wormgears is not practical. According to AGMA 370.01, Design Manual for Fine-Pitch Gearing, a minimum of 50 per cent initial area of contact is suitable for most fine-pitch worm gearing, although in some cases, such as when the load fluctuates widely, a more restricted initial area of contact may be desirable.

Except where single-pointed lathe tools, end mills, or cutters of special shape are used in the manufacture of worms, the pressure angle and profile produced by the cutter are different from those of the cutter itself. The amounts of these differences depend on several factors, namely, diameter and lead angle of the worm, thickness and depth of the worm thread, and diameter of the cutter or grinding wheel. The accompanying diagram shows the curvature and pressure angle effects produced in the worm by cutters and grinding wheels, and how the amount of variation in worm profile and pressure angle is influenced by the diameter of the cutting tool used.

Materials for Worm Gearing.—Worm gearing, especially for power transmission, should have steel worms and phosphor bronze wormgears. This combination is used extensively. The worms should be hardened and ground to obtain accuracy and a smooth finish.

The phosphor bronze wormgears should contain from 10 to 12 per cent of tin. The S.A.E. phosphor gear bronze (No. 65) contains 88–90% copper, 10–12% tin, 0.50% lead, 0.50% zinc (but with a maximum total lead, zinc and nickel content of 1.0 per cent), phosphorous 0.10–0.30%, aluminum 0.005%. The S.A.E. nickel phosphor gear bronze (No. 65 + Ni) contains 87% copper, 11% tin, 2% nickel and 0.2% phosphorous.

Single-thread Worms.—The ratio of the worm speed to the wormgear speed may range from 1.5 or even less up to 100 or more. Worm gearing having high ratios are not very efficient as transmitters of power; nevertheless high as well as low ratios often are required. Since the ratio equals the number of wormgear teeth divided by the number of threads or "starts" on the worm, single-thread worms are used to obtain a high ratio. As a general rule, a ratio of 50 is about the maximum recommended for a single worm and wormgear combination, although ratios up to 100 or higher are possible. When a high ratio is required, it may be preferable to use, in combination, two sets of worm gearing of the multi-thread type in preference to one set of the single-thread type in order to obtain the same total reduction and a higher combined efficiency.

Single-thread worms are comparatively inefficient because of the effect of the low lead angle; consequently, single-thread worms are not used when the primary purpose is to transmit power as efficiently as possible but they may be employed either when a large speed reduction with one set of gearing is necessary, or possibly as a means of adjustment, especially if "mechanical advantage" or self-locking are important factors.

Multi-thread Worms.—When worm gearing is designed primarily for transmitting power efficiently, the lead angle of the worm should be as high as is consistent with other requirements and preferably between, say, 25 or 30 and 45 degrees. This means that the worm must be multi-threaded. To obtain a given ratio, some number of wormgear teeth divided by some number of worm threads must equal the ratio. Thus, if the ratio is 6, combinations such as the following might be used:

$$\frac{24}{4}, \frac{30}{5}, \frac{36}{6}, \frac{42}{7}$$

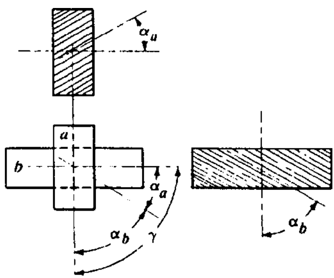
The numerators represent the number of wormgear teeth and the denominators, the number of worm threads or "starts." The number of wormgear teeth may not be an exact multiple of the number of threads on a multi-thread worm in order to obtain a "hunting tooth" action.

Number of Threads or "Starts" on Worm: The number of threads on the worm ordinarily varies from one to six or eight, depending upon the ratio of the gearing. As the ratio is increased, the number of worm threads is reduced, as a general rule. In some cases, however, the higher of two ratios may also have a larger number of threads. For example, a ratio of $6\frac{1}{5}$ would have 5 threads whereas a ratio of $6\frac{5}{6}$ would have 6 threads. Whenever the ratio is fractional, the number of threads on the worm equals the denominator of the fractional part of the ratio.

HELICAL GEARING

Basic Rules and Formulas for Helical Gear Calculations.—The rules and formulas in the following table and elsewhere in this article are basic to helical gear calculations. The notation used in the formulas is: P_n = normal diametral pitch of cutter; D = pitch diameter; N = number of teeth; α = helix angle; γ = center angle or angle between shafts; C = center distance; N' = number of teeth for which to select a formed cutter for milled teeth; L = lead of tooth helix; S = addendum; W = whole depth; T_n = normal tooth thickness at pitch line; and O = outside diameter.

Rules and Formulas for Helical Gear Calculations

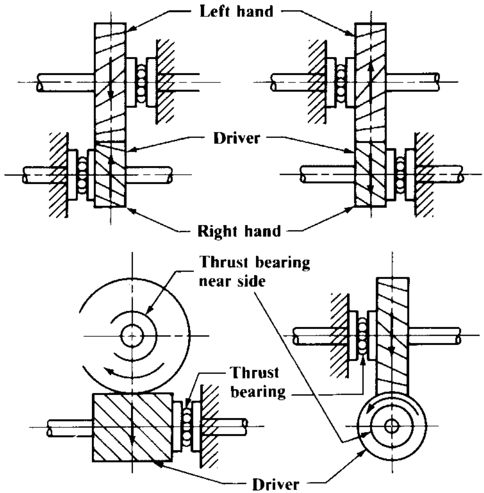


In the formulas, the symbols D, N, L, and α are for either the gear or the pinion. Subscripts a and b refer to the pinion or gear, respectively, in a pair of gears a and b.

No.	To Find	Rule	Formula
1	Pitch Diameter	Divide the number of teeth by the product of the normal diameter pitch and the cosine of the helix angle.	$D = \dfrac{N}{P_n \cos\alpha}$
2	Center Distance	Add together the two pitch diameters and divide by 2.	$C = \dfrac{D_a + D_b}{2}$
3	Lead of Tooth Helix	Multiply the pitch diameter by 3.1416 by the cotangent of the helix angle.	$L = \pi D \cot\alpha$
4	Addendum	Divide 1 by the normal diametral pitch.	$S = \dfrac{1}{P_n}$
5	Whole Depth of tooth	Divide 2.157 by the normal diametral pitch.	$W = \dfrac{2.157}{P_n}$
6	Normal Tooth Thickness at Pitch Line	Divide 1.5708 by the normal diametral pitch.	$T_n = \dfrac{1.5708}{P_n}$
7	OutsideDiameter	Add twice the addendum to the pitch diameter.	$O = D + 2S$

Determining Direction of Thrust.—The first step in helical gear design is to determine the desired direction of the thrust. When the direction of the thrust has been determined and the relative positions of the driver and driven gears are known, then the direction of helix (right- or left-hand) may be found from the accompanying thrust diagrams, *Directions of rotation and resulting thrust for parallel shaft and 90 degree shaft angle helical gears*. The diagrams show the directions of rotation and the resulting thrust for parallel-shaft and 90-

degree shaft angle helical gears. The thrust bearings are located so as to take the thrust caused by the tooth loads. The direction of the thrust depends on the direction of the helix, the relative positions of driver and driven gears, and the direction of rotation. The thrust may be changed to the opposite direction by changing any one of the three conditions, namely, by changing the hand of the helix, by reversing the direction of rotation, or by exchanging of driver and driven gear positions.



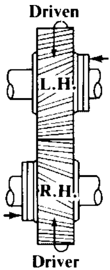
Directions of rotation and resulting thrust for parallel shaft and 90 degree shaft angle helical gears

Determining Helix Angles.—The following rules should be observed for helical gears with shafts at any given angle. If each helix angle is less than the shaft angle, then the sum of the helix angles of the two gears will equal the angle between the shafts, and the helix angle is of the same hand for both gears; if the helix angle of one of the gears is larger than the shaft angle, then the difference between the helix angles of the two gears will be equal to the shaft angle, and the gears will be of opposite hand.

Pitch of Cutter to be Used.—The thickness of the cutter at the pitchline for cutting helical gears should equal one-half the *normal* circular pitch. The normal pitch varies with the helix angle, hence, the helix angle must be considered when selecting a cutter. The cutter should be of the same pitch as the *normal* diametral pitch of the gear. This normal pitch is found by dividing the transverse diametral pitch of the gear by the cosine of the helix angle. To illustrate, if the pitch diameter of a helical gear is 6.718 and there are 38 teeth having a helix angle of 45 degrees, the transverse diametral pitch equals 38 divided by 6.718 = 5.656; then the normal diametral pitch equals 5.656 divided by 0.707 = 8. A cutter, then, of 8 diametral pitch is the one to use for this particular gear.

Helical gears should preferably be cut on a generating-type gear cutting machine such as a hobber or shaper. Milling machines are used in some shops when hobbers or shapers are not available or when single, replacement gears are being made. In such instances, the pitch of the formed cutter used in milling a helical gear must not only conform to the normal diametral pitch of the gear, but the cutter number must also be determined. See *Selecting Cutter for Milling Helical Gears* starting on page 2077.

1. Shafts Parallel, Center Distance Approximate.—Given or assumed:



1) Position of gear having right- or left-hand helix, depending upon rotation and direction in which thrust is to be received
2) C_a = approximate center distance
3) P_n = normal diametral pitch
4) N = number of teeth in large gear
5) n = number of teeth in small gear
6) α = angle of helix

To find:

1) D = pitch diameter of large gear = $\dfrac{N}{P_n \cos\alpha}$

2) d = pitch diameter of small gear = $\dfrac{n}{P_n \cos\alpha}$

3) O = outside diameter of large gear = $D + \dfrac{2}{P_n}$

4) o = outside diameter of small gear = $d + \dfrac{2}{P_n}$

5) T = number of teeth marked on formed milling cutter (large gear) = $\dfrac{N}{\cos^3\alpha}$

6) t = number of teeth marked on formed milling cutter (small gear) = $\dfrac{n}{\cos^3\alpha}$

7) L = lead of helix on large gear = $\pi D \cot\alpha$
8) l = lead of helix on small gear = $\pi d \cot\alpha$
9) C = center distance (if not right, vary α) = $\frac{1}{2}(D + d)$

Example

Given or assumed:
1) See illustration 2) $C_a = 17$ inches 3) $P_n = 2$ 4) $N = 48$ 5) $n = 20$ 6) $\alpha = 20$

To find:

1) $D = \dfrac{N}{P_n \cos\alpha} = \dfrac{48}{2 \times 0.9397} = 25.541$ inches

2) $d = \dfrac{n}{P_n \cos\alpha} = \dfrac{20}{2 \times 0.9397} = 10.642$ inches

3) $O = \dfrac{2}{P_n} = 25.541 + \dfrac{2}{2} = 26.541$ inches

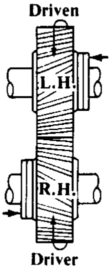
4) $o = d + \dfrac{2}{P_n} = 10.642 + \dfrac{2}{2} = 11.642$ inches

5) $T = \dfrac{N}{\cos^3\alpha} = \dfrac{48}{(0.9397)^3} = 57.8$, say 58 teeth

6) $t = \dfrac{n}{\cos^3\alpha} = \dfrac{20}{(0.9397)^3} = 24.1$, say 24 teeth

7) $L = \pi D \cot\alpha = 3.1416 \times 25.541 \times 2.747 = 220.42$ inches
8) $l = \pi d \cot\alpha = 3.1416 \times 10.642 \times 2.747 = 91.84$ inches
9) $C = \frac{1}{2}(D + d) = \frac{1}{2}(25.541 + 10.642) = 18.091$ inches

2. Shafts Parallel, Center Distance Exact.—Given or assumed:



1) Position of gear having right- or left-hand helix, depending upon rotation and direction in which thrust is to be received
2) C = exact center distance
3) P_n = normal diametral pitch (pitch of cutter)
4) N = number of teeth in large gear
5) n = number of teeth in small gear

To find:

1) $\cos\alpha = \dfrac{N+n}{2P_nC}$

2) D = pitch diameter of large gear = $\dfrac{N}{P_n\cos\alpha}$

3) d = pitch diameter of small gear = $\dfrac{n}{P_n\cos\alpha}$

4) O = outside diameter of large gear = $D + \dfrac{2}{P_n}$

5) o = outside diameter of small gear = $d + \dfrac{2}{P_n}$

6) T = number of teeth marked on formed milling cutter (large gear) = $\dfrac{N}{\cos^3\alpha}$

7) t = number of teeth marked on formed milling cutter (small gear) = $\dfrac{n}{\cos^3\alpha}$

8) L = lead of helix (large gear) = $\pi D\cot\alpha$
9) l = lead of helix (small gear) = $\pi d\cot\alpha$

Example

Given or assumed:
1) See illustration 2) C = 18.75 inches 3) $P_n = 4$ 4) $N = 96$ 5) $n = 48$

To find:

1) $\cos\alpha = \dfrac{N+n}{2P_nC} = \dfrac{96+48}{2\times4\times18.75} = 0.96$, or $\alpha = 16°16'$

2) $D = \dfrac{N}{P_n\cos\alpha} = \dfrac{96}{4\times0.96} = 25$ inches

3) $d = \dfrac{n}{P_n\cos\alpha} = \dfrac{48}{4\times0.96} = 12.5$ inches

4) $O = D + \dfrac{2}{P_n} = 25 + \dfrac{2}{4} = 25.5$ inches

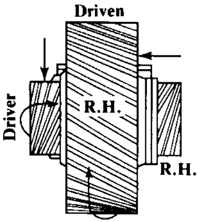
5) $o = d + \dfrac{2}{P_n} = 12.5 + \dfrac{2}{4} = 13$ inches

6) $T = \dfrac{N}{\cos^3\alpha} = \dfrac{96}{(0.96)^3} = 108$ teeth

7) $t = \dfrac{n}{\cos^3\alpha} = \dfrac{48}{(0.96)^3} = 54$ teeth

8) $L = \pi D \cot\alpha = 3.1416 \times 25 \times 3.427 = 269.15$ inches
9) $l = \pi d \cot\alpha = 3.1416 \times 12.5 \times 3.427 = 134.57$ inches

3. Shafts at Right Angles, Center Distance Approx.—Sum of helix angles of gear and pinion must equal 90 degrees.



Given or assumed:
1) Position of gear having right- or left-hand helix, depending on rotation and direction in which thrust is to be received
2) C_a = approximate center distance
3) P_n = normal diametral pitch (pitch of cutter)
4) R = ratio of gear to pinion size
5) n = number of teeth in pinion = $\frac{1.41 C_a P_n}{R+1}$ for 45 degrees;
and $\frac{2 C_a P_n \cos\alpha \cos\beta}{R\cos\beta + \cos\alpha}$ for any angle
6) N = number of teeth in gear = nR
7) α = angle of helix of gear
8) β = angle of helix of pinion

To find:
A) When helix angles are 45 degrees,
1) D = pitch diameter of gear = $\frac{N}{0.70711 P_n}$
2) d = pitch diameter of pinion = $\frac{n}{0.70711 P_n}$
3) O = outside diameter of gear = $D + \frac{2}{P_n}$
4) o = outside diameter of pinion = $d + \frac{2}{P_n}$
5) T = number of formed cutter (gear) = $\frac{N}{0.353}$
6) t = number of formed cutter (pinion) = $\frac{n}{0.353}$
7) L = lead of helix of gear = πD
8) l = lead of helix of pinion = πd
9) C = center distance (exact) = $\frac{D+d}{2}$

B) When helix angles are other than 45 degrees
1) $D = \frac{N}{P_n \cos\alpha}$ 2) $d = \frac{n}{P_n \cos\beta}$ 3) $T = \frac{N}{\cos^3\alpha}$
4) $t = \frac{n}{\cos^3\beta}$ 5) $L = \pi D \cot\alpha$ 6) $l = \pi d \cot\beta$

Example

Given or assumed:
1) See illustration 2) $C_a = 3.2$ inches 3) $P_n = 10$ 4) $R = 1.5$

5) $n = \frac{1.41 C_a P_n}{R+1} = \frac{1.41 \times 3.2 \times 10}{1.5 + 1}$ = say 18 teeth

6) $N = nR = 18 \times 1.5 = 27$ teeth

7) $\alpha = 45$ degrees

8) $\beta = 45$ degrees

To find:

1) $D = \frac{N}{0.70711 P_n} = \frac{27}{0.70711 \times 10} = 3.818$ inches

2) $d = \frac{n}{0.70711 P_n} = \frac{18}{0.70711 \times 10} = 2.545$ inches

3) $O = D + \frac{2}{P_n} = 3.818 + \frac{2}{10} = 4.018$ inches

4) $o = d + \frac{2}{P_n} = 2.545 + \frac{2}{10} = 2.745$ inches

5) $T = \frac{N}{0.353} = \frac{27}{0.353} = 76.5$, say 76 teeth

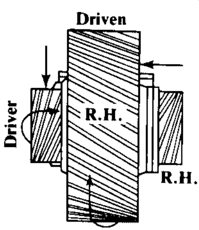
6) $t = \frac{n}{0.353} = \frac{18}{0.353} = 51$ teeth

7) $L = \pi D = 3.1416 \times 3.818 = 12$ inches

8) $l = \pi d = 3.1416 \times 2.545 = 8$ inches

9) $C = \frac{D + d}{2} = \frac{3.818 + 2.545}{2} = 3.182$ inches

4A. Shafts at Right Angles, Center Distance Exact.—Gears have same direction of helix. Sum of the helix angles will equal 90 degrees.



Given or assumed:

1) Position of gear having right- or left-hand helix depending on rotation and direction in which thrust is to be received

2) P_n = normal diametral pitch (pitch of cutter)

3) R = ratio of number of teeth in large gear to number of teeth in small gear

4) α_a = approximate helix angle of large gear

5) C = exact center distance

To find:

1) n = number of teeth in small gear nearest = $2\,CP_n \sin \alpha_a \div 1 + R \tan \alpha_a$

2) N = number of teeth in large gear = Rn

3) α = exact helix angle of large gear, found by trial from $R \sec \alpha + \operatorname{cosec} \alpha = 2\,CP_n \div n$

4) β = exact helix angle of small gear = $90° - \alpha$

5) D = pitch diameter of large gear = $\frac{N}{P_n \cos \alpha}$

6) d = pitch diameter of small gear = $\dfrac{n}{P_n \cos\beta}$

7) O = outside diameter of large gear = $D + \dfrac{2}{P_n}$

8) o = outside diameter of small gear = $d + \dfrac{2}{P_n}$

9) N' and n' = numbers of teeth marked on cuttters for large and small gears (see page 2077)

10) L = lead of helix on large gear = $\pi D \cot\alpha$

11) l = lead of helix on small gear = $\pi d \cot\beta$

Example

Given or assumed:

1) See illustration 2) $P_n = 8$ 3) $R = 3$ 4) $\alpha_a = 45$ degrees 5) $C = 10$ in

To find:

1) $n = \dfrac{2CP_n \sin\alpha_a}{1 + R\tan\alpha_a} = \dfrac{2 \times 10 \times 8 \times 0.70711}{1 + 3} = 28.25$, say 28 teeth

2) $N = Rn = 3 \times 28 = 84$ teeth

3) $R\sec\alpha + \operatorname{cosec}\alpha = \dfrac{2CP_n}{n} = \dfrac{2 \times 10 \times 8}{28} = 5.714$, or $\alpha = 46°6'$

4) $\beta = 90° - \alpha = 90° - 46°6' = 43°54'$

5) $D = \dfrac{N}{P_n \cos\alpha} = \dfrac{84}{8 \times 0.6934} = 15.143$ inches

6) $d = \dfrac{n}{P_n \cos\beta} = \dfrac{28}{8 \times 0.72055} = 4.857$ inches

7) $O = D + \dfrac{2}{P_n} = 15.143 + 0.25 = 15.393$ inches

8) $o = d + \dfrac{2}{P_n} = 4.857 + 0.25 = 5.107$ inches

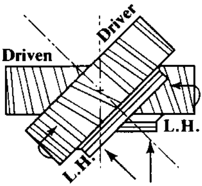
9) $N' = 275$; $n' = 94$ (see page 2077)

10) $L = \pi D \cot\alpha = 3.1416 \times 15.143 \times 0.96232 = 45.78$ inches

11) $l = \pi d \cot\beta = 3.1416 \times 4.857 \times 1.0392 = 15.857$ inches

4B. Shafts at Right Angles, Any Ratio, Helix Angle for Minimum Center Distance.— Diagram similar to 4A. Gears have same direction of helix. The sum of the helix angles will equal 90 degrees.

For any given ratio of gearing R there is a helix angle α for the larger gear and a helix angle $\beta = 90° - \alpha$ for the smaller gear that will make the center distance C a minimum. Helix angle α is found from the formula $\cot\alpha = R^{1/3}$. As an example, using the data found in Case 4A, helix angles α and β for minimum center distance would be: $\cot\alpha = R^{1/3} = 1.4422$; $\alpha = 34°44'$ and $\beta = 90° - 34°44' = 55°16'$. Using these helix angles, $D = 12.777$; $d = 6.143$; and $C = 9.460$ from the formulas for D and d given under Case 4A.



5. Shafts at Any Angle, Center Distance Approx.— The sum of the helix angles of the two gears equals the shaft angle, and the gears are of the same hand, if each angle is less than the shaft angle. The difference between the helix angles equals the shaft angle, and the gears are of opposite hand, if either angle is greater than the shaft angle.

Given or assumed:

1) Hand of helix, depending on rotation and direction in which thrust is to be received

2) C_a = center distance

3) P_n = normal diametral pitch (pitch of cutter)

4) R = ratio of gear to pinion = $\frac{N}{n}$

5) α = angle of helix, gear

6) β = angle of helix, pinion

7) n = number of teeth in pinion nearest $\frac{2C_aP_n\cos\alpha\cos\beta}{R\cos\beta + \cos\alpha}$

for any angle, and $\frac{2C_aP_n\cos\alpha}{R+1}$ when both angles are equal

8) N = number of teeth in gear = Rn

To find:

1) D = pitch diameter of gear = $\frac{N}{P_n\cos\alpha}$

2) d = pitch diameter of pinion = $\frac{n}{P_n\cos\beta}$

3) O = outside diameter of gear = $D + \frac{2}{P_n}$

4) o = outside diameter of pinion = $d + \frac{2}{P_n}$

5) T = number of teeth marked on cutter for gear = $\frac{N}{\cos^3\alpha}$

6) t = number of teeth marked on cutter for pinion = $\frac{n}{\cos^3\beta}$

7) L = lead of helix on gear = $\pi D\cot\alpha$

8) l = lead of helix on pinion = $\pi d\cot\beta$

9) C = actual center distance = $\frac{D+d}{2}$

Example

Given or assumed (angle of shafts, 60 degrees):

1) See illustration 2) C_a = 12 inches 3) $P_n = 8$

4) $R = 4$ 5) α = 30 degrees 6) β = 30 degrees

7) $n = \frac{2C_aP_n\cos\alpha}{R+1} = \frac{2\times 12\times 8\times 0.86603}{4+1} = 33$ teeth

8) $N = 4 \times 33 = 132$ teeth

To find:

1) $D = \frac{N}{P_n \cos\alpha} = \frac{132}{8 \times 0.86603} = 19.052$ inches

2) $d = \frac{n}{P_n \cos\beta} = \frac{33}{8 \times 0.86603} = 4.763$ inches

3) $O = D + \frac{2}{P_n} = 19.052 + \frac{2}{8} = 19.302$ inches

4) $o = d + \frac{2}{P_n} = 4.763 + \frac{2}{8} = 5.010000000000000003$ inches

5) $T = \frac{N}{\cos^3\alpha} = \frac{132}{0.65} = 203$ teeth

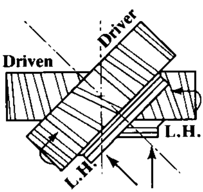
6) $t = \frac{n}{\cos^3\beta} = \frac{33}{0.65} = 51$ teeth

7) $L = \pi D \cot\alpha = \pi \times 19.052 \times 1.732 = 103.66$ inches

8) $l = \pi d \cot\beta = \pi \times 4.763 \times 1.732 = 25.92$ inches

9) $C = \frac{D + d}{2} = \frac{19.052 + 4.763}{2} = 11.9075$ inches

6. Shafts at Any Angle, Center Distance Exact.—The sum of the helix angles of the two gears equals the shaft angle, and the gears are of the same hand, if each angle is less than the shaft angle. The difference between the helix angles equals the shaft angle, and the gears are of opposite hand, if either angle is greater than the shaft angle.



Given or assumed:

1) Hand of helix, depending on rotation and direction in which thrust is to be received

2) C = center distance

3) P_n = normal diametral pitch (pitch of cutter)

4) α_a = approximate helix angle of gear

5) β_a = approximate helix angle of pinion

6) R = ratio of gear to pinion size = $\frac{N}{n}$

7) n = number of pinion teeth nearest $\frac{2CP_n \cos\alpha_a \cos\beta_a}{R\cos\beta_a + \cos\alpha_a}$

8) N = number of gear teeth = Rn

To find:

1) α and β, exact helix angles, found by trial from $R \sec\alpha + \sec\beta = \frac{2CP_n}{n}$

2) D = pitch diameter of gear = $\dfrac{N}{P_n \cos\alpha}$

3) d = pitch diameter of pinion = $\dfrac{n}{P_n \cos\beta}$

4) O = outside diameter of gear = $D + \dfrac{2}{P_n}$

5) o = outside diameter of pinion = $d + \dfrac{2}{P_n}$

6) N' = number of teeth marked on formed cutter for gear (see below)
7) n' = number of teeth marked on formed cutter for pinion (see below)
8) L = lead of helix on gear = $\pi D \cot\alpha$
9) l = lead of helix on pinion = $\pi d \cot\beta$

Selecting Cutter for Milling Helical Gears.—The proper milling cutter to use for *spur* gears depends on the pitch of the teeth and also upon the number of teeth as explained on page 2021 but a cutter for milling helical gears is not selected with reference to the actual number of teeth in the gear, as in spur gearing, but rather with reference to a calculated number N' that takes into account the effect on the tooth profile of lead angle, normal diametral pitch, and cutter diameter.

In the helical gearing examples starting on page 2070 the number of teeth N' on which to base the selection of the cutter has been determined using the approximate formula $N' = N \div \cos^3\alpha$ or $N' = N\sec^3\alpha$, where N = the actual number of teeth in the helical gear and α = the helix angle. However, the use of this formula may, where a combination of high helix angle and low tooth number is involved, result in the selection of a higher number of cutter than should actually be used for greatest accuracy. This condition is most likely to occur when the aforementioned formula is used to calculate N' for gears of high helix angle and low number of teeth.

To avoid the possibility of error in choice of cutter number, the following formula, which gives theoretically correct results for all combinations of helix angle and tooth numbers, is to be preferred:

$$N' = N\sec^3\alpha + P_n D_c \tan^2\alpha \qquad (1)$$

where: N' = number of teeth on which to base selection of cutter number from table on page 2023; N = actual number of teeth in helical gear; α = helix angle; P_n = normal diametral pitch of gear and cutter; and D_c = pitch diameter of cutter.

Factors for Selecting Cutters for Milling Helical Gears

Helix Angle, α	K	K′	Helix Angle, α	K	K′	Helix Angle, α	K	K′	Helix Angle, α	K	K′
0	1.000	0	16	1.127	0.082	32	1.640	0.390	48	3.336	1.233
1	1.001	0	17	1.145	0.093	33	1.695	0.422	49	3.540	1.323
2	1.002	0.001	18	1.163	0.106	34	1.755	0.455	50	3.767	1.420
3	1.004	0.003	19	1.182	0.119	35	1.819	0.490	51	4.012	1.525
4	1.007	0.005	20	1.204	0.132	36	1.889	0.528	52	4.284	1.638
5	1.011	0.008	21	1.228	0.147	37	1.963	0.568	53	4.586	1.761
6	1.016	0.011	22	1.254	0.163	38	2.044	0.610	54	4.925	1.894
7	1.022	0.015	23	1.282	0.180	39	2.130	0.656	55	5.295	2.039
8	1.030	0.020	24	1.312	0.198	40	2.225	0.704	56	5.710	2.198
9	1.038	0.025	25	1.344	0.217	41	2.326	0.756	57	6.190	2.371
10	1.047	0.031	26	1.377	0.238	42	2.436	0.811	58	6.720	2.561
11	1.057	0.038	27	1.414	0.260	43	2.557	0.870	59	7.321	2.770
12	1.068	0.045	28	1.454	0.283	44	2.687	0.933	60	8.000	3.000
13	1.080	0.053	29	1.495	0.307	45	2.828	1	61	8.780	3.254
14	1.094	0.062	30	1.540	0.333	46	2.983	1.072	62	9.658	3.537
15	1.110	0.072	31	1.588	0.361	47	3.152	1.150	63	10.687	3.852

$K = 1 \div \cos^3 \alpha = \sec^3 \alpha$; $K' = \tan^2 \alpha$

Outside and Pitch Diameters of Standard Involute-form Milling Cutters

Normal Diametral Pitch, P_n	Outside Dia., D_o	Pitch Dia., D_c	$Q = P_n D_c$	Normal Diametral Pitch, P_n	Outside Dia., D_o	Pitch Dia., D_c	$Q = P_n D_c$	Normal Diametral Pitch, P_n	Outside Dia., D_o	Pitch Dia., D_c	$Q = P_n D_c$
1	8.500	6.18	6.18	6	3.125	2.76	16.56	20	2.000	1.89	37.80
1¼	7.750	5.70	7.12	7	2.875	2.54	17.78	24	1.750	1.65	39.60
1½	7.000	5.46	8.19	8	2.875	2.61	20.88	28	1.750	1.67	46.76
1¾	6.500	5.04	8.82	9	2.750	2.50	22.50	32	1.750	1.68	53.76
2	5.750	4.60	9.20	10	2.375	2.14	21.40	36	1.750	1.69	60.84
2½	5.750	4.83	12.08	12	2.250	2.06	24.72	40	1.750	1.70	68.00
3	4.750	3.98	11.94	14	2.125	1.96	27.44	48	1.750	1.70	81.60
4	4.250	3.67	14.68	16	2.125	1.98	31.68	..	…	…	…
5	3.750	3.29	16.45	18	2.000	1.87	33.66	..	…	…	…

Pitch diameters shown in the table are computed from the formula: $D_c = D_o - 2(1.57 \div P_n)$. This same formula may be used to compute the pitch diameter of a non-standard outside diameter cutter when the normal diametral pitch P_n and the outside diameter D_o are known.

To simplify calculations, Formula (1) may be written as follows:

$$N' = NK + QK' \qquad (2)$$

In this formula, K, K' and Q are constants obtained from the tables on page 2078.

Example: Helix angle = 30 degrees; number of teeth in helical gear = 15; and normal diametral pitch = 20. From the tables on page 2078 K, K', and Q are, respectively, 1.540, 0.333, and 37.80.

$$\begin{aligned} N' &= (15 \times 1.540) + (37.80 \times 0.333) = 23.10 + 12.60 \\ &= 35.70, \text{ say, } 36 \end{aligned}$$

Hence, from page 2023 select a number 3 cutter. Had the approximate formula been used, then a number 5 cutter would have been selected on the basis of $N' = 23$.

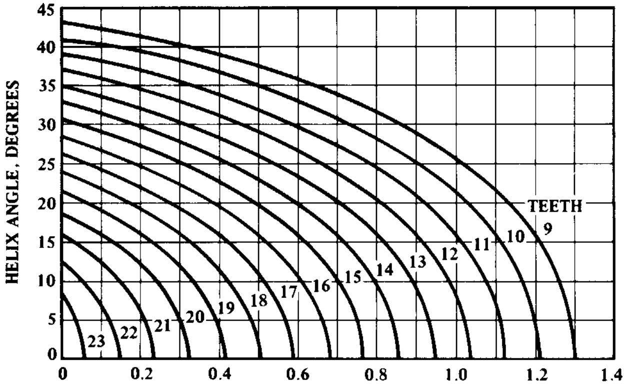
Milling the Helical Teeth.—The teeth of a helical gear are proportioned from the normal pitch and not the circular pitch. The whole depth of the tooth can be found by dividing 2.157 by the normal diametral pitch of the gear, which corresponds to the pitch of the cutter. The thickness of the tooth at the pitch line equals 1.571 divided by the normal diametral pitch. After a tooth space has been milled, the cutter should be prevented from dragging through it when being returned for another cut. This can be done by lowering the blank slightly, or by stopping the machine and turning the cutter to such a position that the teeth will not touch the work. If the gear has teeth coarser than 10 or 12 diametral pitch, it is well to take a roughing and a finishing cut. When pressing a helical gear blank on the arbor, it should be remembered that it is more likely to slip when being milled than a spur gear, because the pressure of the cut, being at an angle, tends to rotate the blank on the arbor.

Angular Position of Table: When cutting a helical gear on a milling machine, the table is set to the helix angle of the gear. If the lead of the helical gear is known, but not the helix angle, the helix angle is determined by multiplying the pitch diameter of the gear by 3.1416 and dividing this product by the lead; the result is the tangent of the lead angle which may be obtained from trigonometric tables or a calculator.

American National Standard Fine-Pitch Teeth For Helical Gears.—This Standard, ANSI B6.7-1977, provides a 20-degree tooth form for both spur and helical gears of 20 diametral pitch and finer. Formulas for tooth parts are given on page 2008.

Enlargement of Helical Pinions of 20-Degree Normal Pressure Angle: Formula (4) and the accompanying graph are based on the use of hobs having sharp corners at their top lands. Pinions cut by shaper cutters may not require as much modification as indicated by Formula (4) or the graph. The number 2.1 appearing in (4) results from the use of a standard tooth thickness rack having an addendum of $1.05/P_n$ which will start contact at a roll angle 5 degrees above the base radius. The roll angle of 5 degrees is also reflected in Formula (4).

To avoid undercutting of the teeth and to provide more favorable contact conditions near the base of the tooth, it is recommended that helical pinions with less than 24 teeth be enlarged in accordance with the following graph and formulas. As with enlarged spur pinions, when an enlarged helical pinion is used it is necessary either to reduce the diameter of the mating gear or to increase the center distance. In the formulas that follow, ϕ_n = normal pressure angle; ϕ_t = transverse pressure angle; ψ = helix angle of pinion; P_n = normal diametral pitch; P_t = transverse diametral pitch; d = pitch diameter of pinion; d_o = outside diameter of enlarged pinion, K_h = enlargement for full depth pinions of 1 normal diametral pitch; and n = number of teeth in pinion.



ENLARGEMENT K_h, IN INCHES, FOR 1 DIAMETRAL PITCH, 20° PRESSURE ANGLE PINIONS

To eliminate the need for making the calculations indicated in Formulas (3) and (4), the accompanying graph may be used to obtain the value of K_h directly for full-depth pinions of 20-degree normal pressure angle.

$$P_t = P_n \cos\psi \quad (1)$$

$$d = n \div P_t \quad (2)$$

$$tan \psi_t = \tan\phi_n \div \cos\psi \quad (3)$$

$$K_h = 2.1 - \frac{n}{\cos\psi}(\sin\phi_t - \cos\phi_t \tan 5°) \sin\phi_t \quad (4)$$

$$d_o = d + \frac{2 + K_h}{P_n} \quad (5)$$

Example: Find the outside diameter of a helical pinion having 12 teeth, 32 normal diametral pitch, 20-degree pressure angle, and 18-degree helix angle.

$$P_t = P_n \cos\psi = 32 \cos 18° = 32 \times 0.95106 = 30.4339$$

$$d = n \div P_t = 12 \div 30.4339 = 0.3943 \text{ inch}$$

$$K_h = 0.851 \text{ (from graph)}$$

$$d_o = 0.3943 + \frac{2 + 0.851}{32} = 0.4834$$

Center Distance at Which Modified Mating Helical Gears Will Mesh with no Backlash.—If the helical pinion in the previous example on page 2080 had been made to standard dimensions, that is, not enlarged, and was in tight mesh with a standard 24-tooth mating gear, the center distance for tight mesh could be calculated from the formula on page 2008:

$$C = \frac{n + N}{2P_n \cos\psi} = \frac{12 + 24}{2 \times 32 \times \cos 18^\circ} = 0.5914 \text{ inch} \quad (1)$$

However, if the pinion is enlarged as in the example and meshed with the same standard 24-tooth gear, then the center distance for tight mesh will be increased. To calculate the new center distance, the following formulas and calculations are required:

First, calculate the transverse pressure angle ϕ_t using Formula (2):

$$\tan\phi_t = \tan\phi_n \div \cos\psi = \tan 20^\circ \div \cos 18^\circ = 0.38270 \quad (2)$$

and from a calculator the angle ϕ_t is found to be 20° 56′ 30″. In the table on page 98, inv ϕ_t is found to be 0.017196, and the cosine from a calculator as 0.93394.

Next, using Formula (3), calculate the pressure angle ϕ at which the gears are in tight mesh:

$$\text{inv}\,\phi = \text{inv}\,\phi_t + \frac{(t_{nP} + t_{nG}) - \pi}{n + N} \quad (3)$$

In this formula, the value for t_{nP} for 1 diametral pitch is that found in Table 3c on page 2025, for a 12-tooth pinion, in the fourth column: 1.94703. The value of t_{nG} for 1 diametral pitch for a standard gear is always 1.5708.

$$\text{inv}\,\phi = 0.017196 + \frac{(1.94703 + 1.5708) - \pi}{12 + 24} = 0.027647$$

From the table on page 99, or a calculator, 0.027647 is the involute of 24° 22′ 7″ and the cosine corresponding to this angle is 0.91091.

Finally, using Formula (4), the center distance for tight mesh, C' is found:

$$C' = \frac{C\cos\phi_t}{\cos\phi} = \frac{0.5914 \times 0.93394}{0.91091} = 0.606 \text{ inch} \quad (4)$$

Change-gears for Helical Gear Hobbing.—If a gear-hobbing machine is not equipped with a differential, there is a fixed relation between the index and feed gears and it is necessary to compensate for even slight errors in the index gear ratio, to avoid excessive lead errors. This may be done readily (as shown by the example to follow) by modifying the ratio of the feed gears slightly, thus offsetting the index gear error and making very accurate leads possible.

Machine Without Differential: The formulas which follow may be applied in computing the index gear ratio.

R = index-gear ratio
L = lead of gear, inches
F = feed per gear revolution, inch
K = machine constant
T = number of threads on hob
N = number of teeth on gear
P_n = normal diametral pitch
P_{nc} = normal circular pitch
A = helix angle, relative to axis
M = feed gear constant

$$R = \frac{L \div F}{(L \div F) \pm 1} \times \frac{KT}{N} = \frac{L}{L \pm F} \times \frac{KT}{N} = \frac{\text{Driving gear sizes}}{\text{Driven gear sizes}} \quad (1)$$

Use minus (−) sign in Formulas (1) and (2) when gear and hob are the same "hand" and plus (+) sign when they are of opposite hand; when *climb* hobbing is to be used, reverse this rule.

$$R = \frac{KT}{N \pm \frac{P_n \times \sin A \times F}{\pi}} = \frac{KT}{N \pm \frac{\sin A \times F}{P_{nc}}} \quad (2)$$

$$\text{Ratio of feed gears} = \frac{F}{M} \qquad F = \frac{L(NR - KT)}{NR} \tag{3}$$

$$L = \frac{FNR}{(NR - KT)} = \text{lead obtained with available index and feed gears} \tag{4}$$

Note: If gear and hob are of opposite hand, then in Formulas (3) and (4) change $(NR - KT)$ to $(KT - NR)$. This change is also made if gear and hob are of same hand but *climb* hobbing is used.

Example: A right-hand helical gear with 48 teeth of 10 normal diametral pitch, has a lead of 44.0894 inches. The feed is to be 0.035 inch, with whatever slight adjustment may be necessary to compensate for the error in available index gears. $K = 30$ and $M = 0.075$. A single-thread right-hand hob is to be used.

$$R = \frac{44.0894}{44.0894 - 0.035} \times \frac{30 \times 1}{48} = 0.62549654$$

Using the method of *Conjugate Fractions* beginning on page 14, several suitable ratios close to 0.62549654 were found. One of these, (34 × 53)/(43 × 67) = 0.625477264839 will be used as the index ratio. Other usable ratios and their decimal values were found to be as follows:

$$\frac{32 \times 38}{27 \times 72} = 0.6255144 \qquad \frac{27 \times 42}{42 \times 37} = 0.62548263$$

$$\frac{44 \times 29}{34 \times 60} = 0.6254902 \qquad \frac{26 \times 97}{96 \times 42} = 0.62549603$$

$$\frac{20 \times 41}{23 \times 57} = 0.62547674$$

Index ratio error = 0.62549654 − 0.62547726 = 0.00001928.

Now use Formula (3) to find slight change required in rate of feed. This change compensates sufficiently for the error in available index gears.

Change in Feed Rate: Insert in Formula (3) obtainable index ratio.

$$F = \frac{44.0894 \times (48 \times 0.62547726 - 30)}{48 \times 0.62547726} = 0.0336417$$

$$\text{Modified feed gear ratio} = \frac{F}{M} = \frac{0.0336417}{0.075} = 0.448556$$

$$\text{Log } 0.448556 = \bar{1}.651817 \qquad \text{log of reciprocal} = 0.348183$$

To find close approximation to modified feed gear ratio, proceed as in finding suitable gears for index ratio, thus obtaining $\frac{106}{71} \times \frac{112}{75}$. Inverting, modified feed gear ratio = $\frac{71}{106} \times \frac{75}{112} = 0.448534$.

Modified feed F = obtainable modified feed ratio × M = 0.448534 × 0.075 = 0.03364 inch. If the feed rate is not modified, even a small error in the index gear ratio may result in an excessive lead error.

Checking Accuracy of Lead: The modified feed and obtainable index ratio are inserted in Formula (4). Desired lead = 44.0894 inches. Lead obtained = 44.087196 inches; hence the computed error = 44.0894 − 44.087196 = 0.002204 inch or about 0.00005 inch per inch of lead.

Machine with Differential: If a machine is equipped with a differential, the *lead gears* are computed in order to obtain the required helix angle and lead. The instructions of the hobbing machine manufacturer should be followed in computing the lead gears, because the ratio formula is affected by the location of the differential gears. If these gears are *ahead* of the index gears, the lead gear ratio is not affected by a change in the number of teeth to be cut (see Formula (5)); hence, the same lead gears are used when, for example, a gear and pinion are cut on the same machine. In the formulas which follow, the notation is the same as previously given, with these exceptions: R_d = lead gear ratio for machine with differential; P_a = axial or linear pitch of helical gear = distance from center of one tooth to center of next tooth measured parallel to gear axis = total lead $L \div$ number of teeth N.

$$R_d = \frac{P_a \times T}{K} = \frac{L \times T}{N \times K} = \frac{\pi \times \operatorname{cosec} A \times T}{P_n \times K} = \frac{\text{Driven gear sizes}}{\text{Driving gear sizes}} \qquad (5)$$

The number of hob threads T is included in the formula because double-thread hobs are used sometimes, especially for roughing in order to reduce the hobbing time. Lead gears having a ratio sufficiently close to the required ratio may be determined by using the table of gear ratio logarithms as previously described in connection with the non-differential type of machine. When using a machine equipped with a differential, the effect of a lead-gear ratio error upon the lead of the gear is small in comparison with the effect of an index gear error when using a non-differential type of machine. The lead obtained with a given or obtainable lead gear ratio may be determined by the following formula: $L = (R_d NK) \div T$. In this formula, R_d represents the ratio obtained with available gears. If the given lead is 44.0894 inches, as in the preceding example, then the desired ratio as obtained with Formula (5) would be 0.9185292 if $K = 1$. Assume that the lead gears selected by using logs of ratios have a ratio of 0.9184704; then this ratio error of 0.0000588 would result in a computed lead error of only 0.000065 inch per inch.

Formula (5), as mentioned, applies to machines having the differential located *ahead* of the index gears. If the differential is located after the index gears, it is necessary to change lead gears whenever the index gears are changed for hobbing a different number of teeth, as indicated by the following formula which gives the lead gear ratio. In this formula, D = pitch diameter.

$$R_d = \frac{L \times T}{K} = \frac{D \times \pi \times T}{K \times \tan A} = \frac{\text{Driven gear sizes}}{\text{Driving gear sizes}} \qquad (6)$$

General Remarks on Helical Gear Hobbing.—In cutting teeth having large angles, it is desirable to have the direction of helix of the hob the same as the direction of helix of the gear, or in other words, the gear and the hob of the same "hand." Then the direction of the cut will come against the movement of the blank. At ordinary angles, however, one hob will cut both right- and left-hand gears. In setting up the hobbing machine for helical gears, care should be taken to see that the vertical feed does not trip until the machine has been stopped or the hob has fed down past the finished gear.

Herringbone Gears

Double helical or herringbone gears are commonly used in parallel-shaft transmissions, especially when a smooth, continuous action (due to the gradual overlapping engagement of the teeth) is essential, as in high-speed drives where the pitch-line velocity may range from about 1000 to 3000 feet per minute in commercial gearing and up to 12,000 feet per minute or higher in more specialized installations. These relatively high speeds are encountered in marine reduction gears, in certain speed-reducing and speed-increasing units, and in various other transmissions, particularly in connection with steam turbine and electric motor drives.

General Classes of Helical Gear Problems.—There are two general classes of problems. In one, the problem is to design gears capable of transmitting a given amount of power at a given speed, safely and without excessive wear; hence, the required proportions must be determined. In the second, the proportions and speed are known and the power-transmitting capacity is required. The first is the more difficult and the more common problem.

Causes of Herringbone Gear Failures.—Where failure occurs in a herringbone gear transmission, it is rarely due to tooth breakage but usually to excessive wear or sub-surface failures, such as pitting and spalling; hence, it is common practice to base the design of such gears upon durability, or upon tooth pressures which are within the allowable limits for wear. In this connection, it seems to have been well established by tests of both spur gears and herringbone gears, that there is a critical surface pressure value for teeth having given physical properties and coefficient of friction. According to these tests, pressures above the critical value result in rapid wear and a short gear life, whereas when pressures are below the critical, wear is negligible. The yield point or endurance limit of the material marks the critical loading point, and in practical designing a reasonable factor of safety would, of course, be employed.

Planetary Gearing

Planetary or epicyclic gearing provides means of obtaining a compact design of transmission, with driving and driven shafts in line, and a large speed reduction when required. Typical arrangements of planetary gearing are shown by the following diagrams which are accompanied by speed ratio formulas. When planetary gears are arranged as shown by Fig. 5, 6, 9 and 12, the speed of the follower relative to the driver is increased, whereas Fig. 7, 8, 10, and 11 illustrate speed-reducing mechanisms.

Direction of Rotation.—In using the following formulas, if the final result is preceded by a minus sign (negative), this indicates that the driver and follower will rotate in opposite directions; otherwise, both will rotate in the same direction.

Compound Drive.—The formulas accompanying Figs. 19 through 22 are for obtaining the speed ratios when there are *two* driving members rotating at different speeds. For example, in Fig. 19, the central shaft with its attached link is one driver. The internal gear z, instead of being fixed, is also rotated. In Fig. 22, if $z = 24$, $B = 60$ and $S = 3\frac{1}{2}$, with both drivers rotating in the same direction, then $F = 0$, thus indicating, in this case, the point where a larger value of S will reverse follower rotation.

Planetary Bevel Gears.—Two forms of planetary gears of the bevel type are shown in Fig. 23 and 24. The planet gear in Fig. 23 rotates about a fixed bevel gear at the center of which is the driven shaft. Fig. 24 illustrates the Humpage reduction gear. This is sometimes referred to as cone-pulley back-gearing because of its use within the cone pulleys of certain types of machine tools.

D = rotation of *driver* per revolution of follower or driven member

F = rotation of *follower* or driven member per revolution of driver. (In Figs. 1 through 4 F = rotation of planet type follower about its axis.)

A = size of driving gear (use either number of teeth or pitch diameter). Note: When follower derives its motion both from A and from a secondary driving member, A = size of *initial* driving gear, and formula gives speed relationship between A and follower.

B = size of *driven gear or follower* (use either pitch diameter or number of teeth)

C = size of *fixed gear* (use either pitch diameter or number of teeth)

x = size of *planet gear* as shown by diagram (use either pitch diameter or number of teeth)

y = size of *planet gear* as shown by diagram (use either pitch diameter or number of teeth)

z = size of secondary or *auxiliary driving gear*, when follower derives its motion from two driving members

S = rotation of *secondary driver*, per revolution of *initial driver*. S is negative when secondary and initial drivers rotate in opposite directions. (Formulas in which S is used, give speed relationship between follower and the initial driver.)

Note: In all cases, if D is known, $F = 1 \div D$, or, if F is known, $D = 1 \div F$.

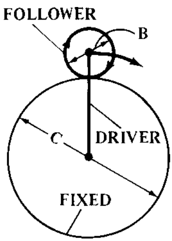


Fig. 1.

$$F = 1 + \frac{C}{B}$$

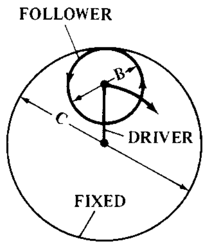


Fig. 2.

$$F = 1 - \frac{C}{B}$$

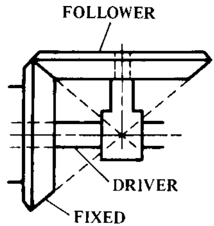


Fig. 3.

$$F = \frac{C}{B}$$

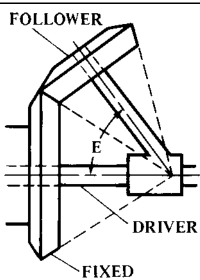


Fig. 4.

$$F = \cos E + \frac{C}{B}$$

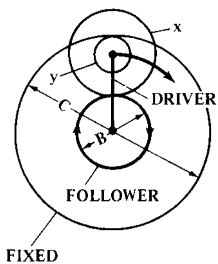


Fig. 5.

$$F = 1 + \frac{x \times C}{y \times B}$$

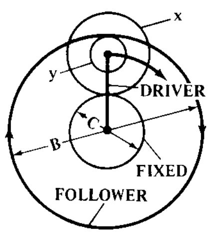


Fig. 6.

$$F = 1 + \frac{y \times C}{x \times B}$$

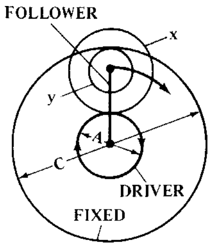


Fig. 7.

$$D = 1 + \frac{x \times C}{y \times A}$$

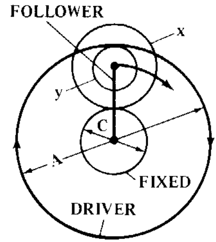


Fig. 8.

$$D = 1 + \frac{y \times C}{x \times A}$$

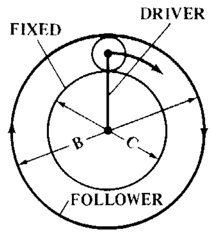


Fig. 9.

$$F = 1 + \frac{C}{B}$$

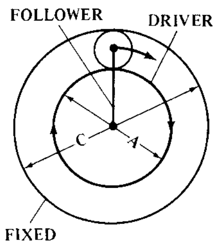


Fig. 10.

$$D = 1 + \frac{C}{A}$$

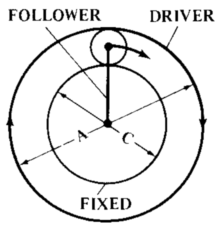


Fig. 11.

$$D = 1 + \frac{C}{A}$$

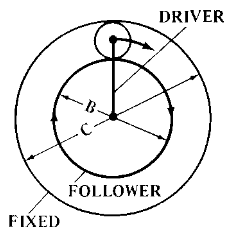


Fig. 12.

$$F = 1 + \frac{C}{B}$$

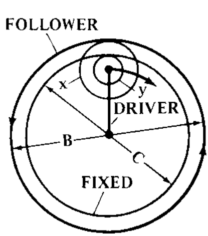


Fig. 13.

$$F = 1 - \left(\frac{C \times x}{y \times B}\right)$$

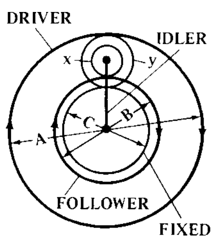


Fig. 14.

$$D = \frac{1 + \frac{C}{A}}{1 - \left(\frac{C \times x}{y \times B}\right)}$$

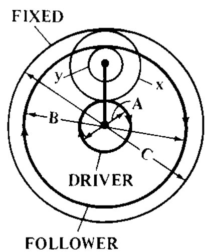


Fig. 15.

$$D = \frac{1 + \frac{C}{A}}{1 - \left(\frac{C \times y}{x \times B}\right)}$$

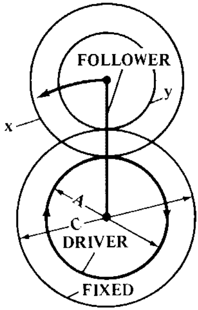


Fig. 16.

$$D = 1 - \left(\frac{C \times x}{y \times A}\right)$$

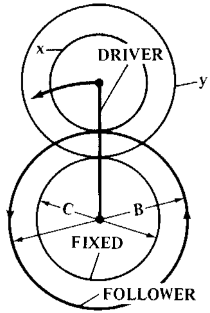


Fig. 17.

$$F = 1 - \left(\frac{C \times x}{y \times B}\right)$$

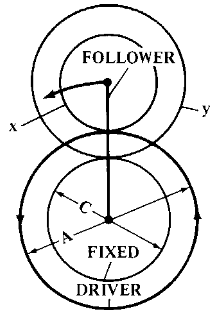


Fig. 18.

$$D = 1 - \left(\frac{C \times x}{y \times A}\right)$$

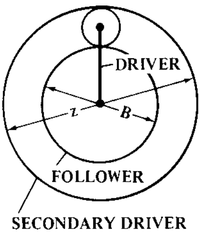


Fig. 19.

$$F = 1 + \frac{z \times (1 - S)}{B}$$

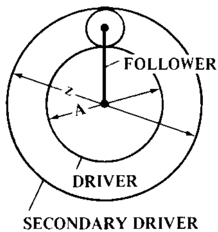


Fig. 20.

$$D = \frac{A + z}{A + (S \times z)}$$

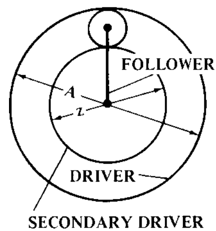


Fig. 21.

$$D = \frac{A + z}{A + (S \times z)}$$

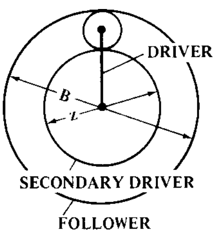


Fig. 22.

$$F = 1 + \frac{z \times (1 - S)}{B}$$

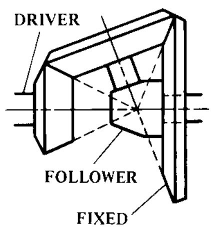


Fig. 23.

$$D = 1 + \frac{C}{A}$$

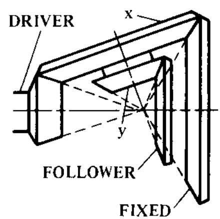


Fig. 24.

$$D = \frac{1 + \frac{C}{A}}{1 - \left(\frac{C \times y}{x \times B}\right)}$$

Ratchet Gearing

Ratchet gearing may be used to transmit intermittent motion, or its only function may be to prevent the ratchet wheel from rotating backward. Ratchet gearing of this latter form is commonly used in connection with hoisting mechanisms of various kinds, to prevent the hoisting drum or shaft from rotating in a reverse direction under the action of the load.

Types of Ratchet Gearing

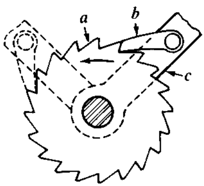


Fig. a.

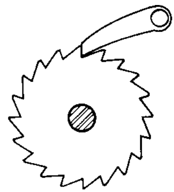


Fig. b.

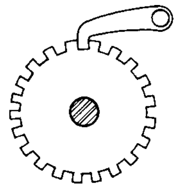


Fig. c.

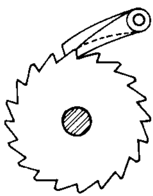


Fig. d.

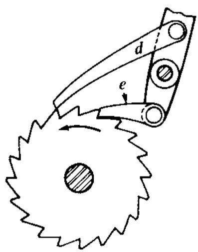


Fig. e.

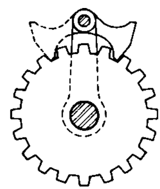


Fig. f.

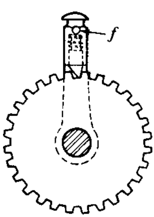


Fig. g.

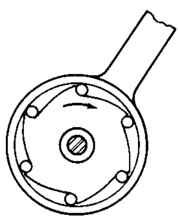


Fig. h.

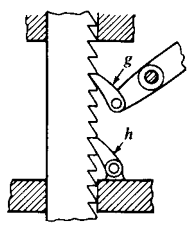


Fig. i.

Ratchet gearing in its simplest form consists of a toothed ratchet wheel *a* (see Fig. a), and a pawl or detent *b*, and it may be used to transmit intermittent motion or to prevent relative motion between two parts except in one direction. The pawl *b* is pivoted to lever *c* which, when given an oscillating movement, imparts an intermittent rotary movement to ratchet wheel *a*. Fig. b illustrates another application of the ordinary ratchet and pawl mechanism. In this instance, the pawl is pivoted to a stationary member and its only function is to prevent the ratchet wheel from rotating backward. With the stationary design, illustrated at

Fig. c, the pawl prevents the ratchet wheel from rotating in either direction, so long as it is in engagement with the wheel.

The principle of *multiple-pawl ratchet gearing* is illustrated at Fig. d, which shows the use of two pawls. One of these pawls is longer than the other, by an amount equal to one-half the pitch of the ratchet-wheel teeth, so that the practical effect is that of reducing the pitch one-half. By placing a number of driving pawls side byside and proportioning their lengths according to the pitch of the teeth, a very fine feed can be obtained with a ratchet wheel of comparatively coarse pitch.

This method of obtaining a fine feed from relatively coarse-pitch ratchets may be preferable to the use of single ratchets of fine pitch which, although providing the feed required, may have considerably weaker teeth.

The type of ratchet gearing shown at Fig. e is sometimes employed to impart a rotary movement to the ratchet wheel for both the forward and backward motions of the lever to which the two pawls are attached.

A simple form of *reversing ratchet* is illustrated at Fig. f. The teeth of the wheel are so shaped that either side may be used for driving by simply changing the position of the double-ended pawl, as indicated by the full and dotted lines.

Another form of reversible ratchet gearing for shapers is illustrated at Fig. g. The pawl, in this case, instead of being a pivoted latch, is in the form of a plunger which is free to move in the direction of its axis, but is normally held into engagement with the ratchet wheel by a small spring. When the pawl is lifted and turned one-half revolution, the driving face then engages the opposite sides of the teeth and the ratchet wheel is given an intermittent rotary motion in the opposite direction.

The *frictional type* of ratchet gearing differs from the designs previously referred to, in that there is no positive engagement between the driving and driven members of the ratchet mechanism, the motion being transmitted by frictional resistance. One type of frictional ratchet gearing is illustrated at Fig. h. Rollers or balls are placed between the ratchet wheel and an outer ring which, when turned in one direction, causes the rollers or balls to wedge between the wheel and ring as they move up the inclined edges of the teeth.

Fig. i illustrates one method of utilizing ratchet gearing for moving the driven member in a straight line, as in the case of a lifting jack. The pawl g is pivoted to the operating lever of the jack and does the lifting, whereas the pawl h holds the load while the lifting pawl g is being returned preparatory to another lifting movement.

Shape of Ratchet Wheel Teeth.—When designing ratchet gearing, it is important to so shape the teeth that the pawl will remain in engagement when a load is applied. The faces of the teeth which engage the end of the pawl should be in such relation with the center of the pawl pivot that a line perpendicular to the face of the engaging tooth will pass somewhere between the center of the ratchet wheel and the center of the pivot about which the pawl swings. This is true if the pawl *pushes* the ratchet wheel, or if the ratchet wheel *pushes* the pawl. However, if the pawl *pulls* the ratchet wheel or if the ratchet wheel *pulls* the pawl, the perpendicular from the face of the ratchet teeth should fall outside the pawl pivot center. Ratchet teeth may be either cut by a milling cutter having the correct angle, or hobbed in a gear-hobbing machine by the use of a special hob.

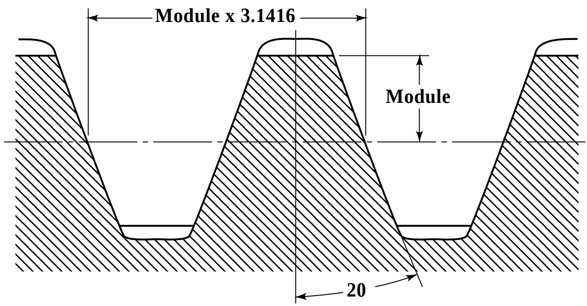
Pitch of Ratchet Wheel Teeth.—The pitch of ratchet wheels used for holding suspended loads may be calculated by the following formula, in which P = circular pitch, in inches, measured at the outside circumference; M = turning moment acting upon the ratchet wheel shaft, in inch-pounds; L = length of tooth face, in inches (thickness of ratchet gear); S = safe stress (for steel, 2500 pounds per square inch when subjected to shock, and 4000 pounds per square inch when not subjected to shock); N = number of teeth in ratchet wheel; F = a factor the value of which is 50 for ratchet gears with 12 teeth or less, 35 for gears having from 12 to 20 teeth, and 20 for gears having over 20 teeth:

$$P = \sqrt{\frac{FM}{LSN}}$$

This formula has been used in the calculation of ratchet gears for crane design.

Gear Design Based upon Module System.—The *module* of a gear is equal to the pitch diameter divided by the number of teeth, whereas *diametral pitch* is equal to the number of teeth divided by the pitch diameter. The module system (see accompanying table and diagram) is in general use in countries that have adopted the metric system; hence, the term module is usually understood to mean the pitch diameter *in millimeters* divided by the number of teeth. The module system, however, may also be based on inch measurements and then it is known as the English module to avoid confusion with the metric module. Module is an actual dimension, whereas diametral pitch is only a ratio. Thus, if the pitch diameter of a gear is 50 millimeters and the number of teeth 25, the module is 2, which means that there are 2 millimeters of pitch diameter for each tooth. The table *Tooth Dimensions Based Upon Module System* shows the relation among module, diametral pitch, and circular pitch.

German Standard Tooth Form for Spur and Bevel Gears *DIN 867*



The flanks or sides are straight (involute system) and the pressure angle is 20 degrees. The shape of the root clearance space and the amount of clearance depend upon the method of cutting and special requirements. The amount of clearance may vary from 0.1 × module to 0.3 × module.

To Find	Module Known	Circular Pitch Known
Addendum	Equals module	0.31823 × Circular pitch
Dedendum	1.157 × module* 1.167 × module**	0.3683 × Circular pitch* 0.3714 × Circular pitch**
Working Depth	2 × module	0.6366 × Circular pitch
Total Depth	2.157 × module* 2.167 × module**	0.6866 × Circular pitch* 0.6898 × Circulate pitch**
Tooth Thickness on Pitch Line	1.5708 × module	0.5 × Circular pitch

Formulas for dedendum and total depth, marked (*) are used when clearance equals 0.157 × module. Formulas marked (**) are used when clearance equals one-sixth module. It is common practice among American cutter manufacturers to make the clearance of metric or module cutters equal to 0.157 × module.

Tooth Dimensions Based Upon Module System

Module, *DIN* Standard Series	Equivalent Diametral Pitch	Circular Pitch		Addendum, Millimeters	Dedendum, Millimeters[a]	Whole Depth,[a] Millimeters	Whole Depth,[b] Millimeters
		Millimeters	Inches				
0.3	84.667	0.943	0.0371	0.30	0.35	0.650	0.647
0.4	63.500	1.257	0.0495	0.40	0.467	0.867	0.863
0.5	50.800	1.571	0.0618	0.50	0.583	1.083	1.079
0.6	42.333	1.885	0.0742	0.60	0.700	1.300	1.294
0.7	36.286	2.199	0.0865	0.70	0.817	1.517	1.510
0.8	31.750	2.513	0.0989	0.80	0.933	1.733	1.726
0.9	28.222	2.827	0.1113	0.90	1.050	1.950	1.941
1	25.400	3.142	0.1237	1.00	1.167	2.167	2.157
1.25	20.320	3.927	0.1546	1.25	1.458	2.708	2.697
1.5	16.933	4.712	0.1855	1.50	1.750	3.250	3.236
1.75	14.514	5.498	0.2164	1.75	2.042	3.792	3.774
2	12.700	6.283	0.2474	2.00	2.333	4.333	4.314
2.25	11.289	7.069	0.2783	2.25	2.625	4.875	4.853
2.5	10.160	7.854	0.3092	2.50	2.917	5.417	5.392
2.75	9.236	8.639	0.3401	2.75	3.208	5.958	5.932
3	8.466	9.425	0.3711	3.00	3.500	6.500	6.471
3.25	7.815	10.210	0.4020	3.25	3.791	7.041	7.010
3.5	7.257	10.996	0.4329	3.50	4.083	7.583	7.550
3.75	6.773	11.781	0.4638	3.75	4.375	8.125	8.089
4	6.350	12.566	0.4947	4.00	4.666	8.666	8.628
4.5	5.644	14.137	0.5566	4.50	5.25	9.750	9.707
5	5.080	15.708	0.6184	5.00	5.833	10.833	10.785
5.5	4.618	17.279	0.6803	5.50	6.416	11.916	11.864
6	4.233	18.850	0.7421	6.00	7.000	13.000	12.942
6.5	3.908	20.420	0.8035	6.50	7.583	14.083	14.021
7	3.628	21.991	0.8658	7.	8.166	15.166	15.099
8	3.175	25.132	0.9895	8.	9.333	17.333	17.256
9	2.822	28.274	1.1132	9.	10.499	19.499	19.413
10	2.540	31.416	1.2368	10.	11.666	21.666	21.571
11	2.309	34.558	1.3606	11.	12.833	23.833	23.728
12	2.117	37.699	1.4843	12.	14.000	26.000	25.884
13	1.954	40.841	1.6079	13.	15.166	28.166	28.041
14	1.814	43.982	1.7317	14.	16.332	30.332	30.198
15	1.693	47.124	1.8541	15.	17.499	32.499	32.355
16	1.587	50.266	1.9790	16.	18.666	34.666	34.512
18	1.411	56.549	2.2263	18.	21.000	39.000	38.826
20	1.270	62.832	2.4737	20.	23.332	43.332	43.142
22	1.155	69.115	2.7210	22.	25.665	47.665	47.454
24	1.058	75.398	2.9685	24.	28.000	52.000	51.768
27	0.941	84.823	3.339	27.	31.498	58.498	58.239
30	0.847	94.248	3.711	30.	35.000	65.000	64.713
33	0.770	103.673	4.082	33.	38.498	71.498	71.181
36	0.706	113.097	4.453	36.	41.998	77.998	77.652
39	0.651	122.522	4.824	39.	45.497	84.497	84.123
42	0.605	131.947	5.195	42.	48.997	90.997	90.594
45	0.564	141.372	5.566	45.	52.497	97.497	97.065
50	0.508	157.080	6.184	50.	58.330	108.330	107.855
55	0.462	172.788	6.803	55.	64.163	119.163	118.635
60	0.423	188.496	7.421	60.	69.996	129.996	129.426
65	0.391	204.204	8.040	65.	75.829	140.829	140.205
70	0.363	219.911	8.658	70.	81.662	151.662	150.997
75	0.339	235.619	9.276	75.	87.495	162.495	161.775

[a] Dedendum and total depth when clearance = 0.1666 × module, or one-sixth module.
[b] Total depth equivalent to American standard full-depth teeth. (Clearance = 0.157 × module.)

Rules for Module System of Gearing

To Find	Rule
Metric Module	*Rule 1:* To find the metric module, divide the pitch diameter in millimeters by the number of teeth. *Example 1:* The pitch diameter of a gear is 200 millimeters and the number of teeth, 40; then $\text{Module} = \frac{200}{40} = 5$ *Rule 2:* Multiply circular pitch in millimeters by 0.3183. *Example 2:* (Same as Example 1. Circular pitch of this gear equals 15.708 millimeters.) $\text{Module} = 15.708 \times 0.3183 = 5$ *Rule 3:* Divide outside diameter in millimeters by the number of teeth plus 2.
English Module	*Note:* The module system is usually applied when gear dimensions are expressed in millimeters, but module may also be based on inch measurements. *Rule:* To find the English module, divide pitch diameter in inches by the number of teeth. *Example:* A gear has 48 teeth and a pitch diameter of 12 inches. $\text{Module} = \frac{12}{48} = \frac{1}{4}$ module or 4 diametral pitch
Metric Module Equivalent to Diametral Pitch	*Rule:* To find the metric module equivalent to a given diametral pitch, divide 25.4 by the diametral pitch. *Example:* Determine metric module equivalent to 10 diameteral pitch. $\text{Equivalent module} = \frac{25.4}{10} = 2.54$ *Note:* The nearest standard module is 2.5.
Diametral Pitch Equivalent to Metric Module	*Rule:* To find the diametral pitch equivalent to a given module, divide 25.4 by the module. (25.4 = number of millimeters per inch.) *Example:* The module is 12; determine equivalent diametral pitch. $\text{Equivalent diametral pitch} = \frac{25.4}{12} = 2.117$ *Note:* A diametral pitch of 2 is the nearest *standard* equivalent.
Pitch Diameter	*Rule:* Multiply number of teeth by module. *Example:* The metric module is 8 and the gear has 40 teeth; then $D = 40 \times 8 = 320 \text{ millimeters} = 12.598 \text{ inches}$
OutsideDiameter	*Rule:* Add 2 to the number of teeth and multiply sum by the module. *Example:* A gear has 40 teeth and module is 6. Find outside or blank diameter. $\text{Outside diameter} = (40 + 2) \times 6 = 252 \text{ millimeters}$

For tooth dimensions, see table *Tooth Dimensions Based Upon Module System*; also formulas in *German Standard Tooth Form for Spur and Bevel Gears DIN 867*.

Equivalent Diametral Pitches, Circular Pitches, and Metric Modules

Commonly Used Pitches and Modules in Bold Type

Diametral Pitch	Circular Pitch, Inches	Module Millimeters	Diametral Pitch	Circular Pitch, Inches	Module Millimeters	Diametral Pitch	Circular Pitch, Inches	Module Millimeters
1/2	6.2832	50.8000	2.2848	**1 3/8**	11.1170	10.0531	**5/16**	2.5266
0.5080	6.1842	**50**	2.3091	1.3605	**11**	10.1600	0.3092	**2 1/2**
0.5236	**6**	48.5104	**2 1/2**	1.2566	10.1600	**11**	0.2856	2.3091
0.5644	5.5658	**45**	2.5133	**1 1/4**	10.1063	**12**	0.2618	2.1167
0.5712	**5 1/2**	44.4679	2.5400	1.2368	**10**	12.5664	**1/4**	2.0213
0.6283	**5**	40.4253	**2 3/4**	1.1424	9.2364	12.7000	0.2474	**2**
0.6350	4.9474	**40**	2.7925	**1 1/8**	9.0957	**13**	0.2417	1.9538
0.6981	**4 1/2**	36.3828	2.8222	1.1132	**9**	**14**	0.2244	1.8143
0.7257	4.3290	**35**	**3**	1.0472	8.4667	**15**	0.2094	1.6933
3/4	4.1888	33.8667	3.1416	**1**	8.0851	**16**	0.1963	1.5875
0.7854	**4**	32.3403	3.1750	0.9895	**8**	16.7552	**3/16**	1.5160
0.8378	**3 3/4**	30.3190	3.3510	**15/16**	7.5797	16.9333	0.1855	**1 1/2**
0.8467	3.7105	**30**	**3 1/2**	0.8976	7.2571	**17**	0.1848	1.4941
0.8976	**3 1/2**	28.2977	3.5904	7/8	7.0744	**18**	0.1745	1.4111
0.9666	**3 1/4**	26.2765	3.6286	0.8658	**7**	**19**	0.1653	1.3368
1	3.1416	25.4000	3.8666	**13/16**	6.5691	**20**	0.1571	1.2700
1.0160	3.0921	**25**	3.9078	0.8040	**6 1/2**	**22**	0.1428	1.1545
1.0472	**3**	24.2552	**4**	0.7854	6.3500	**24**	0.1309	1.0583
1.1424	**2 3/4**	22.2339	4.1888	**3/4**	6.0638	**25**	0.1257	1.0160
1 1/4	2.5133	20.3200	4.2333	0.7421	**6**	25.1328	**1/8**	1.0106
1.2566	**2 1/2**	20.2127	4.5696	**11/16**	5.5585	25.4000	0.1237	**1**
1.2700	2.4737	**20**	4.6182	0.6803	5 1/2	**26**	0.1208	0.9769
1.3963	**2 1/4**	18.1914	**5**	0.6283	5.0800	**28**	0.1122	0.9071
1.4111	2.2263	**18**	5.0265	**5/8**	5.0532	**30**	0.1047	0.8467
1 1/2	2.0944	16.9333	5.0800	0.6184	**5**	**32**	0.0982	0.7937
1.5708	**2**	16.1701	5.5851	**9/16**	4.5478	**34**	0.0924	0.7470
1.5875	1.9790	**16**	5.6443	0.5566	**4 1/2**	**36**	0.0873	0.7056
1.6755	**1 7/8**	15.1595	**6**	0.5236	4.2333	**38**	0.0827	0.6684
1.6933	1.8553	**15**	6.2832	**1/2**	4.0425	**40**	0.0785	0.6350
1 3/4	1.7952	14.5143	6.3500	0.4947	**4**	**42**	0.0748	0.6048
1.7952	**1 3/4**	14.1489	**7**	0.4488	3.6286	**44**	0.0714	0.5773
1.8143	1.7316	**14**	7.1808	**7/16**	3.5372	**46**	0.0683	0.5522
1.9333	**1 5/8**	13.1382	7.2571	0.4329	**3 1/2**	**48**	0.0654	0.5292
1.9538	1.6079	**13**	**8**	0.3927	3.1750	**50**	0.0628	0.5080
2	1.5708	12.7000	8.3776	**3/8**	3.0319	50.2656	**1/16**	0.5053
2.0944	**1 1/2**	12.1276	8.4667	0.3711	**3**	50.8000	0.0618	**1/2**
2.1167	1.4842	**12**	**9**	0.3491	2.8222	**56**	0.0561	0.4536
2 1/4	1.3963	11.2889	**10**	0.3142	2.5400	**60**	0.0524	0.4233

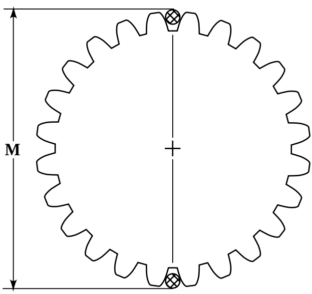
The module of a gear is the pitch diameter divided by the number of teeth. The module may be expressed in any units; but when no units are stated, it is understood to be in millimeters. The metric module, therefore, equals the pitch diameter in millimeters divided by the number of teeth. To find the metric module equivalent to a given diametral pitch, divide 25.4 by the diametral pitch. To find the diametral pitch equivalent to a given module, divide 25.4 by the module. (25.4 = number of millimeters per inch.)

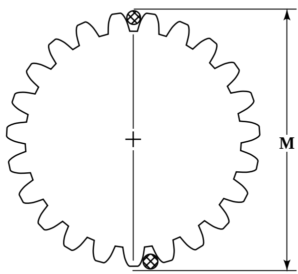
CHECKING GEAR SIZES

Checking Gear Size by Measurement Over Wires or Pins

The wire or pin method of checking gear sizes is accurate, easily applied, and especially useful in shops with limited inspection equipment. Two cylindrical wires or pins of predetermined diameter are placed in diametrically opposite tooth spaces (see diagram). If the gear has an odd number of teeth, the wires are located as nearly opposite as possible, as shown by the diagram at the right. The overall measurement *M* is checked by using any sufficiently accurate method of measurement. The value of measurement *M* when the pitch diameter is correct can be determined easily and quickly by means of the calculated values in the accompanying tables.

Measurements for Checking External Spur Gears when Wire Diameter Equals 1.728 Divided by Diametral Pitch.—Tables 1 and 2 give measurements *M*, in inches, for checking the pitch diameters of external spur gears of 1 diametral pitch. For any other diametral pitch, divide the measurement given in the table by whatever diametral pitch is required. The result shows what measurement *M* should be when the pitch diameter is correct *and there is no allowance for backlash.* The procedure for obtaining a given amount of backlash will be explained later. Tables 1 through 4 inclusive are based on wire sizes conforming to the Van Keuren standard. For external spur gears, the wire size equals 1.728 divided by the diametral pitch. The wire diameters for various diametral pitches will be found in the left-hand section of Table 5.





Even Number of Teeth: Table 1 is for even numbers of teeth. To illustrate the use of the table, assume that a spur gear has 32 teeth of 4 diametral pitch and a pressure angle of 20 degrees. Table 1 shows that the measurement for 1 diametral pitch is 34.4130; hence, for 4 diametral pitch, the measurement equals 34.4130 ÷ 4 = 8.6032 inches. This dimension is the measurement over the wires when the pitch diameter is correct, provided there is no allowance for backlash. The wire diameter here equals 1.728 ÷ 4 = 0.432 inch (Table 5).

Measurement for even numbers of teeth above 170 and not in Table 1 may be determined as shown by the following example: Assume that number of teeth = 240 and pressure angle = 14½ degrees; then, for 1 diametral pitch, figure at left of decimal point = given No. of teeth + 2 = 240 + 2 = 242. Figure at right of decimal point lies between decimal values given in table for 200 teeth and 300 teeth and is obtained by interpolation. Thus, 240 – 200 = 40 (change to 0.40); 0.5395 – 0.5321 = 0.0074 = difference between decimal values for 300 and 200 teeth; hence, decimal required = 0.5321 + (0.40 × 0.0074) = 0.53506. Total dimension = 242.53506 divided by the diametral pitch required.

Odd Number of Teeth: Table 2 is for odd numbers of teeth. Measurement for odd numbers above 171 and not in Table 2 may be determined as shown by the following example: Assume that number of teeth = 335 and pressure angle = 20 degrees; then, for 1 diametral pitch, figure at left of decimal point = given No. of teeth + 2 = 335 + 2 = 337. Figure at right of decimal point lies between decimal values given in table for 301 and 401 teeth. Thus, 335 − 301 = 34 (change to 0.34); 0.4565 − 0.4538 = 0.0027; hence, decimal required = 0.4538 + (0.34 × 0.0027) = 0.4547. Total dimension = 337.4547.

Table 1. Checking External Spur Gear Sizes by Measurement Over Wires

EVEN NUMBERS OF TEETH

Dimensions in table are for 1 diametral pitch and Van Keuren standard wire sizes. For any other diametral pitch, divide dimension in table by given pitch.

$$\text{Wire or pin diameter} = \frac{1.728}{\text{Diametral Pitch}}$$

No. of Teeth	Pressure Angle				
	14½°	17½°	20°	25°	30°
6	8.2846	8.2927	8.3032	8.3340	8.3759
8	10.3160	10.3196	10.3271	10.3533	10.3919
10	12.3399	12.3396	12.3445	12.3667	12.4028
12	14.3590	14.3552	14.3578	14.3768	14.4108
14	16.3746	16.3677	16.3683	16.3846	16.4169
16	18.3877	18.3780	18.3768	18.3908	18.4217
18	20.3989	20.3866	20.3840	20.3959	20.4256
20	22.4087	22.3940	22.3900	22.4002	22.4288
22	24.4172	24.4004	24.3952	24.4038	24.4315
24	26.4247	26.4060	26.3997	26.4069	26.4339
26	28.4314	28.4110	28.4036	28.4096	28.4358
28	30.4374	30.4154	30.4071	30.4120	30.4376
30	32.4429	32.4193	32.4102	32.4141	32.4391
32	34.4478	34.4228	34.4130	34.4159	34.4405
34	36.4523	36.4260	36.4155	36.4176	36.4417
36	38.4565	38.4290	38.4178	38.4191	38.4428
38	40.4603	40.4317	40.4198	40.4205	40.4438
40	42.4638	42.4341	42.4217	42.4217	42.4447
42	44.4671	44.4364	44.4234	44.4228	44.4455
44	46.4701	46.4385	46.4250	46.4239	46.4463
46	48.4729	48.4404	48.4265	48.4248	48.4470
48	50.4756	50.4422	50.4279	50.4257	50.4476
50	52.4781	52.4439	52.4292	52.4265	52.4482
52	54.4804	54.4454	54.4304	54.4273	54.4487
54	56.4826	56.4469	56.4315	56.4280	56.4492
56	58.4847	58.4483	58.4325	58.4287	58.4497
58	60.4866	60.4496	60.4335	60.4293	60.4501
60	62.4884	62.4509	62.4344	62.4299	62.4506
62	64.4902	64.4520	64.4352	64.4304	64.4510
64	66.4918	66.4531	66.4361	66.4309	66.4513
66	68.4933	68.4542	68.4369	68.4314	68.4517
68	70.4948	70.4552	70.4376	70.4319	70.4520
70	72.4963	72.4561	72.4383	72.4323	72.4523
72	74.4977	74.4570	74.4390	74.4327	74.4526
74	76.4990	76.4578	76.4396	76.4331	76.4529
76	78.5002	78.4586	78.4402	78.4335	78.4532
78	80.5014	80.4594	80.4408	80.4339	80.4534
80	82.5026	82.4601	82.4413	82.4342	82.4536
82	84.5037	84.4608	84.4418	84.4345	84.4538
84	86.5047	86.4615	86.4423	86.4348	86.4540
86	88.5057	88.4621	88.4428	88.4351	88.4542
88	90.5067	90.4627	90.4433	90.4354	90.4544

Table 1. *(Continued)* **Checking External Spur Gear Sizes by Measurement Over Wires**

EVEN NUMBERS OF TEETH

Dimensions in table are for 1 diametral pitch and Van Keuren standard wire sizes. For any other diametral pitch, divide dimension in table by given pitch.

$$\text{Wire or pin diameter} = \frac{1.728}{\text{Diametral Pitch}}$$

No. of Teeth	Pressure Angle				
	14½°	17½°	20°	25°	30°
90	92.5076	92.4633	92.4437	92.4357	92.4546
92	94.5085	94.4639	94.4441	94.4359	94.4548
94	96.5094	96.4644	96.4445	96.4362	96.4550
96	98.5102	98.4649	98.4449	98.4364	98.4552
98	100.5110	100.4655	100.4453	100.4367	100.4554
100	102.5118	102.4660	102.4456	102.4369	102.4555
102	104.5125	104.4665	104.4460	104.4370	104.4557
104	106.5132	106.4669	106.4463	106.4372	106.4558
106	108.5139	108.4673	108.4466	108.4374	108.4560
108	110.5146	110.4678	110.4469	110.4376	110.4561
110	112.5152	112.4682	112.4472	112.4378	112.4562
112	114.5159	114.4686	114.4475	114.4380	114.4563
114	116.5165	116.4690	116.4478	116.4382	116.4564
116	118.5171	118.4693	118.4481	118.4384	118.4565
118	120.5177	120.4697	120.4484	120.4385	120.4566
120	122.5182	122.4701	122.4486	122.4387	122.4567
122	124.5188	124.4704	124.4489	124.4388	124.4568
124	126.5193	126.4708	126.4491	126.4390	126.4569
126	128.5198	128.4711	128.4493	128.4391	128.4570
128	130.5203	130.4714	130.4496	130.4393	130.4571
130	132.5208	132.4717	132.4498	132.4394	132.4572
132	134.5213	134.4720	134.4500	134.4395	134.4573
134	136.5217	136.4723	136.4502	136.4397	136.4574
136	138.5221	138.4725	138.4504	138.4398	138.4575
138	140.5226	140.4728	140.4506	140.4399	140.4576
140	142.5230	142.4730	142.4508	142.4400	142.4577
142	144.5234	144.4733	144.4510	144.4401	144.4578
144	146.5238	146.4736	146.4512	146.4402	146.4578
146	148.5242	148.4738	148.4513	148.4403	148.4579
148	150.5246	150.4740	150.4515	150.4404	150.4580
150	152.5250	152.4742	152.4516	152.4405	152.4580
152	154.5254	154.4745	154.4518	154.4406	154.4581
154	156.5257	156.4747	156.4520	156.4407	156.4581
156	158.5261	158.4749	158.4521	158.4408	158.4582
158	160.5264	160.4751	160.4523	160.4409	160.4582
160	162.5267	162.4753	162.4524	162.4410	162.4583
162	164.5270	164.4755	164.4526	164.4411	164.4584
164	166.5273	166.4757	166.4527	166.4411	166.4584
166	168.5276	168.4759	168.4528	168.4412	168.4585
168	170.5279	170.4760	170.4529	170.4413	170.4585
170	172.5282	172.4761	172.4531	172.4414	172.4586
180	182.5297	182.4771	182.4537	182.4418	182.4589
190	192.5310	192.4780	192.4542	192.4421	192.4591
200	202.5321	202.4786	202.4548	202.4424	202.4593
300	302.5395	302.4831	302.4579	302.4443	302.4606
400	402.5434	402.4854	402.4596	402.4453	402.4613
500	502.5458	502.4868	502.4606	502.4458	502.4619

Table 2. Checking External Spur Gear Sizes by Measurement Over Wires

ODD NUMBERS OF TEETH

Dimensions in table are for 1 diametral pitch and Van Keuren standard wire sizes. For any other diametral pitch, divide dimension in table by given pitch.

$$\text{Wire or pin diameter} = \frac{1.728}{\text{Diametral Pitch}}$$

No. of Teeth	Pressure Angle				
	14½°	17½°	20°	25°	30°
7	9.1116	9.1172	9.1260	9.1536	9.1928
9	11.1829	11.1844	11.1905	11.2142	11.2509
11	13.2317	13.2296	13.2332	13.2536	13.2882
13	15.2677	15.2617	15.2639	15.2814	15.3142
15	17.2957	17.2873	17.2871	17.3021	17.3329
17	19.3182	19.3072	19.3053	19.3181	19.3482
19	21.3368	21.3233	21.3200	21.3310	21.3600
21	23.3524	23.3368	23.3321	23.3415	23.3696
23	25.3658	25.3481	25.3423	25.3502	25.3775
25	27.3774	27.3579	27.3511	27.3576	27.3842
27	29.3876	29.3664	29.3586	29.3640	29.3899
29	31.3966	31.3738	31.3652	31.3695	31.3948
31	33.4047	33.3804	33.3710	33.3743	33.3991
33	35.4119	35.3863	35.3761	35.3786	35.4029
35	37.4185	37.3916	37.3807	37.3824	37.4063
37	39.4245	39.3964	39.3849	39.3858	39.4094
39	41.4299	41.4007	41.3886	41.3889	41.4120
41	43.4348	43.4047	43.3920	43.3917	43.4145
43	45.4394	45.4083	45.3951	45.3942	45.4168
45	47.4437	47.4116	47.3980	47.3965	47.4188
47	49.4477	49.4147	49.4007	49.3986	49.4206
49	51.4514	51.4175	51.4031	51.4006	51.4223
51	53.4547	53.4202	53.4053	53.4024	53.4239
53	55.4579	55.4227	55.4074	55.4041	55.4254
55	57.4609	57.4249	57.4093	57.4056	57.4267
57	59.4637	59.4271	59.4111	59.4071	59.4280
59	61.4664	61.4291	61.4128	61.4084	61.4292
61	63.4689	63.4310	63.4144	63.4097	63.4303
63	65.4712	65.4328	65.4159	65.4109	65.4313
65	67.4734	67.4344	67.4173	67.4120	67.4323
67	69.4755	69.4360	69.4186	69.4130	69.4332
69	71.4775	71.4375	71.4198	71.4140	71.4341
71	73.4795	73.4389	73.4210	73.4150	73.4349
73	75.4813	75.4403	75.4221	75.4159	75.4357
75	77.4830	77.4416	77.4232	77.4167	77.4364
77	79.4847	79.4428	79.4242	79.4175	79.4371
79	81.4863	81.4440	81.4252	81.4183	81.4378
81	83.4877	83.4451	83.4262	83.4190	83.4384
83	85.4892	85.4462	85.4271	85.4196	85.4390
85	87.4906	87.4472	87.4279	87.4203	87.4395
87	89.4919	89.4481	89.4287	89.4209	89.4400
89	91.4932	91.4490	91.4295	91.4215	91.4405
91	93.4944	93.4499	93.4303	93.4221	93.4410
93	95.4956	95.4508	95.4310	95.4227	95.4415

Table 2. *(Continued)* **Checking External Spur Gear Sizes by Measurement Over Wires**

ODD NUMBERS OF TEETH

Dimensions in table are for 1 diametral pitch and Van Keuren standard wire sizes. For any other diametral pitch, divide dimension in table by given pitch.

$$\text{Wire or pin diameter} = \frac{1.728}{\text{Diametral Pitch}}$$

No. of Teeth	Pressure Angle				
	14½°	17½°	20°	25°	30°
95	97.4967	97.4516	97.4317	97.4232	97.4420
97	99.4978	99.4524	99.4323	99.4237	99.4424
99	101.4988	101.4532	101.4329	101.4242	101.4428
101	103.4998	103.4540	103.4335	103.4247	103.4432
103	105.5008	105.4546	105.4341	105.4252	105.4436
105	107.5017	107.4553	107.4346	107.4256	107.4440
107	109.5026	109.4559	109.4352	109.4260	109.4443
109	111.5035	111.4566	111.4357	111.4264	111.4447
111	113.5044	113.4572	113.4362	113.4268	113.4450
113	115.5052	115.4578	115.4367	115.4272	115.4453
115	117.5060	117.4584	117.4372	117.4275	117.4456
117	119.5068	119.4589	119.4376	119.4279	119.4459
119	121.5075	121.4594	121.4380	121.4282	121.4462
121	123.5082	123.4599	123.4384	123.4285	123.4465
123	125.5089	125.4604	125.4388	125.4288	125.4468
125	127.5096	127.4609	127.4392	127.4291	127.4471
127	129.5103	129.4614	129.4396	129.4294	129.4473
129	131.5109	131.4619	131.4400	131.4297	131.4476
131	133.5115	133.4623	133.4404	133.4300	133.4478
133	135.5121	135.4628	135.4408	135.4302	135.4480
135	137.5127	137.4632	137.4411	137.4305	137.4483
137	139.5133	139.4636	139.4414	139.4307	139.4485
139	141.5139	141.4640	141.4418	141.4310	141.4487
141	143.5144	143.4644	143.4421	143.4312	143.4489
143	145.5149	145.4648	145.4424	145.4315	145.4491
145	147.5154	147.4651	147.4427	147.4317	147.4493
147	149.5159	149.4655	149.4430	149.4319	149.4495
149	151.5164	151.4658	151.4433	151.4321	151.4497
151	153.5169	153.4661	153.4435	153.4323	153.4498
153	155.5174	155.4665	155.4438	155.4325	155.4500
155	157.5179	157.4668	157.4440	157.4327	157.4502
157	159.5183	159.4671	159.4443	159.4329	159.4504
159	161.5188	161.4674	161.4445	161.4331	161.4505
161	163.5192	163.4677	163.4448	163.4333	163.4507
163	165.5196	165.4680	165.4450	165.4335	165.4508
165	167.5200	167.4683	167.4453	167.4337	167.4510
167	169.5204	169.4686	169.4455	169.4338	169.4511
169	171.5208	171.4688	171.4457	171.4340	171.4513
171	173.5212	173.4691	173.4459	173.4342	173.4514
181	183.5230	183.4704	183.4469	183.4350	183.4520
191	193.5246	193.4715	193.4478	193.4357	193.4526
201	203.5260	203.4725	203.4487	203.4363	203.4532
301	303.5355	303.4790	303.4538	303.4402	303.4565
401	403.5404	403.4823	403.4565	403.4422	403.4582
501	503.5433	503.4843	503.4581	503.4434	503.4592

Table 3. Checking Internal Spur Gear Sizes by Measurement Between Wires

EVEN NUMBERS OF TEETH

Dimensions in table are for 1 diametral pitch and Van Keuren standard wire sizes. For any other diametral pitch, divide dimension in table by given pitch.

$$\text{Wire or pin diameter} = \frac{1.44}{\text{Diametral Pitch}}$$

No. of Teeth	Pressure Angle				
	14½°	17½°	20°	25°	30°
10	8.8337	8.7383	8.6617	8.5209	8.3966
12	10.8394	10.7404	10.6623	10.5210	10.3973
14	12.8438	12.7419	12.6627	12.5210	12.3978
16	14.8474	14.7431	14.6630	14.5210	14.3982
18	16.8504	16.7441	16.6633	16.5210	16.3985
20	18.8529	18.7449	18.6635	18.5211	18.3987
22	20.8550	20.7456	20.6636	20.5211	20.3989
24	22.8569	22.7462	22.6638	22.5211	22.3991
26	24.8585	24.7467	24.6639	24.5211	24.3992
28	26.8599	26.7471	26.6640	26.5211	26.3993
30	28.8612	28.7475	28.6641	28.5211	28.3994
32	30.8623	30.7478	30.6642	30.5211	30.3995
34	32.8633	32.7481	32.6642	32.5211	32.3995
36	34.8642	34.7483	34.6643	34.5212	34.3996
38	36.8650	36.7486	36.6642	36.5212	36.3996
40	38.8658	38.7488	38.6644	38.5212	38.3997
42	40.8665	40.7490	40.6644	40.5212	40.3997
44	42.8672	42.7492	42.6645	42.5212	42.3998
46	44.8678	44.7493	44.6645	44.5212	44.3998
48	46.8683	46.7495	46.6646	46.5212	46.3999
50	48.8688	48.7496	48.6646	48.5212	48.3999
52	50.8692	50.7497	50.6646	50.5212	50.3999
54	52.8697	52.7499	52.6647	52.5212	52.4000
56	54.8701	54.7500	54.6647	54.5212	54.4000
58	56.8705	56.7501	56.6648	56.5212	56.4001
60	58.8709	58.7502	58.6648	58.5212	58.4001
62	60.8712	60.7503	60.6648	60.5212	60.4001
64	62.8715	62.7504	62.6648	62.5212	62.4001
66	64.8718	64.7505	64.6649	64.5212	64.4001
68	66.8721	66.7505	66.6649	66.5212	66.4001
70	68.8724	68.7506	68.6649	68.5212	68.4001
72	70.8727	70.7507	70.6649	70.5212	70.4002
74	72.8729	72.7507	72.6649	72.5212	72.4002
76	74.8731	74.7508	74.6649	74.5212	74.4002
78	76.8734	76.7509	76.6649	76.5212	76.4002
80	78.8736	78.7509	78.6649	78.5212	78.4002
82	80.8738	80.7510	80.6649	80.5212	80.4002
84	82.8740	82.7510	82.6649	82.5212	82.4002
86	84.8742	84.7511	84.6650	84.5212	84.4002
88	86.8743	86.7511	86.6650	86.5212	86.4003
90	88.8745	88.7512	88.6650	88.5212	88.4003
92	90.8747	90.7512	90.6650	90.5212	90.4003
94	92.8749	92.7513	92.6650	92.5212	92.4003

Table 3. *(Continued)* **Checking Internal Spur Gear Sizes by Measurement Between Wires**

EVEN NUMBERS OF TEETH

Dimensions in table are for 1 diametral pitch and Van Keuren standard wire sizes. For any other diametral pitch, divide dimension in table by given pitch.

$$\text{Wire or pin diameter} = \frac{1.44}{\text{Diametral Pitch}}$$

No. of Teeth	Pressure Angle				
	14½°	17½°	20°	25°	30°
96	94.8750	94.7513	94.6650	94.5212	94.4003
98	96.8752	96.7513	96.6650	96.5212	96.4003
100	98.8753	98.7514	98.6650	98.5212	98.4003
102	100.8754	100.7514	100.6650	100.5212	100.4003
104	102.8756	102.7514	102.6650	102.5212	102.4003
106	104.8757	104.7515	104.6650	104.5212	104.4003
108	106.8758	106.7515	106.6650	106.5212	106.4003
110	108.8759	108.7515	108.6651	108.5212	108.4004
112	110.8760	110.7516	110.6651	110.5212	110.4004
114	112.8761	112.7516	112.6651	112.5212	112.4004
116	114.8762	114.7516	114.6651	114.5212	114.4004
118	116.8763	116.7516	116.6651	116.5212	116.4004
120	118.8764	118.7517	118.6651	118.5212	118.4004
122	120.8765	120.7517	120.6651	120.5212	120.4004
124	122.8766	122.7517	122.6651	122.5212	122.4004
126	124.8767	124.7517	124.6651	124.5212	124.4004
128	126.8768	126.7518	126.6651	126.5212	126.4004
130	128.8769	128.7518	128.6652	128.5212	128.4004
132	130.8769	130.7518	130.6652	130.5212	130.4004
134	132.8770	132.7518	132.6652	132.5212	132.4004
136	134.8771	134.7519	134.6652	134.5212	134.4004
138	136.8772	136.7519	136.6652	136.5212	136.4004
140	138.8773	138.7519	138.6652	138.5212	138.4004
142	140.8773	140.7519	140.6652	140.5212	140.4004
144	142.8774	142.7519	142.6652	142.5212	142.4004
146	144.8774	144.7520	144.6652	144.5212	144.4004
148	146.8775	146.7520	146.6652	146.5212	146.4004
150	148.8775	148.7520	148.6652	148.5212	148.4005
152	150.8776	150.7520	150.6652	150.5212	150.4005
154	152.8776	152.7520	152.6652	152.5212	152.4005
156	154.8777	154.7520	154.6652	154.5212	154.4005
158	156.8778	156.7520	156.6652	156.5212	156.4005
160	158.8778	158.7520	158.6652	158.5212	158.4005
162	160.8779	160.7520	160.6652	160.5212	160.4005
164	162.8779	162.7521	162.6652	162.5212	162.4005
166	164.8780	164.7521	164.6652	164.5212	164.4005
168	166.8780	166.7521	166.6652	166.5212	166.4005
170	168.8781	168.7521	168.6652	168.5212	168.4005
180	178.8783	178.7522	178.6652	178.5212	178.4005
190	188.8785	188.7522	188.6652	188.5212	188.4005
200	198.8788	198.7523	198.6652	198.5212	198.4005
300	298.8795	298.7525	298.6654	298.5212	298.4005
400	398.8803	398.7527	398.6654	398.5212	398.4006
500	498.8810	498.7528	498.6654	498.5212	498.4006

Table 4. Checking Internal Spur Gear Sizes by Measurement Between Wires

ODD NUMBERS OF TEETH

Dimensions in table are for 1 diametral pitch and Van Keuren standard wire sizes. For any other diametral pitch, divide dimensions in table by given pitch.

$$\text{Wire or pin diameter} = \frac{1.44}{\text{Diametral Pitch}}$$

No. of Teeth	Pressure Angle				
	14½°	17½°	20°	25°	30°
7	5.6393	5.5537	5.4823	5.3462	5.2232
9	7.6894	7.5976	7.5230	7.3847	7.2618
11	9.7219	9.6256	9.5490	9.4094	9.2867
13	11.7449	11.6451	11.5669	11.4265	11.3040
15	13.7620	13.6594	13.5801	13.4391	13.3167
17	15.7752	15.6703	15.5902	15.4487	15.3265
19	17.7858	17.6790	17.5981	17.4563	17.3343
21	19.7945	19.6860	19.6045	19.4625	19.3405
23	21.8017	21.6918	21.6099	21.4676	21.3457
25	23.8078	23.6967	23.6143	23.4719	23.3501
27	25.8130	25.7009	25.6181	25.4755	25.3538
29	27.8176	27.7045	27.6214	27.4787	27.3571
31	29.8216	29.7076	29.6242	29.4814	29.3599
33	31.8251	31.7104	31.6267	31.4838	31.3623
35	33.8282	33.7128	33.6289	33.4860	33.3645
37	35.8311	35.7150	35.6310	35.4879	35.3665
39	37.8336	37.7169	37.6327	37.4896	37.3682
41	39.8359	39.7187	39.6343	39.4911	39.3698
43	41.8380	41.7203	41.6357	41.4925	41.3712
45	43.8399	43.7217	43.6371	43.4938	43.3725
47	45.8416	45.7231	45.6383	45.4950	45.3737
49	47.8432	47.7243	47.6394	47.4960	47.3748
51	49.8447	49.7254	49.6404	49.4970	49.3758
53	51.8461	51.7265	51.6414	51.4979	51.3768
55	53.8474	53.7274	53.6422	53.4988	53.3776
57	55.8486	55.7283	55.6431	55.4996	55.3784
59	57.8497	57.7292	57.6438	57.5003	57.3792
61	59.8508	59.7300	59.6445	59.5010	59.3799
63	61.8517	61.7307	61.6452	61.5016	61.3806
65	63.8526	63.7314	63.6458	63.5022	63.3812
67	65.8535	65.7320	65.6464	65.5028	65.3818
69	67.8543	67.7327	67.6469	67.5033	67.3823
71	69.8551	69.7332	69.6475	69.5038	69.3828
73	71.8558	71.7338	71.6480	71.5043	71.3833
75	73.8565	73.7343	73.6484	73.5048	73.3838
77	75.8572	75.7348	75.6489	75.5052	75.3842
79	77.8573	77.7352	77.6493	77.5056	77.3846
81	79.8584	79.7357	79.6497	79.5060	79.3850
83	81.8590	81.7361	81.6501	81.5064	81.3854
85	83.8595	83.7365	83.6505	83.5067	83.3858
87	85.8600	85.7369	85.6508	85.5071	85.3861
89	87.8605	87.7373	87.6511	87.5074	87.3864
91	89.8610	89.7376	89.6514	89.5077	89.3867
93	91.8614	91.7379	91.6517	91.5080	91.3870
95	93.8619	93.7383	93.6520	93.5082	93.3873
97	95.8623	95.7386	95.6523	95.5085	95.3876
99	97.8627	97.7389	97.6526	97.5088	97.3879
101	99.8631	99.7391	99.6528	99.5090	99.3881
103	101.8635	101.7394	101.6531	101.5093	101.3883
105	103.8638	103.7397	103.6533	103.5095	103.3886
107	105.8642	105.7399	105.6535	105.5097	105.3888
109	107.8645	107.7402	107.6537	107.5099	107.3890
111	109.8648	109.7404	109.6539	109.5101	109.3893

Table 4. *(Continued)* Checking Internal Spur Gear Sizes by Measurement Between Wires

ODD NUMBERS OF TEETH

Dimensions in table are for 1 diametral pitch and Van Keuren standard wire sizes. For any other diametral pitch, divide dimensions in table by given pitch.

$$\text{Wire or pin diameter} = \frac{1.44}{\text{Diametral Pitch}}$$

No. of Teeth	Pressure Angle				
	14½°	17½°	20°	25°	30°
113	111.8651	111.7406	111.6541	111.5103	111.3895
115	113.8654	113.7409	113.6543	113.5105	113.3897
117	115.8657	115.7411	115.6545	115.5107	115.3899
119	117.8660	117.7413	117.6547	117.5109	117.3900
121	119.8662	119.7415	119.6548	119.5110	119.3902
123	121.8663	121.7417	121.6550	121.5112	121.3904
125	123.8668	123.7418	123.6552	123.5114	123.3905
127	125.8670	125.7420	125.6554	125.5115	125.3907
129	127.8672	127.7422	127.6556	127.5117	127.3908
131	129.8675	129.7424	129.6557	129.5118	129.3910
133	131.8677	131.7425	131.6559	131.5120	131.3911
135	133.8679	133.7427	133.6560	133.5121	133.3913
137	135.8681	135.7428	135.6561	135.5123	135.3914
139	137.8683	137.7430	137.6563	137.5124	137.3916
141	139.8685	139.7431	139.6564	139.5125	139.3917
143	141.8687	141.7433	141.6565	141.5126	141.3918
145	143.8689	143.7434	143.6566	143.5127	143.3919
147	145.8691	145.7436	145.6568	145.5128	145.3920
149	147.8693	147.7437	147.6569	147.5130	147.3922
151	149.8694	149.7438	149.6570	149.5131	149.3923
153	151.8696	151.7439	151.6571	151.5132	151.3924
155	153.8698	153.7441	153.6572	153.5133	153.3925
157	155.8699	155.7442	155.6573	155.5134	155.3926
159	157.8701	157.7443	157.6574	157.5135	157.3927
161	159.8702	159.7444	159.6575	159.5136	159.3928
163	161.8704	161.7445	161.6576	161.5137	161.3929
165	163.8705	163.7446	163.6577	163.5138	163.3930
167	165.8707	165.7447	165.6578	165.5139	165.3931
169	167.8708	167.7448	167.6579	167.5139	167.3932
171	169.8710	169.7449	169.6580	169.5140	169.3933
181	179.8717	179.7453	179.6584	179.5144	179.3937
191	189.8721	189.7458	189.6588	189.5148	189.3940
201	199.8727	199.7461	199.6591	199.5151	199.3944
301	299.8759	299.7485	299.6612	299.5171	299.3965
401	399.8776	399.7496	399.6623	399.5182	399.3975
501	499.8786	499.7504	499.6629	499.5188	499.3981

Table 5. Van Keuren Wire Diameters for Gears

External Gears Wire Dia. = 1.728 ÷ D.P.				Internal Gears Wire Dia. = 1.44 ÷ D.P.			
D.P.	Dia.	D.P.	Dia.	D.P.	Dia.	D.P.	Dia.
2	0.86400	16	0.10800	2	0.72000	16	0.09000
2½	0.69120	18	0.09600	2½	0.57600	18	0.08000
3	0.57600	20	0.08640	3	0.48000	20	0.07200
4	0.43200	22	0.07855	4	0.36000	22	0.06545
5	0.34560	24	0.07200	5	0.28800	24	0.06000
6	0.28800	28	0.06171	6	0.24000	28	0.05143
7	0.24686	32	0.05400	7	0.20571	32	0.04500
8	0.21600	36	0.04800	8	0.18000	36	0.04000
9	0.19200	40	0.04320	9	0.16000	40	0.03600
10	0.17280	48	0.03600	10	0.14400	48	0.03000
11	0.15709	64	0.02700	11	0.13091	64	0.02250
12	0.14400	72	0.02400	12	0.12000	72	0.02000
14	0.12343	80	0.02160	14	0.10286	80	0.01800

Measurements for Checking Internal Gears when Wire Diameter Equals 1.44 Divided by Diametral Pitch.—Tables 3 and 4 give measurements between wires for checking internal gears of 1 diametral pitch. For any other diametral pitch, divide the measurement given in the table by the diametral pitch required. These measurements are based upon the Van Keuren standard wire size, which, for internal spur gears, equals 1.44 divided by the diametral pitch (see Table 5).

Even Number of Teeth: For an even number of teeth above 170 and not in Table 3, proceed as shown by the following example: Assume that the number of teeth = 380 and pressure angle is 14½ degrees; then, for 1 diametral pitch, figure at left of decimal point = given number of teeth – 2 = 380 – 2 = 378. Figure at right of decimal point lies between decimal values given in table for 300 and 400 teeth and is obtained by interpolation. Thus, 380 – 300 = 80 (change to 0.80); 0.8803 – 0.8795 = 0.0008; hence, decimal required = 0.8795 + (0.80 × 0.0008) 0.88014. Total dimension = 378.88014.

Odd Number of Teeth: Table 4 is for internal gears having odd numbers of teeth. For tooth numbers above 171 and not in the table, proceed as shown by the following example: Assume that number of teeth = 337 and pressure angle is 14½ degrees; then, for 1 diametral pitch, figure at left of decimal point = given No. of teeth – 2 = 337 – 2 = 335. Figure at right of decimal point lies between decimal values given in table for 301 and 401 teeth and is obtained by interpolation. Thus, 337 – 301 = 36 (change to 0.36); 0.8776 – 0.8759 = 0.0017; hence, decimal required = 0.8759 + (0.36 × 0.0017) = 0.8765. Total dimension = 335.8765.

Measurements for Checking External Spur Gears when Wire Diameter Equals 1.68 Divided by Diametral Pitch.—Tables 7 and 8 give measurements *M*, in inches, for checking the pitch diameters of external spur gears of 1 diametral pitch. For any other diametral pitch, divide the measurement given in the table by whatever diametral pitch is required. The result shows what measurement *M* should be when the pitch diameter is correct and there is no allowance for backlash. The procedure for checking for a given amount of backlash when the diameter of the measuring wires equals 1.68 divided by the diametral pitch is explained under a subsequent heading. Tables 7 and 8 are based upon wire sizes equal to 1.68 divided by the diametral pitch. The corresponding wire diameters for various diametral pitches are given in Table 6.

Table 6. Wire Diameters for Spur and Helical Gears Based upon 1.68 Constant

Diametral or Normal Diametral Pitch	Wire Diameter	Diametral or Normal Diametral Pitch	Wire Diameter	Diametral or Normal Diametral Pitch	Wire Diameter	Diametral or Normal Diametral Pitch	Wire Diameter
2	0.840	8	0.210	18	0.09333	40	0.042
2½	0.672	9	0.18666	20	0.084	48	0.035
3	0.560	10	0.168	22	0.07636	64	0.02625
4	0.420	11	0.15273	24	0.070	72	0.02333
5	0.336	12	0.140	28	0.060	80	0.021
6	0.280	14	0.120	32	0.0525	…	…
7	0.240	16	0.105	36	0.04667	…	…

Pin diameter = 1.68 ÷ diametral pitch for spur gears and 1.68 ÷ normal diametral pitch for helical gears.

To find measurement *M* of an external spur gear using wire sizes equal to 1.68 inches divided by the diametral pitch, the same method is followed in using Tables 7 and 8 as that outlined for Tables 1 and 2.

Table 7. Checking External Spur Gear Sizes by Measurement Over Wires

EVEN NUMBERS OF TEETH

Dimensions in table are for 1 diametral pitch and 1.68-inch series wire sizes (a Van Keuren standard). For any other diametral pitch, divide dimension in table by given pitch.

$$\text{Wire or pin diameter} = \frac{1.68}{\text{Diametral Pitch}}$$

No. of Teeth	Pressure Angle				
	14½°	17½°	20°	25°	30°
6	8.1298	8.1442	8.1600	8.2003	8.2504
8	10.1535	10.1647	10.1783	10.2155	10.2633
10	12.1712	12.1796	12.1914	12.2260	12.2722
12	14.1851	14.1910	14.2013	14.2338	14.2785
14	16.1964	16.2001	16.2091	16.2397	16.2833
16	18.2058	18.2076	18.2154	18.2445	18.2871
18	20.2137	20.2138	20.2205	20.2483	20.2902
20	22.2205	22.2190	22.2249	22.2515	22.2927
22	24.2265	24.2235	24.2286	24.2542	24.2949
24	26.2317	26.2275	26.2318	26.2566	26.2967
26	28.2363	28.2309	28.2346	28.2586	28.2982
28	30.2404	30.2339	30.2371	30.2603	30.2996
30	32.2441	32.2367	32.2392	32.2619	32.3008
32	34.2475	34.2391	34.2412	34.2632	34.3017
34	36.2505	36.2413	36.2430	36.2644	36.3026
36	38.2533	38.2433	38.2445	38.2655	38.3035
38	40.2558	40.2451	40.2460	40.2666	40.3044
40	42.2582	42.2468	42.2473	42.2675	42.3051
42	44.2604	44.2483	44.2485	44.2683	44.3057
44	46.2624	46.2497	46.2496	46.2690	46.3063
46	48.2642	48.2510	48.2506	48.2697	48.3068
48	50.2660	50.2522	50.2516	50.2704	50.3073
50	52.2676	52.2534	52.2525	52.2710	52.3078
52	54.2691	54.2545	54.2533	54.2716	54.3082
54	56.2705	56.2555	56.2541	56.2721	56.3086
56	58.2719	58.2564	58.2548	58.2726	58.3089
58	60.2731	60.2572	60.2555	60.2730	60.3093
60	62.2743	62.2580	62.2561	62.2735	62.3096
62	64.2755	64.2587	64.2567	64.2739	64.3099
64	66.2765	66.2594	66.2572	66.2742	66.3102
66	68.2775	68.2601	68.2577	68.2746	68.3104
68	70.2785	70.2608	70.2582	70.2749	70.3107
70	72.2794	72.2615	72.2587	72.2752	72.3109
72	74.2803	74.2620	74.2591	74.2755	74.3111
74	76.2811	76.2625	76.2596	76.2758	76.3113
76	78.2819	78.2631	78.2600	78.2761	78.3115
78	80.2827	80.2636	80.2604	80.2763	80.3117
80	82.2834	82.2641	82.2607	82.2766	82.3119
82	84.2841	84.2646	84.2611	84.2768	84.3121
84	86.2847	86.2650	86.2614	86.2771	86.3123
86	88.2854	88.2655	88.2617	88.2773	88.3124
88	90.2860	90.2659	90.2620	90.2775	90.3126
90	92.2866	92.2662	92.2624	92.2777	92.3127
92	94.2872	94.2666	94.2626	94.2779	94.3129
94	96.2877	96.2670	96.2629	96.2780	96.3130
96	98.2882	98.2673	98.2632	98.2782	98.3131
98	100.2887	100.2677	100.2635	100.2784	100.3132
100	102.2892	102.2680	102.2638	102.2785	102.3134
102	104.2897	104.2683	104.2640	104.2787	104.3135

Table 7. *(Continued)* **Checking External Spur Gear Sizes by Measurement Over Wires**

EVEN NUMBERS OF TEETH

Dimensions in table are for 1 diametral pitch and 1.68-inch series wire sizes (a Van Keuren standard). For any other diametral pitch, divide dimension in table by given pitch.

$$\text{Wire or pin diameter} = \frac{1.68}{\text{Diametral Pitch}}$$

No. of Teeth	Pressure Angle				
	14½°	17½°	20°	25°	30°
104	106.2901	106.2685	106.2642	106.2788	106.3136
106	108.2905	108.2688	108.2644	108.2789	108.3137
108	110.2910	110.2691	110.2645	110.2791	110.3138
110	112.2914	112.2694	112.2647	112.2792	112.3139
112	114.2918	114.2696	114.2649	114.2793	114.3140
114	116.2921	116.2699	116.2651	116.2794	116.3141
116	118.2925	118.2701	118.2653	118.2795	118.3142
118	120.2929	120.2703	120.2655	120.2797	120.3142
120	122.2932	122.2706	122.2656	122.2798	122.3143
122	124.2936	124.2708	124.2658	124.2799	124.3144
124	126.2939	126.2710	126.2660	126.2800	126.3145
126	128.2941	128.2712	128.2661	128.2801	128.3146
128	130.2945	130.2714	130.2663	130.2802	130.3146
130	132.2948	132.2716	132.2664	132.2803	132.3147
132	134.2951	134.2718	134.2666	134.2804	134.3147
134	136.2954	136.2720	136.2667	136.2805	136.3148
136	138.2957	138.2722	138.2669	138.2806	138.3149
138	140.2960	140.2724	140.2670	140.2807	140.3149
140	142.2962	142.2725	142.2671	142.2808	142.3150
142	144.2965	144.2727	144.2672	144.2808	144.3151
144	146.2967	146.2729	146.2674	146.2809	146.3151
146	148.2970	148.2730	148.2675	148.2810	148.3152
148	150.2972	150.2732	150.2676	150.2811	150.3152
150	152.2974	152.2733	152.2677	152.2812	152.3153
152	154.2977	154.2735	154.2678	154.2812	154.3153
154	156.2979	156.2736	156.2679	156.2813	156.3154
156	158.2981	158.2737	158.2680	158.2813	158.3155
158	160.2983	160.2739	160.2681	160.2814	160.3155
160	162.2985	162.2740	162.2682	162.2815	162.3155
162	164.2987	164.2741	164.2683	164.2815	164.3156
164	166.2989	166.2742	166.2684	166.2816	166.3156
166	168.2990	168.2744	168.2685	168.2816	168.3157
168	170.2992	170.2745	170.2686	170.2817	170.3157
170	172.2994	172.2746	172.2687	172.2818	172.3158
180	182.3003	182.2752	182.2691	182.2820	182.3160
190	192.3011	192.2757	192.2694	192.2823	192.3161
200	202.3018	202.2761	202.2698	202.2825	202.3163
300	302.3063	302.2790	302.2719	302.2839	302.3173
400	402.3087	402.2804	402.2730	402.2845	402.3178
500	502.3101	502.2813	502.2736	502.2850	502.3181

Allowance for Backlash: Tables 1, 2, 7, and 8 give measurements over wires when the pitch diameters are correct and there is no allowance for backlash or play between meshing teeth. Backlash is obtained by cutting the teeth somewhat deeper than standard, thus reducing the thickness. Usually, the teeth of both mating gears are reduced in thickness an amount equal to one-half of the total backlash desired. However, if the pinion is small, it is common practice to reduce the gear teeth the full amount of backlash and the pinion is made to standard size. The changes in measurements M over wires, for obtaining backlash in external spur gears, are listed in Table 9.

Table 8. Checking External Spur Gear Sizes by Measurement Over Wires

ODD NUMBERS OF TEETH

Dimensions in table are for 1 diametral pitch and 1.68-inch series wire sizes (a Van Keuren standard). For any other diametral pitch, divide dimension in table by given pitch.

$$\text{Wire or pin diameter} = \frac{1.68}{\text{Diametral Pitch}}$$

No. of Teeth	Pressure Angle				
	14½°	17 ½°	20°	25°	30°
5	6.8485	6.8639	6.8800	6.9202	6.9691
7	8.9555	8.9679	8.9822	9.0199	9.0675
9	11.0189	11.0285	11.0410	11.0762	11.1224
11	13.0615	13.0686	13.0795	13.1126	13.1575
13	15.0925	15.0973	15.1068	15.1381	15.1819
15	17.1163	17.1190	17.1273	17.1570	17.1998
17	19.1351	19.1360	19.1432	19.1716	19.2136
19	21.1505	21.1498	21.1561	21.1832	21.2245
21	23.1634	23.1611	23.1665	23.1926	23.2334
23	25.1743	25.1707	25.1754	25.2005	25.2408
25	27.1836	27.1788	27.1828	27.2071	27.2469
27	29.1918	29.1859	29.1892	29.2128	29.2522
29	31.1990	31.1920	31.1948	31.2177	31.2568
31	33.2053	33.1974	33.1997	33.2220	33.2607
33	35.2110	35.2021	35.2041	35.2258	35.2642
35	37.2161	37.2065	37.2079	37.2292	37.2674
37	39.2208	39.2104	39.2115	39.2323	39.2702
39	41.2249	41.2138	41.2147	41.2349	41.2726
41	43.2287	43.2170	43.2174	43.2374	43.2749
43	45.2323	45.2199	45.2200	45.2396	45.2769
45	47.2355	47.2226	47.2224	47.2417	47.2788
47	49.2385	49.2251	49.2246	49.2435	49.2805
49	51.2413	51.2273	51.2266	51.2452	51.2820
51	53.2439	53.2294	53.2284	53.2468	53.2835
53	55.2463	55.2313	55.2302	55.2483	55.2848
55	57.2485	57.2331	57.2318	57.2497	57.2861
57	59.2506	59.2348	59.2333	59.2509	59.2872
59	61.2526	61.2363	61.2347	61.2521	61.2883
61	63.2545	63.2378	63.2360	63.2532	63.2893
63	65.2562	65.2392	65.2372	65.2543	65.2902
65	67.2579	67.2406	67.2383	67.2553	67.2911
67	69.2594	69.2419	69.2394	69.2562	69.2920
69	71.2609	71.2431	71.2405	71.2571	71.2928
71	73.2623	73.2442	73.2414	73.2579	73.2935
73	75.2636	75.2452	75.2423	75.2586	75.2942
75	77.2649	77.2462	77.2432	77.2594	77.2949
77	79.2661	79.2472	79.2440	79.2601	79.2955
79	81.2673	81.2481	81.2448	81.2607	81.2961
81	83.2684	83.2490	83.2456	83.2614	83.2967
83	85.2694	85.2498	85.2463	85.2620	85.2972
85	87.2704	87.2506	87.2470	87.2625	87.2977
87	89.2714	89.2514	89.2476	89.2631	89.2982
89	91.2723	91.2521	91.2482	91.2636	91.2987
91	93.2732	93.2528	93.2489	93.2641	93.2991
93	95.2741	95.2534	95.2494	95.2646	95.2996

Table 8. *(Continued)* **Checking External Spur Gear Sizes by Measurement Over Wires**

ODD NUMBERS OF TEETH

Dimensions in table are for 1 diametral pitch and 1.68-inch series wire sizes (a Van Keuren standard). For any other diametral pitch, divide dimension in table by given pitch.

$$\text{Wire or pin diameter} = \frac{1.68}{\text{Diametral Pitch}}$$

No. of Teeth	Pressure Angle				
	14½°	17 ½°	20°	25°	30°
95	97.2749	97.2541	97.2500	97.2650	97.3000
97	99.2757	99.2547	99.2506	99.2655	99.3004
99	101.2764	101.2553	101.2511	101.2659	101.3008
101	103.2771	103.2558	103.2516	103.2663	103.3011
103	105.2778	105.2563	105.2520	105.2667	105.3015
105	107.2785	107.2568	107.2525	107.2671	107.3018
107	109.2791	109.2573	109.2529	109.2674	109.3021
109	111.2798	111.2578	111.2533	111.2678	111.3024
111	113.2804	113.2583	113.2537	113.2681	113.3027
113	115.2809	115.2588	115.2541	115.2684	115.3030
115	117.2815	117.2592	117.2544	117.2687	117.3033
117	119.2821	119.2596	119.2548	119.2690	119.3036
119	121.2826	121.2601	121.2552	121.2693	121.3038
121	123.2831	123.2605	123.2555	123.2696	123.3041
123	125.2836	125.2608	125.2558	125.2699	125.3043
125	127.2841	127.2612	127.2562	127.2702	127.3046
127	129.2846	129.2615	129.2565	129.2704	129.3048
129	131.2851	131.2619	131.2568	131.2707	131.3050
131	133.2855	133.2622	133.2571	133.2709	133.3053
133	135.2859	135.2626	135.2574	135.2712	135.3055
135	137.2863	137.2629	137.2577	137.2714	137.3057
137	139.2867	139.3632	139.2579	139.2716	139.3059
139	141.2871	141.2635	141.2582	141.2718	141.3060
141	143.2875	143.2638	143.2584	143.2720	143.3062
143	145.2879	145.2641	145.2587	145.2722	145.3064
145	147.2883	147.2644	147.2589	147.2724	147.3066
147	149.2887	149.2647	149.2591	149.2726	149.3068
149	151.2890	151.2649	151.2594	151.2728	151.3069
151	153.2893	153.2652	153.2596	153.2730	153.3071
153	155.2897	155.2654	155.2598	155.2732	155.3073
155	157.2900	157.2657	157.2600	157.2733	157.3074
157	159.2903	159.2659	159.2602	159.2735	159.3076
159	161.2906	161.2661	161.2604	161.2736	161.3077
161	163.2909	163.2663	163.2606	163.2738	163.3078
163	165.2912	165.2665	165.2608	165.2740	165.3080
165	167.2915	167.2668	167.2610	167.2741	167.3081
167	169.2917	169.2670	169.2611	169.2743	169.3083
169	171.2920	171.2672	171.2613	171.2744	171.3084
171	173.2922	173.2674	173.2615	173.2746	173.3085
181	183.2936	183.2684	183.2623	183.2752	183.3091
191	193.2947	193.2692	193.2630	193.2758	193.3097
201	203.2957	203.2700	203.2636	203.2764	203.3101
301	303.3022	303.2749	303.2678	303.2798	303.3132
401	403.3056	403.2774	403.2699	403.2815	403.3147
501	503.3076	503.2789	503.2711	503.2825	503.3156

Table 9. Backlash Allowances for External and Internal Spur Gears

External Gears: For each 0.001 inch reduction in pitch-line tooth thickness, *reduce* measurement over wires obtained from Tables 1, 2, 7, or 8 by the amount shown below.

Internal Gears: For each 0.001 inch reduction in pitch-line tooth thickness, *increase* measurement between wires obtained from Tables 3 or 4 by the amounts shown below.

Backlash on pitch line equals double tooth thickness reduction when teeth of *both* mating gears are reduced. If teeth of *one* gear only are reduced, backlash on pitch line equals amount of reduction.

Example: For a 30-tooth, 10-diametral pitch, 20-degree pressure angle, external gear the measurement over wires from Table 1 is 32.4102 ÷ 10. For a backlash of 0.002 this measurement must be reduced by 2 × 0.0024 to 3.2362 or (3.2410 – 0.0048).

	14½°		17½°		20°		25°		30°	
No. of Teeth	Ext.	Int.	Ext.	Int.	Ext.	Int.	Ext.	Int.	Ext.	Int.
5	.0019	.0024	.0018	.0024	.0017	.0023	.0015	.0021	.0013	.0019
10	.0024	.0029	.0022	.0027	.0020	.0026	.0017	.0022	.0015	.0018
20	.0028	.0032	.0025	.0029	.0023	.0027	.0019	.0022	.0016	.0018
30	.0030	.0034	.0026	.0030	.0024	.0027	.0020	.0022	.0016	.0018
40	.0031	.0035	.0027	.0030	.0025	.0027	.0020	.0022	.0017	.0018
50	.0032	.0036	.0028	.0031	.0025	.0027	.0020	.0022	.0017	.0018
100	.0035	.0037	.0030	.0031	.0026	.0027	.0021	.0022	.0017	.0017
200	.0036	.0038	.0031	.0031	.0027	.0027	.0021	.0022	.0017	.0017

Measurements for Checking Helical Gears using Wires or Balls.—Helical gears may be checked for size by using one wire, or ball; two wires, or balls; and three wires, depending on the case at hand. Three wires may be used for measurement of either even or odd tooth numbers provided that the face width and helix angle of the gear permit the arrangement of two wires in adjacent tooth spaces on one side of the gear and a third wire on the opposite side. The wires should be held between flat, parallel plates. The measurement between these plates, and perpendicular to the gear axis, will be the same for both even and odd numbers of teeth because the axial displacement of the wires with the odd numbers of teeth does not affect the perpendicular measurement between the plates. The calculation of measurements over three wires is the same as described for measurements over two wires for even numbers of teeth.

Measurements over One Wire or One Ball for Even or Odd Numbers of Teeth: This measurement is calculated by the method for measurement over two wires for even numbers of teeth and the result divided by two to obtain the measurement from over the wire or ball to the center of the gear mounted on an arbor.

Measurement over Two Wires or Two Balls for Even Numbers of Teeth: The measurement over two wires (or two balls kept in the same plane by holding them against a surface parallel to the face of the gear) is calculated as follows: First, calculate the pitch diameter of the helical gear from the formula D = Number of teeth divided by the product of the normal diametral pitch and the cosine of the helix angle, $D = N \div (P_n \times \cos\psi)$. Next, calculate the number of teeth, N_e, there would be in a spur gear for it to have the same tooth curvature as the helical gear has in the normal plane: $N_e = N/\cos^3\psi$. Next, refer to Table 7 for spur gears with even tooth numbers and find, by interpolation, the *decimal* value of the constant for this number of teeth under the given *normal* pressure angle. Finally, add 2 to this decimal value and divide the sum by the normal diametral pitch P_n. The result of this calculation, added to the pitch diameter D, is the measurement over two wires or balls.

Example: A helical gear has 32 teeth of 6 normal diametral pitch, 20 degree pressure angle, and 23 degree helix angle. Determine the measurement over two wires, M, without allowance for backlash.

$D = 32 \div 6 \times \cos 23° = 5.7939$; $N_e = 32 \div \cos^3 23° = 41.027$; and in Table 7, fourth column, the decimal part of the measurement for 40 teeth is .2473 and that for 42 teeth is .2485. The decimal part for 41.027 teeth is, by interpolation, $\frac{(41.027-40)}{(42-40)} \times (.2485-.2473)+.2473$ $= 0.2479$; $(0.2479+2) \div 6 = 0.3747$; and $M = 0.3747+5.7939 = 6.1686$.

This measurement over wires or balls is based upon the use of $1.68/P_n$ wires or balls. If measurements over $1.728/P_n$ diameter wires or balls are preferred, use Table 1 to find the decimal part described above instead of Table 7.

Measurement over Two Wires or Two Balls for Odd Numbers of Teeth: The procedure is similar to that for two wire or two ball measurement for even tooth numbers except that a correction is made in the final *M* value to account for the wires or balls not being diametrically opposite by one-half tooth interval. In addition, care must be taken to ensure that the balls or wires are kept in a plane of the gear's rotation as described previously.

Example: A helical gear has 13 teeth of 8 normal diametral pitch, 14½ degree pressure angle, and 45 degree helix angle. Determine measurement *M* without allowance for backlash based upon the use of $1.728/P_n$ balls or wires.

As before, $D = 13/8 \times \cos 45° = 2.2981$; $N_e = 13/\cos^3 45° = 36.770$; and in the second column of Table 1 the *decimal* part of the measurement for 36 teeth is .4565 and that for 38 teeth is .4603. The decimal part for 36.770 teeth is, by interpolation, $\frac{(36.770-36)}{(38-36)} \times$ $(.4603 - .4565) + .4565 = 0.4580$; $(0.4580 + 2)/8 = 0.3073$; and $M = 0.3073 + 2.2981 = 2.6054$. This measurement is correct for three-wire measurements but, for two balls or wires held in the plane of rotation of the gear, *M* must be corrected as follows:

$$M \text{ corrected} = (M - \text{Ball Diam.}) \times \cos(90°/N) + \text{Ball Diam.}$$
$$= (2.6054 - 1.728/8) \times \cos(90°/13) + 1.728/8 = 2.5880$$

Checking Spur Gear Size by Chordal Measurement Over Two or More Teeth.— Another method of checking gear sizes, that is generally available, is illustrated by the diagram accompanying Table 10. A vernier caliper is used to measure the distance *M* over two or more teeth. The diagram illustrates the measurement over two teeth (or with one intervening tooth space), but three or more teeth might be included, depending upon the pitch. The jaws of the caliper are merely held in contact with the sides or profiles of the teeth and perpendicular to the axis of the gear. Measurement *M* for involute teeth of the correct size is determined as follows

General Formula for Checking External and Internal Spur Gears by Measurement Over Wires: The following formulas may be used for pressure angles or wire sizes not covered by the tables. In these formulas, *M* = measurement *over* wires for external gears or measurement *between* wires for internal gears; *D* = pitch diameter; *T* = arc tooth thickness on pitch circle; *W* = wire diameter; *N* = number of gear teeth; *A* = pressure angle of gear; *a* = angle, the cosine of which is required in Formulas (2) and (3).

First determine the involute function of angle *a* (inv *a*); then the corresponding angle *a* is found by referring to the tables of involute functions beginning on page 98,

$$\text{inv}\,a = \text{inv}\,A \pm \frac{T}{D} \pm \frac{W}{D\cos A} \mp \frac{\pi}{N} \qquad (1)$$

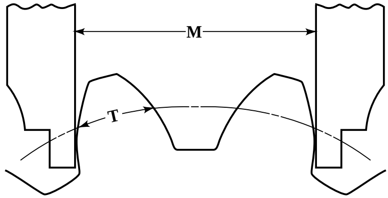
$$\text{For even numbers of teeth, } M = \frac{D\cos A}{\cos a} \pm W \qquad (2)$$

$$\text{For odd numbers of teeth, } M = \left(\frac{D\cos A}{\cos a}\right)\left(\cos\frac{90°}{N}\right) \pm W \qquad (3)$$

Note: In Formulas (1), (2), and (3), use the upper sign for *external* and the lower sign for *internal* gears wherever a ± or ∓ appears in the formulas.

Table 10. Chordal Measurements over Spur Gear Teeth of 1 Diametral Pitch

Find value of *M* under pressure angle and opposite number of teeth; divide *M* by diametral pitch of gear to be measured and then subtract one-half total backlash to obtain a measurement *M* equivalent to given pitch and backlash. The number of teeth to gage or measure over is shown by Table 11.



Number of Gear Teeth	*M* in Inches for 1 D.P.	Number of Gear Teeth	*M* in Inches for 1 D.P.	Number of Gear Teeth	*M* in Inches for 1 D.P.	Number of Gear Teeth	*M* in Inches for 1 D.P.
Pressure Angle, 14½ Degrees							
12	4.6267	37	7.8024	62	14.0197	87	20.2370
13	4.6321	38	10.8493	63	17.0666	88	23.2838
14	4.6374	39	10.8547	64	17.0720	89	23.2892
15	4.6428	40	10.8601	65	17.0773	90	23.2946
16	4.6482	41	10.8654	66	17.0827	91	23.2999
17	4.6536	42	10.8708	67	17.0881	92	23.3053
18	4.6589	43	10.8762	68	17.0934	93	23.3107
19	7.7058	44	10.8815	69	17.0988	94	23.3160
20	7.7112	45	10.8869	70	17.1042	95	23.3214
21	7.7166	46	10.8923	71	17.1095	96	23.3268
22	7.7219	47	10.8976	72	17.1149	97	23.3322
23	7.7273	48	10.9030	73	17.1203	98	23.3375
24	7.7326	49	10.9084	74	17.1256	99	23.3429
25	7.7380	50	10.9137	75	17.1310	100	23.3483
26	7.7434	51	13.9606	76	20.1779	101	26.3952
27	7.7488	52	13.9660	77	20.1833	102	26.4005
28	7.7541	53	13.9714	78	20.1886	103	26.4059
29	7.7595	54	13.9767	79	20.1940	104	26.4113
30	7.7649	55	13.9821	80	20.1994	105	26.4166
31	7.7702	56	13.9875	81	20.2047	106	26.4220
32	7.7756	57	13.9929	82	20.2101	107	26.4274
33	7.7810	58	13.9982	83	20.2155	108	26.4327
34	7.7683	59	14.0036	84	20.2208	109	26.4381
35	7.7917	60	14.0090	85	20.2262	110	26.4435
36	7.7971	61	14.0143	86	20.2316	…	…
Pressure Angle, 20 Degrees							
12	4.5963	30	10.7526	48	16.9090	66	23.0653
13	4.6103	31	10.7666	49	16.9230	67	23.0793
14	4.6243	32	10.7806	50	16.9370	68	23.0933
15	4.6383	33	10.7946	51	16.9510	69	23.1073
16	4.6523	34	10.8086	52	16.9650	70	23.1214
17	4.6663	35	10.8226	53	16.9790	71	23.1354
18	4.6803	36	10.8366	54	16.9930	72	23.1494
19	7.6464	37	13.8028	55	19.9591	73	26.1155
20	7.6604	38	13.8168	56	19.9731	74	26.1295
21	7.6744	39	13.8307	57	19.9872	75	26.1435
22	7.6884	40	13.8447	58	20.0012	76	26.1575
23	7.7024	41	13.8587	59	20.0152	77	26.1715
24	7.7165	42	13.8727	60	20.0292	78	26.1855
25	7.7305	43	13.8867	61	20.0432	79	26.1995
26	7.7445	44	13.9007	62	20.0572	80	26.2135
27	7.7585	45	13.9147	63	20.0712	81	26.2275
28	10.7246	46	16.8810	64	23.0373	…	…
29	10.7386	47	16.8950	65	23.0513	…	…

Table for Determining the Chordal Dimension: Table 10 gives the chordal dimensions for one diametral pitch when measuring over the number of teeth indicated in Table 11. To obtain any chordal dimension, it is simply necessary to divide chord *M* in the table (opposite the given number of teeth) by the diametral pitch of the gear to be measured and then subtract from the quotient one-half the total backlash between the mating pair of gears. In cases where a small pinion is used with a large gear and all of the backlash is to be obtained by reducing the gear teeth, the total amount of backlash is subtracted from the chordal dimension of the gear and nothing from the chordal dimension of the pinion. The application of the tables will be illustrated by an example.

Table 11. Number of Teeth Included in Chordal Measurement

Tooth Range for 14½° Pressure Angle	Tooth Range for 20° Pressure Angle	Number of Teeth to Gage Over	Tooth Range for 14½° Pressure Angle	Tooth Range for 20° Pressure Angle	Number of Teeth to Gage Over
12 to 18	12 to 18	2	63 to 75	46 to 54	6
19 to 37	19 to 27	3	76 to 87	55 to 63	7
38 to 50	28 to 36	4	88 to 100	64 to 72	8
51 to 62	37 to 45	5	101 to 110	73 to 81	9

This table shows the number of teeth to be included between the jaws of the vernier caliper in measuring dimension *M* as explained in connection with Table 10.

Example: Determine the chordal dimension for checking the size of a gear having 30 teeth of 5 diametral pitch and a pressure angle of 20 degrees. A total backlash of 0.008 inch is to be obtained by reducing equally the teeth of both mating gears.

Table 10 shows that the chordal distance for 30 teeth of one diametral pitch and a pressure angle of 20 degrees is 10.7526 inches; one-half of the backlash equals 0.004 inch; hence,

$$\text{Chordal dimension} = \frac{10.7526}{5} - 0.004 = 2.1465 \text{ inches}$$

Table 11 shows that this is the chordal dimension when the vernier caliper spans four teeth, this being the number of teeth to gage over whenever gears of 20-degree pressure angle have any number of teeth from 28 to 36, inclusive.

If it is considered necessary to leave enough stock on the gear teeth for a shaving or finishing cut, this allowance is simply added to the chordal dimension of the finished teeth to obtain the required measurement over the teeth for the roughing operation. It may be advisable to place this chordal dimension for rough machining on the detail drawing.

Formula for Chordal Dimension *M*.—The required measurement *M* over spur gear teeth may be obtained by the following formula in which *R* = pitch radius of gear, *A* = pressure angle, *T* = tooth thickness along pitch circle, *N* = number of gear teeth, *S* = number of tooth *spaces* between caliper jaws, *F* = a factor depending on the pressure angle = 0.01109 for 14½°; = 0.01973 for 17½°; = 0.0298 for 20°; = 0.04303 for 22½°; = 0.05995 for 25°. This factor *F* equals twice the involute function of the pressure angle.

$$M = R \times \cos A \times \left(\frac{T}{R} + \frac{6.2832 \times S}{N} + F\right)$$

Example: A spur gear has 30 teeth of 6 diametral pitch and a pressure angle of 14½ degrees. Determine measurement *M* over three teeth, there being two intervening tooth spaces.

The pitch radius = 2½ inches, the arc tooth thickness equivalent to 6 diametral pitch is 0.2618 inch (if no allowance is made for backlash) and factor *F* for 14½ degrees = 0.01109 inch.

$$M = 2.5 \times 0.96815 \times \left(\frac{0.2618}{2.5} + \frac{6.2832 \times 2}{30} + 0.01109\right) = 1.2941 \text{ inches}$$

Checking Enlarged Pinions by Measuring Over Pins or Wires.—When the teeth of small spur gears or pinions would be undercut if generated by an unmodified straight-sided rack cutter or hob, it is common practice to make the outside diameter larger than standard. The amount of increase in outside diameter varies with the pressure angle and number of teeth, as shown by Table 1 on page 2019. The teeth are always cut to standard depth on a generating type of machine such as a gear hobber or gear shaper; and because the number of teeth and pitch are not changed, the pitch diameter also remains unchanged. The tooth thickness on the pitch circle, however, is increased and wire sizes suitable for standard gears are not large enough to extend above the tops of these enlarged gears or pinions; hence, the Van Keuren wire size recommended for these enlarged pinions equals 1.92 ÷ diametral pitch. Table 12 gives measurements over wires of this size, for checking full-depth involute gears of 1 diametral pitch. For any other pitch, merely divide the measurement given in the table by the diametral pitch. Table 12 applies to pinions that have been enlarged by the same amounts as given in tables 1 and 2, starting on page 2019. These enlarged pinions will mesh with standard gears; but if the standard center distance is to be maintained, reduce the gear diameter below the standard size by as much as the pinion diameter is increased.

Table 12. Checking Enlarged Spur Pinions by Measurement Over Wires

Measurements over wires are given in table for 1 diametral pitch. For any other diametral pitch, divide measurement in table by given pitch. Wire size equals 1.92 ÷ diametral pitch.

Number of Teeth	Outside or Major Diameter (Note 1)	Circular Tooth Thickness (Note 2)	Measure-ment Over Wires	Number of Teeth	Outside or Major Diameter (Note 1)	Circular Tooth Thickness (Note 2)	Measure-ment Over Wires
14½-degree full-depth involute teeth:				20-degree full-depth involute teeth:			
10	13.3731	1.9259	13.6186	10	12.936	1.912	13.5039
11	14.3104	1.9097	14.4966	11	13.818	1.868	14.3299
12	15.2477	1.8935	15.6290	12	14.702	1.826	15.4086
13	16.1850	1.8773	16.5211	13	15.584	1.783	16.2473
14	17.1223	1.8611	17.6244	14	16.468	1.741	17.2933
15	18.0597	1.8449	18.5260	15	17.350	1.698	18.1383
16	18.9970	1.8286	19.6075	16	18.234	1.656	19.1596
17	19.9343	1.8124	20.5156	17	19.116	1.613	20.0080
18	20.8716	1.7962	21.5806				
19	21.8089	1.7800	22.4934				
20	22.7462	1.7638	23.5451				
21	23.6835	1.7476	24.4611				
22	24.6208	1.7314	25.5018				
23	25.5581	1.7151	26.4201				
24	26.4954	1.6989	27.4515				
25	27.4328	1.6827	28.3718				
26	28.3701	1.6665	29.3952				
27	29.3074	1.6503	30.3168				
28	30.2447	1.6341	31.3333				
29	31.1820	1.6179	32.2558				
30	32.1193	1.6017	33.2661				
31	33.0566	1.5854	34.1889				

Note 1: These enlargements, which are to improve the tooth form and avoid undercut, conform to those given in tables 1 and 2, starting on page 2019 where data will be found on the minimum number of teeth in the mating gear.

Note 2: The circular or arc thickness is at the standard pitch diameter. The corresponding chordal thickness may be found as follows: Multiply arc thickness by 90 and then divide product by 3.1416 × pitch radius; find sine of angle thus obtained and multiply it by pitch diameter.

GEAR MATERIALS

Classification of Gear Steels.—Gear steels may be divided into two general classes — the plain carbon and the alloy steels. Alloy steels are used to some extent in the industrial field, but heat-treated plain carbon steels are far more common. The use of untreated alloy steels for gears is seldom, if ever, justified, and then, only when heat-treating facilities are lacking. The points to be considered in determining whether to use heat-treated plain carbon steels or heat-treated alloy steels are: Does the service condition or design require the superior characteristics of the alloy steels, or, if alloy steels are not required, will the advantages to be derived offset the additional cost? For most applications, plain carbon steels, heat-treated to obtain the best of their qualities for the service intended, are satisfactory and quite economical. The advantages obtained from using heat-treated alloy steels in place of heat-treated plain carbon steels are as follows:

1) Increased surface hardness and depth of hardness penetration for the same carbon content and quench.

2) Ability to obtain the same surface hardness with a less drastic quench and, in the case of some of the alloys, a lower quenching temperature, thus giving less distortion.

3) Increased toughness, as indicated by the higher values of yield point, elongation, and reduction of area.

4) Finer grain size, with the resulting higher impact toughness and increased wear resistance.

5) In the case of some of the alloys, better machining qualities or the possibility of machining at higher hardnesses.

Use of Casehardening Steels.—Each of the two general classes of gear steels may be further subdivided as follows: 1) Casehardening steels; 2) full-hardening steels; and

3) steels that are heat-treated and drawn to a hardness that will permit machining.

The first two — casehardening and full-hardening steels — are interchangeable for some kinds of service, and the choice is often a matter of personal opinion. Casehardening steels with their extremely hard, fine-grained (when properly treated) case and comparatively soft and ductile core are generally used when resistance to wear is desired. Casehardening alloy steels have a fairly tough core, but not as tough as that of the full-hardening steels. In order to realize the greatest benefits from the core properties, casehardened steels should be double-quenched. This is particularly true of the alloy steels, because the benefits derived from their use seldom justify the additional expense, unless the core is refined and toughened by a second quench. The penalty that must be paid for the additional refinement is increased distortion, which may be excessive if the shape or design does not lend itself to the casehardening process.

Use of "Thru-Hardening" Steels.—Thru-hardening steels are used when great strength, high endurance limit, toughness, and resistance to shock are required. These qualities are governed by the kind of steel and treatment used. Fairly high surface hardnesses are obtainable in this group, though not so high as those of the casehardening steels. For that reason, the resistance to wear is not so great as might be obtained, but when wear resistance combined with great strength and toughness is required, this type of steel is superior to the others. Thru-hardening steels become distorted to some extent when hardened, the amount depending upon the steel and quenching medium used. For that reason, thru-hardening steels are not suitable for high-speed gearing where noise is a factor, or for gearing where accuracy is of paramount importance, except, of course, in cases where grinding of the teeth is practicable. The medium and high-carbon percentages require an oil quench, but a water quench may be necessary for the lower carbon contents, in order to obtain the highest physical properties and hardness. The distortion, however, will be greater with the water quench.

Heat-Treatment that Permits Machining.—When the grinding of gear teeth is not practicable and a high degree of accuracy is required, hardened steels may be drawn or tempered to a hardness that will permit the cutting of the teeth. This treatment gives a highly refined structure, great toughness, and, in spite of the low hardness, excellent wearing qualities. The lower strength is somewhat compensated for by the elimination of the increment loads due to the impacts which are caused by inaccuracies. When steels that have a low degree of hardness penetration from surface to core are treated in this manner, the design cannot be based on the physical properties corresponding to the hardness at the surface. Since the physical properties are determined by the hardness, the drop in hardness from surface to core will give lower physical properties at the root of the tooth, where the stress is greatest. The quenching medium may be either oil, water, or brine, depending on the steel used and hardness penetration desired. The amount of distortion, of course, is immaterial, because the machining is done after heat-treating.

Making Pinion Harder than Gear to Equalize Wear.—Beneficial results from a wear standpoint are obtained by making the pinion harder than the gear. The pinion, having a lesser number of teeth than the gear, naturally does more work per tooth, and the differential in hardness between the pinion and the gear (the amount being dependent on the ratio) serves to equalize the rate of wear. The harder pinion teeth correct the errors in the gear teeth to some extent by the initial wear and then seem to burnish the teeth of the gear and increase its ability to withstand wear by the greater hardness due to the cold-working of the surface. In applications where the gear ratio is high and there are no severe shock loads, a casehardened pinion running with an oil-treated gear, treated to a Brinell hardness at which the teeth may be cut after treating, is an excellent combination. The pinion, being relatively small, is distorted but little, and distortion in the gear is circumvented by cutting the teeth after treatment.

Forged and Rolled Carbon Steels for Gears.—These compositions cover steel for gears in three groups, according to heat treatment, as follows: D) case-hardened gears;

E) unhardened gears, not heat treated after machining; and F) hardened and tempered gears.

Forged and rolled carbon gear steels are purchased on the basis of the requirements as to chemical composition specified in Table 1. Class N steel will normally be ordered in ten point carbon ranges within these limits. Requirements as to physical properties have been omitted, but when they are called for the requirements as to carbon shall be omitted. The steels may be made by either or both the open hearth and electric furnace processes.

Table 1. Compositions of Forged and Rolled Carbon Steels for Gears

Heat Treatment	Class	Carbon	Manganese	Phosphorus	Sulfur
Case-hardened	C	0.15–0.25	0.40–0.70	0.045 max	0.055 max
Untreated	N	0.25–0.50	0.50–0.80	0.045 max	0.055 max
Hardened (or untreated)	H	0.40–0.50	0.40–0.70	0.045 max	0.055 max

Forged and Rolled Alloy Steels for Gears.—These compositions cover alloy steel for gears, in two classes according to heat treatment, as follows: G) casehardened gears; and

H) hardened and tempered gears.

Forged and rolled alloy gear steels are purchased on the basis of the requirements as to chemical composition specified in Table 2. Requirements as to physical properties have been omitted. The steel shall be made by either or both the open hearth and electric furnace process.

Table 2. Compositions of Forged and Rolled Alloy Steels for Gears

Steel Specification	Chemical Composition[a]					
	C	Mn	Si	Ni	Cr	Mo
AISI 4130	0.28–0.30	0.40–0.60	0.20–0.35	…	0.80–1.1	0.15–0.25
AISI 4140	0.38–0.43	0.75–1.0	0.20–0.35	…	0.80–1.1	0.15–0.25
AISI 4340	0.38–0.43	0.60–0.80	0.20–0.35	1.65–2.0	0.70–90	0.20–0.30
AISI 4615	0.13–0.18	0.45–0.65	0.20–0.35	1.65–2.0	…	0.20–0.30
AISI 4620	0.17–0.22	0.45–0.65	0.20–0.35	1.65–2.0	…	0.20–0.30
AISI 8615	0.13–0.18	0.70–0.90	0.20–0.35	0.40–0.70	0.40–0.60	0.15–0.25
AISI 8620	0.18–0.23	0.70–0.90	0.20–0.35	0.40–0.70	0.40–0.60	0.15–0.25
AISI 9310	0.08–0.13	0.45–0.65	0.20–0.35	3.0–3.5	1.0–1.4	0.08–0.15
Nitralloy						
Type N[b]	0.20–0.27	0.40–0.70	0.20–0.40	3.2–3.8	1.0–1.3	0.20–0.30
135 Mod.[b]	0.38–0.45	0.40–0.70	0.20–0.40	…	1.4–1.8	0.30–0.45

[a] C = carbon; Mn = manganese; Si = silicon; Ni = nickel; Cr = chromium, and Mo = molybdenum.
[b] Both Nitralloy alloys contain aluminum 0.85–1.2%

Steel Castings for Gears.—It is recommended that steel castings for cut gears be purchased on the basis of chemical analysis and that only two types of analysis be used, one for case-hardened gears and the other for both untreated gears and those which are to be hardened and tempered. The steel is to be made by the open hearth, crucible, or electric furnace processes. The converter process is not recognized. Sufficient risers must be provided to secure soundness and freedom from undue segregation. Risers should not be broken off the unannealed castings by force. Where risers are cut off with a torch, the cut should be at least one-half inch above the surface of the castings, and the remaining metal removed by chipping, grinding, or other noninjurious method.

Steel for use in gears should conform to the requirements for chemical composition indicated in Table 3. All steel castings for gears must be thoroughly normalized or annealed, using such temperature and time as will entirely eliminate the characteristic structure of unannealed castings.

Table 3. Compositions of Cast Steels for Gears

Steel Specification	Chemical Composition[a]			
	C	Mn	Si	
SAE-0022	0.12–0.22	0.50–0.90	0.60 Max.	May be carburized
SAE-0050	0.40–0.50	0.50–0.90	0.80 Max.	Hardenable 210–250

[a] C = carbon; Mn = manganese; and Si = silicon.

Effect of Alloying Metals on Gear Steels.—The effect of the various alloying elements on steel are here summarized to assist in deciding on the particular kind of alloy steel to use for specific purposes. The characteristics outlined apply only to heat-treated steels. When the effect of the addition of an alloying element is stated, it is understood that reference is made to alloy steels of a given carbon content, compared with a plain carbon steel of the same carbon content.

Nickel: The addition of nickel tends to increase the hardness and strength, with but little sacrifice of ductility. The hardness penetration is somewhat greater than that of plain carbon steels. Use of nickel as an alloying element lowers the critical points and produces less distortion, due to the lower quenching temperature. The nickel steels of the case-hardening group carburize more slowly, but the grain growth is less.

Chromium: Chromium increases the hardness and strength over that obtained by the use of nickel, though the loss of ductility is greater. Chromium refines the grain and imparts a greater depth of hardness. Chromium steels have a high degree of wear resistance and are easily machined in spite of the fine grain.

Manganese: When present in sufficient amounts to warrant the use of the term alloy, the addition of manganese is very effective. It gives greater strength than nickel and a higher degree of toughness than chromium. Owing to its susceptibility to cold-working, it is likely to flow under severe unit pressures. Up to the present time, it has never been used to any great extent for heat-treated gears, but is now receiving an increasing amount of attention.

Vanadium: Vanadium has a similar effect to that of manganese—increasing the hardness, strength, and toughness. The loss of ductility is somewhat more than that due to manganese, but the hardness penetration is greater than for any of the other alloying elements. Owing to the extremely fine-grained structure, the impact strength is high; but vanadium tends to make machining difficult.

Molybdenum: Molybdenum has the property of increasing the strength without affecting the ductility. For the same hardness, steels containing molybdenum are more ductile than any other alloy steels, and having nearly the same strength, are tougher; in spite of the increased toughness, the presence of molybdenum does not make machining more difficult. In fact, such steels can be machined at a higher hardness than any of the other alloy steels. The impact strength is nearly as great as that of the vanadium steels.

Chrome-Nickel Steels: The combination of the two alloying elements chromium and nickel adds the beneficial qualities of both. The high degree of ductility present in nickel steels is complemented by the high strength, finer grain size, deep hardening, and wear-resistant properties imparted by the addition of chromium. The increased toughness makes these steels more difficult to machine than the plain carbon steels, and they are more difficult to heat treat. The distortion increases with the amount of chromium and nickel.

Chrome-Vanadium Steels: Chrome-vanadium steels have practically the same tensile properties as the chrome-nickel steels, but the hardening power, impact strength, and wear resistance are increased by the finer grain size. They are difficult to machine and become distorted more easily than the other alloy steels.

Chrome-Molybdenum Steels: This group has the same qualities as the straight molybdenum steels, but the hardening depth and wear resistance are increased by the addition of chromium. This steel is very easily heat treated and machined.

Nickel-Molybdenum Steels: Nickel-molybdenum steels have qualities similar to chrome-molybdenum steel. The toughness is said to be greater, but the steel is somewhat more difficult to machine.

Sintered Materials.—For high production of low and moderately loaded gears, significant production cost savings may be effected by the use of a sintered metal powder. With this material, the gear is formed in a die under high pressure and then sintered in a furnace. The primary cost saving comes from the great reduction in labor cost of machining the gear teeth and other gear blank surfaces. The volume of production must be high enough to amortize the cost of the die and the gear blank must be of such a configuration that it may be formed and readily ejected from the die.

Steels for Industrial Gearing

Case-Hardening Steels			
Material Specification	Hardness		Typical Heat Treatment, Characteristics, and Uses
	Case Rc	Core Bhn	
AISI 1020 AISI 1116	55–60	160–230	Carburize, harden, temper at 350°F. For gears that must be wear-resistant. Normalizedmaterial is easily machined. Core is ductile but has little strength.
AISI 4130 AISI 4140	50–55	270–370	Harden, temper at 900°F, Nitride. For parts requiring greater wear resistance than that of through-hardened steels but cannot tolerate the distortion of carburizing. Case is shallow, core is tough.
AISI 4615 AISI 4620 AISI 8615	55–60	170–260	Carburize, harden, temper at 350°F. For gears requiring high fatigue resistance and strength. The 86xx series has better machinability.
AISI 8620	55–60	200–300	The 20 point steels are used for coarser teeth.
AISI 9310	58–63	250–350	Carburize, harden, temper at 300°F. Primarily for aerospace gears that are highly loaded and operate at high pitch line velocity and for other gears requiring high reliability under extreme operating conditions. This material is not used at high temperatures.
Nitralloy N and Type 135 Mod. (15-N)	90–94	300–370	Harden, temper at 1200°F, Nitride. For gears requiring high strength and wear resistance that cannot tolerate the distortion of thecarburizing process or that operate at high temperatures. Gear teeth are usually finished before nitriding. Care must be exercised in running nitrided gears together to avoid crazing of case-hardened surfaces.
Through-Hardening Steels			
AISI 1045 AISI 1140	24–40	…	Harden and temper to required hardness. Oil quench for lower hardness and water quench for higher hardness. For gears of medium and large size requiring moderate strength and wear resistance. Gears that must have consistent, solid sections to withstand quenching.
AISI 4140 AISI 4340	24–40	…	Harden (oil quench), temper to required hardness. For gears requiring high strength and wear resistance, and high shock loading resistance. Use 41xx series for moderate sections and 43xx series for heavy sections. Gears must have consistent, solid sections to withstand quenching.

Bronze and Brass Gear Castings.—These specifications cover nonferrous metals for spur, bevel, and worm gears, bushings and flanges for composition gears. This material shall be purchased on the basis of chemical composition. The alloys may be made by any approved method.

Spur and Bevel Gears: For spur and bevel gears, hard cast bronze is recommended (ASTM B-10-18; SAE No. 62; and the well-known 88-10-2 mixture) with the following limits as to composition: Copper, 86 to 89; tin, 9 to 11; zinc, 1 to 3; lead (max), 0.20; iron (max), 0.06 per cent. Good castings made from this bronze should have the following minimum physical characteristics: Ultimate strength, 30,000 pounds per square inch; yield point, 15,000 pounds per square inch; elongation in 2 inches, 14 per cent.

Worm Gears: For bronze worm gears, two alternative analyses of phosphor bronze are recommended, SAE No. 65 and No. 63.

SAE No. 65 (called phosphor gear bronze) has the following composition: Copper, 88 to 90; tin, 10 to 12; phosphorus, 0.1 to 0.3; lead, zinc, and impurities (max) 0.5 per cent.

Good castings made of this alloy should have the following minimum physical characteristics: Ultimate strength, 35,000 pounds per square inch; yield point, 20,000 pounds per square inch; elongation in 2 inches, 10 per cent.

The composition of SAE No. 63 (called leaded gun metal) follows: copper, 86 to 89; tin, 9 to 11; lead, 1 to 2.5; phosphorus (max), 0.25; zinc and impurities (max), 0.50 per cent.

Good castings made of this alloy should have the following minimum physical characteristics: Ultimate strength, 30,000 pounds per square inch; yield point, 12,000 pounds per square inch; elongation in 2 inches, 10 per cent.

These alloys, especially No. 65, are adapted to chilling for hardness and refinement of grain. No. 65 is to be preferred for use with worms of great hardness and fine accuracy. No. 63 is to be preferred for use with unhardened worms.

Gear Bushings: For bronze bushings for gears, SAE No. 64 is recommended of the following analysis: copper, 78.5 to 81.5; tin, 9 to 11; lead, 9 to 11; phosphorus, 0.05 to 0.25; zinc (max), 0.75; other impurities (max), 0.25 per cent. Good castings of this alloy should have the following minimum physical characteristics: Ultimate strength, 25,000 pounds per square inch; yield point, 12,000 pounds per square inch; elongation in 2 inches, 8 per cent.

Flanges for Composition Pinions: For brass flanges for composition pinions ASTM B-30-32T, and SAE No. 40 are recommended. This is a good cast red brass of sufficient strength and hardness to take its share of load and wear when the design is such that the flanges mesh with the mating gear. The composition is as follows: copper, 83 to 86; tin, 4.5 to 5.5; lead, 4.5 to 5.5; zinc, 4.5 to 5.5; iron (max) 0.35; antimony (max), 0.25 per cent; aluminum, none. Good castings made from this alloy should have the following minimum physical characteristics: ultimate strength, 27,000 pounds per square inch; yield point, 12,000 pounds per square inch; elongation in 2 inches, 16 per cent.

Materials for Worm Gearing.—The Hamilton Gear & Machine Co. conducted an extensive series of tests on a variety of materials that might be used for worm gears, to ascertain which material is the most suitable. According to these tests chill-cast nickel-phosphor-bronze ranks first in resistance to wear and deformation. This bronze is composed of approximately 87.5 per cent copper, 11 per cent tin, 1.5 per cent nickel, with from 0.1 to 0.2 per cent phosphorus. The worms used in these tests were made from SAE-2315, 3½ per cent nickel steel, case-hardened, ground, and polished. The Shore scleroscope hardness of the worms was between 80 and 90. This nickel alloy steel was adopted after numerous tests of a variety of steels, because it provided the necessary strength, together with the degree of hardness required.

The material that showed up second best in these tests was a No. 65 SAE bronze. Navy bronze (88-10-2) containing 2 per cent zinc, with no phosphorus, and not chilled, performed satisfactorily at speeds of 600 revolutions per minute, but was not sufficiently strong at lower speeds. Red brass (85-5-5) proved slightly better at from 1500 to 1800 revolutions per minute, but would bend at lower speeds, before it would show actual wear.

Non-metallic Gearing.—Non-metallic or composition gearing is used primarily where quietness of operation at high speed is the first consideration. Non-metallic materials are also applied very generally to timing gears and numerous other classes of gearing. Rawhide was used originally for non-metallic gears, but other materials have been introduced that have important advantages. These later materials are sold by different firms under various trade names, such as Micarta, Textolite, Formica, Dilecto, Spauldite, Phenolite, Fibroc, Fabroil, Synthane, Celoron, etc. Most of these gear materials consist of layers of canvas or other material that is impregnated with plastics and forced together under hydraulic pressure, which, in conjunction with the application of heat, forms a dense rigid mass.

Although phenol resin gears in general are resilient, they are self-supporting and require no side plates or shrouds unless subjected to a heavy starting torque. The phenol resinoid element protects these gears from vermin and rodents.

The non-metallic gear materials referred to are generally assumed to have the power-transmitting capacity of cast iron. Although the tensile strength may be considerably less than that of cast iron, the resiliency of these materials enables them to withstand impact and abrasion to a degree that might result in excessive wear of cast-iron teeth. Thus, composition gearing of impregnated canvas has often proved to be more durable than cast iron.

Application of Non-metallic Gears.—The most effective field of use for these non-metallic materials is for high-speed duty. At low speeds, when the starting torque may be high, or when the load may fluctuate widely, or when high shock loads may be encountered, these non-metallic materials do not always prove satisfactory. In general, non-metallic materials should not be used for pitch-line velocities below 600 feet per minute.

Tooth Form: The best tooth form for non-metallic materials is the 20-degree stub-tooth system. When only a single pair of gears is involved and the center distance can be varied, the best results will be obtained by making the non-metallic driving pinion of all-addendum form, and the driven metal gear with standard tooth proportions. Such a drive will carry from 50 to 75 per cent greater loads than one of standard tooth proportions.

Material for Mating Gear: For durability under load, the use of hardened steel (over 400 Brinell) for the mating metal gear appears to give the best results. A good second choice for the material of the mating member is cast iron. The use of brass, bronze, or soft steel (under 400 Brinell) as a material for the mating member of phenolic laminated gears leads to excessive abrasive wear.

Power-Transmitting Capacity of Non-metallic Gears.—The characteristics of gears made of phenolic laminated materials are so different from those of metal gears that they should be considered in a class by themselves. Because of the low modulus of elasticity, most of the effects of small errors in tooth form and spacing are absorbed at the tooth surfaces by the elastic deformation, and have but little effect on the strength of the gears.

If S = safe working stress for a given velocity
S_s = allowable static stress
V = pitch-line velocity in feet per minute

then, according to the recommended practice of the American Gear Manufacturers' Association,

$$S = S_s \times \left(\frac{150}{200 + V} + 0.25\right)$$

The value of S_s for phenolic laminated materials is given as 6000 pounds per square inch. The accompanying table gives the safe working stresses S for different pitch-line velocities. When the value of S is known, the horsepower capacity is determined by substituting the value of S for S_s in the appropriate equations in the section on power-transmitting capacity of plastics gears starting on page 601.

Safe Working Stresses for Non-metallic Gears

Pitch-Line Velocity, Feet per Minute, *V*	Safe Working Stresses	Pitch-Line Velocity, Feet per Minute, *V*	Safe Working Stresses	Pitch-Line Velocity, Feet per Minute, *V*	Safe Working Stresses
600	2625	1800	1950	4000	1714
700	2500	2000	1909	4500	1691
800	2400	2200	1875	5000	1673
900	2318	2400	1846	5500	1653
1000	2250	2600	1821	6000	1645
1200	2143	2800	1800	6500	1634
1400	2063	3000	1781	7000	1622
1600	2000	3500	1743	7500	1617

The tensile strength of the phenolic laminated materials used for gears is slightly less than that of cast iron. These materials are far softer than any metal, and the modulus of elasticity is about one-thirtieth that of steel. In other words, if the tooth load on a steel gear that causes a deformation of 0.001 inch were applied to the tooth of a similar gear made of phenolic laminated material, the tooth of the non-metallic gear would be deformed about $\frac{1}{32}$ inch. Under these conditions, several things will happen. With all gears, regardless of the theoretical duration of contact, one tooth only will carry the load until the load is sufficient to deform the tooth the amount of the error that may be present. On metal gears, when the tooth has been deformed the amount of the error, the stresses set up in the materials may approach or exceed the elastic limit of the material. Hence, for standard tooth forms and those generated from standard basic racks, it is dangerous to calculate their strength as very much greater than that which can safely be carried on a single tooth. On gears made of phenolic laminated materials, on the other hand, the teeth will be deformed the amount of this normal error without setting up any appreciable stresses in the material, so that the load is actually supported by several teeth.

All materials have their own peculiar and distinct characteristics, so that under certain specific conditions, each material has a field of its own where it is superior to any other. Such fields may overlap to some extent, and only in such overlapping fields are different materials directly competitive. For example, steel is more or less ductile, has a high tensile strength, and a high modulus of elasticity. Cast iron, on the other hand, is not ductile, has a low tensile strength, but a high compressive strength, and a low modulus of elasticity. Hence, when stiffness and high tensile strength are essential, steel is far superior to cast iron. On the other hand, when these two characteristics are unimportant, but high compressive strength and a moderate amount of elasticity are essential, cast iron is superior to steel.

Preferred Pitch for Non-metallic Gears.—The pitch of the gear or pinion should bear a reasonable relation either to the horsepower or speed or to the applied torque, as shown by the accompanying table. The upper half of this table is based upon horsepower transmitted at a given pitch-line velocity. The lower half gives the torque in pounds-feet or the torque at a 1-foot radius. This torque *T* for any given horsepower and speed can be obtained from the following formula:

$$T = \frac{5252 \times \text{hp}}{\text{rpm}}$$

Bore Sizes for Non-metallic Gears.—For plain phenolic laminated pinions, that is, pinions without metal end plates, a drive fit of 0.001 inch per inch of shaft diameter should be used. For shafts above 2.5 inches in diameter, the fit should be constant at 0.0025 to 0.003 inch. When metal reinforcing end plates are used, the drive fit should conform to the same standards as used for metal.

Preferred Pitches for Non-metallic Gears

Diametral Pitch for Given Horsepower and Pitch Line Velocities			
Horsepower Transmitted	Pitch Line Velocity up to 1000 Feet per Minute	Pitch Line Velocity from 1000 to 2000 Feet per Minute	Pitch Line Velocity over 2000 Feet per Minute
1⁄4–1	8–10	10–12	12–16
1–2	7–8	8–10	10–12
2–3	6–7	7–8	8–10
3–71⁄2	5–6	6–7	7–8
71⁄2–10	4–5	5–6	6–7
10–15	3–4	4–5	5–6
15–25	21⁄2–3	3–4	4–5
25–60	2–21⁄2	21⁄2–3	3–4
60–100	13⁄4–2	2–21⁄2	21⁄2–3
100–150	11⁄2–13⁄4	13⁄4–2	2–21⁄2

Torque in Pounds-feet for Given Diametral Pitch					
Diametral Pitch	Torque in Pounds-feet		Diametral Pitch	Torque in Pounds-feet	
	Minimum	Maximum		Minimum	Maximum
16	1	2	4	50	100
12	2	4	3	100	200
10	4	8	21⁄2	200	450
8	8	15	2	450	900
6	15	30	11⁄2	900	1800
5	30	50	1	1800	3500

These preferred pitches are applicable both to rawhide and the phenolic laminated types of materials.

The root diameter of a pinion of phenolic laminated type should be such that the minimum distance from the edge of the keyway to the root diameter will be at least equal to the depth of tooth.

Keyway Stresses for Non-metallic Gears.—The keyway stress should not exceed 3000 pounds per square inch on a plain phenolic laminated gear or pinion. The keyway stress is calculated by the formula

$$S = \frac{33,000 \times \text{hp}}{V \times A}$$

where S = unit stress in pounds per square inch

hp = horsepower transmitted

V = peripheral speed of shaft in feet per minute; and

A = square inch area of keyway in pinion (length × height)

If the keyway stress formula is expressed in terms of shaft radius r and revolutions per minute, it will read

$$S = \frac{63,000 \times \text{hp}}{\text{rpm} \times r \times A}$$

When the design is such that the keyway stresses exceed 3000 pounds, metal reinforcing end plates may be used. Such end plates should not extend beyond the root diameter of the teeth. The distance from the outer edge of the retaining bolt to the root diameter of the teeth shall not be less than a full tooth depth. The use of drive keys should be avoided, but if required, metal end plates should be used on the pinion to take the wedging action of the key.

For phenolic laminated pinions, the face of the mating gear should be the same or slightly greater than the pinion face.

Invention of Gear Teeth.—The invention of gear teeth represents a gradual evolution from gearing of primitive form. The earliest evidence we have of an investigation of the problem of *uniform motion* from toothed gearing and the successful solution of that problem dates from the time of Olaf Roemer, the celebrated Danish astronomer, who, in the year 1674, proposed the epicycloidal form to obtain uniform motion. Evidently Robert Willis, professor at the University of Cambridge, was the first to make a practical application of the epicycloidal curve so as to provide for an interchangeable series of gears. Willis gives credit to Camus for conceiving the idea of interchangeable gears, but claims for himself its first application. The involute tooth was suggested as a theory by early scientists and mathematicians, but it remained for Willis to present it in a practical form. Perhaps the earliest conception of the application of this form of teeth to gears was by Philippe de Lahire, a Frenchman, who considered it, in theory, equally suitable with the epicycloidal for tooth outlines. This was about 1695 and not long after Roemer had first demonstrated the epicycloidal form. The applicability of the involute had been further elucidated by Leonard Euler, a Swiss mathematician, born at Basel, 1707, who is credited by Willis with being the first to suggest it. Willis devised the Willis odontograph for laying out involute teeth.

A pressure angle of 14½ degrees was selected for three different reasons. First, because the sine of 14½ degrees is nearly ¼, making it convenient in calculation; second, because this angle coincided closely with the pressure angle resulting from the usual construction of epicycloidal gear teeth; third, because the angle of the straight-sided involute rack is the same as the 29-degree worm thread.

Calculating Replacement-Gear Dimensions from Simple Measurements.—The following tables provide formulas with which to calculate the dimensions needed to produce replacement spur, bevel, and helical gears when only the number of teeth, the outside diameter, and the tooth depth of the gear to be replaced are known.

For helical gears, exact helix angles can be obtained by the following procedure.

1) Using a common protractor, measure the approximate helix angle *A* at the approximate pitch line.

2) Place sample or its mating gear on the arbor of a gear hobbing machine.

3) Calculate the index and lead gears differentially for the angle obtained by the measurements, and set up the machine as though a gear is to be cut.

4) Attach a dial indicator on an adjustable arm to the vertical swivel head, with the indicator plunger in a plane perpendicular to the gear axis and in contact with the tooth face. Contact may be anywhere between the top and the root of the tooth.

5) With the power shut off, engage the starting lever and traverse the indicator plunger axially by means of the handwheel.

6) If angle *A* is correct, the indicator plunger will not move as it traverses the face width of the gear. If it does move from 0, note the amount. Divide the amount of movement by the width of the gear to obtain the tangent of the angle by which to correct angle *A*, plus or minus, depending on the direction of indicator movement.

Spur Gears

Tooth Form and Pressure Angle	Diametral Pitch P	Pitch Diameter D	Circular Pitch P_c	Outside Diameter O	Addendum J
American Standard 14½-and 20-degree full depth	$\frac{N+2}{O}$	$\frac{N}{P}$	$\frac{3.1416}{P}$	$\frac{N+2}{P}$	$\frac{1}{P}$
American Standard 20-degree stub	$\frac{N+1.6}{O}$	$\frac{N}{P}$	$\frac{3.1416}{P}$	$\frac{N+1.6}{P}$	$\frac{0.8}{P}$
Fellows 20-degree stub	Note	$\frac{N}{P_N}$	$\frac{3.1416}{P_N}$	$\frac{N}{P_N}+\frac{2}{P_D}$	$\frac{1}{P_D}$

Tooth Form and Pressure Angle	Dedendum K	Whole Tooth Depth W	Clearance $K-J$	Tooth Thickness on Pitch Circle
American Standard 14½-and 20-degree full depth	$\frac{1.157}{P}$	$\frac{2.157}{P}$	$\frac{0.157}{P}$	$\frac{1.5708}{P}$
American Standard 20-degree stub	$\frac{1}{P}$	$\frac{1.8}{P}$	$\frac{0.2}{P}$	$\frac{1.5708}{P}$
Fellows 20-degree stub	$\frac{1.25}{P_D}$	$\frac{2.25}{P_D}$	$\frac{0.25}{P_D}$	$\frac{1.5708}{P_N}$

N = number of teeth.

In the Fellows stub-tooth system, P_N = diametral pitch in numerator of stub-tooth designation and is used to determine circular pitch and number of teeth, and P_D = diametral pitch in the diameter of stub-tooth designation and is used to determine tooth depth.

Milled Bevel Gears — 90 degree Shafts[a]

Tooth Form and Pressure Angle	Tangent of Pitch Cone Angle of Gear, tan A	Tangent of Pitch Cone Angle of Pinion, tan a	Diametral Pitch of Both Gear and Pinion, P[b]	Outside Diameter of Gear, O, or Pinion, o
American Standard 14½- and 20-degree full depth	$\frac{N_G}{N_P}$	$\frac{N_P}{N_G}$	$\frac{N_a + 2\cos A}{O}$ or $\frac{N_P + 2\cos a}{o}$	$\frac{N_a + 2\cos A}{P}$ or $\frac{N_P + 2\cos a}{P}$

(Continued) **Milled Bevel Gears — 90 degree Shafts**[a]

Tooth Form and Pressure Angle	Tangent of Pitch Cone Angle of Gear, tan *A*	Tangent of Pitch Cone Angle of Pinion, tan *a*	Diametral Pitch of Both Gear and Pinion, *P*[b]	Outside Diameter of Gear, *O*, or Pinion, *o*
American Standard 20-degree stub	$\frac{N_G}{N_P}$	$\frac{N_P}{N_G}$	$\frac{N_a + 1.6\cos A}{O}$ or $\frac{N_P + 1.6\cos a}{o}$	$\frac{N_a + 1.6\cos A}{P}$ or $\frac{N_P + 1.6\cos a}{P}$
Fellows 20-degree stub	$\frac{N_G}{N_P}$	$\frac{N_P}{N_G}$	…	$\frac{N_G}{P_N} + \frac{2\cos A}{P_D}$ or $\frac{N_P}{P_N} + \frac{2\cos a}{P_D}$

[a] These formulas do not apply to Gleason System Gearing.
[b] These values are the same for both gear and pinion.

Tooth Form and Pressure Angle	Pitch-Cone Radius or Cone Distance, *E*[b]	Tangent of Addendum Angle[b]	Tangent of Dedendum Angle[b]	Cosine of Pitch-Cone Angle of Gear, cos *A*[a]
American Standard 14½- and 20-degree full depth	$\frac{D}{2\sin A}$ or $\frac{d}{2\sin a}$	$\frac{2\sin A}{N_a}$ or $\frac{2\sin a}{N_P}$	$\frac{2.314\sin A}{N_G}$ or $\frac{2.314\sin a}{N_P}$	$\frac{(P \times O) - N_G}{2}$
American Standard 20-degree stub	$\frac{D}{2\sin A}$ or $\frac{d}{2\sin a}$	$\frac{1.6\sin A}{N_G}$ or $\frac{1.6\sin a}{N_P}$	$\frac{2\sin A}{N_G}$ or $\frac{2\sin a}{N_P}$	$\frac{(P \times O) - N_G}{1.6}$
Fellows 20-degree stub	$\frac{D}{2\sin A}$ or $\frac{d}{2\sin a}$	$\frac{2P_N\sin A}{N_G \times P_D}$ or $\frac{2P_N\sin a}{N_P \times P_D}$	$\frac{2.5P_N\sin A}{N_G \times P_D}$ or $\frac{2.5P_N\sin a}{N_P \times P_D}$	$\frac{P_D[(O \times P_N) - N_G]}{2P_N}$

[a] The same formulas apply to the pinion, substituting N_P for N_G and *o* for *O*.

N_G = number of teeth in gear; N_P = number of teeth in pinion; *O* = outside diameter of gear; *o* = outside diameter of pinion; *D* = pitch diameter of gear = $N_G \div P$; *d* = pitch diameter of pinion = $N_P \div P$; P_c = circular pitch; *J* = addendum; *K* = dedendum; *W* = whole depth.

See footnote in *Spur Gears* table for meaning of P_N and P_D. The tooth thickness on the pitch circle is found by means of the formulas in the last column under spur gears.

Helical Gears

Tooth Form and Pressure Angle	Normal Diametral Pitch P_N	Diametral Pitch P	Outside Diameter of Blank O	Pitch Diameter D
American Standard 14½- and 20-degree full depth	$\frac{N+2\cos A}{O \times \cos A}$ or $\frac{P}{\cos A}$	$P_N \cos A$ or $\frac{N+2\cos A}{O}$	$\frac{N+2\cos A}{P_N \cos A}$ or $\frac{N+2\cos A}{P}$	$\frac{N}{P_N \cos A}$ or $\frac{N}{P}$
American Standard 20-degree stub	$\frac{N+1.6\cos A}{O \times \cos A}$ or $\frac{P}{\cos A}$	$P_n \cos A$ or $\frac{N+1.6\cos A}{O}$	$\frac{N+1.6\cos A}{P_N \cos A}$ or $\frac{N+1.6\cos A}{P}$	$\frac{N}{P_N \cos A}$ or $\frac{N}{P}$
Fellows 20-degree stub	…	…	$\frac{N}{(P_N)_N \cos A} + \frac{2}{(P_N)_D}$	$\frac{N}{(P_N)_N \cos A}$

Tooth Form and Pressure Angle	Cosine of Helix Angle A	Addendum	Dedendum	Whole Depth
American Standard 14½- and 20-degree full depth	$\frac{P}{P_N}$ or $\frac{N}{O \times P_N - 2}$	$\frac{1}{P_N}$ or $\frac{\cos A}{P}$	$\frac{1.157}{P_N}$ or $\frac{1.157\cos A}{P}$	$\frac{2.157}{P_N}$ or $\frac{2.157\cos A}{P}$
American Standard 20-degree stub	$\frac{P}{P_N}$ or $\frac{N}{O \times P_N - 1.6}$	$\frac{0.8}{P_N}$ or $\frac{0.8\cos A}{P}$	$\frac{1}{P_N}$ or $\frac{\cos A}{P}$	$\frac{1.8}{P_N}$ or $\frac{1.8\cos A}{P}$
Fellows 20-degree stub	$\frac{N}{(P_N)_N\left(O - \frac{2}{(P_N)_D}\right)}$	$\frac{1}{(P_N)_D}$	$\frac{1.25}{(P_N)_D}$	$\frac{2.25}{(P_N)_D}$

P_N = normal diametral pitch; = normal diametral pitch of cutter or hob used to cut teeth

P = diametral pitch

O = outside diameter of blank

D = pitch diameter

A = helix angle

N = number of teeth

$(P_N)_N$ = normal diametral pitch in numerator of stub-tooth designation, which determines thickness of tooth and number of teeth

$(P_N)_D$ = normal diametral pitch in denominator of stub-tooth designation, which determines the addendum, dedendum, and whole depth

SPLINES AND SERRATIONS

A splined shaft is one having a series of parallel keys formed integrally with the shaft and mating with corresponding grooves cut in a hub or fitting; this arrangement is in contrast to a shaft having a series of keys or feathers fitted into slots cut into the shaft. The latter construction weakens the shaft to a considerable degree because of the slots cut into it and consequently, reduces its torque-transmitting capacity.

Splined shafts are most generally used in three types of applications: 1) for coupling shafts when relatively heavy torques are to be transmitted without slippage; 2) for transmitting power to slidably-mounted or permanently-fixed gears, pulleys, and other rotating members; and 3) for attaching parts that may require removal for indexing or change in angular position.

Splines having straight-sided teeth have been used in many applications (see SAE Parallel Side Splines for Soft Broached Holes in Fittings); however, the use of splines with teeth of involute profile has steadily increased since 1) involute spline couplings have greater torque-transmitting capacity than any other type; 2) they can be produced by the same techniques and equipment as is used to cut gears; and 3) they have a self-centering action under load even when there is backlash between mating members.

Involute Splines

American National Standard Involute Splines*.—These splines or multiple keys are similar in form to internal and external involute gears. The general practice is to form the external splines either by hobbing, rolling, or on a gear shaper, and internal splines either by broaching or on a gear shaper. The internal spline is held to basic dimensions and the external spline is varied to control the fit. Involute splines have maximum strength at the base, can be accurately spaced and are self-centering, thus equalizing the bearing and stresses, and they can be measured and fitted accurately.

In American National Standard ANSI B92.1-1970 (R 1993), many features of the 1960 standard are retained; plus the addition of three tolerance classes, for a total of four. The term “involute serration,” formerly applied to involute splines with 45-degree pressure angle, has been deleted and the standard now includes involute splines with 30-, 37.5-, and 45-degree pressure angles. Tables for these splines have been rearranged accordingly. The term “serration” will no longer apply to splines covered by this Standard.

The Standard has only one fit class for all side fit splines; the former Class 2 fit. Class 1 fit has been deleted because of its infrequent use. The major diameter of the flat root side fit spline has been changed and a tolerance applied to include the range of the 1950 and the 1960 standards. The interchangeability limitations with splines made to previous standards are given later in the section entitled “Interchangeability.”

There have been no tolerance nor fit changes to the major diameter fit section.

The Standard recognizes the fact that proper assembly between mating splines is dependent only on the spline being within effective specifications from the tip of the tooth to the form diameter. Therefore, on side fit splines, the internal spline major diameter now is shown as a maximum dimension and the external spline minor diameter is shown as a minimum dimension. The minimum internal major diameter and the maximum external minor diameter must clear the specified form diameter and thus do not need any additional control.

The spline specification tables now include a greater number of tolerance level selections. These tolerance classes were added for greater selection to suit end product needs. The selections differ only in the tolerance as applied to space widthand tooth thickness.

* See American National Standard ANSI B92.2M-1980 (R1989), Metric Module Involute Splines; also see page 2148.

The tolerance class used in ASA B5.15-1960 is the basis and is now designated as tolerance Class 5. The new tolerance classes are based on the following formulas:

$$\text{Tolerance Class 4} = \text{Tolerance Class 5} \times 0.71$$
$$\text{Tolerance Class 6} = \text{Tolerance Class 5} \times 1.40$$
$$\text{Tolerance Class 7} = \text{Tolerance Class 5} \times 2.00$$

All dimensions listed in this standard are for the finished part. Therefore, any compensation that must be made for operations that take place during processing, such as heat treatment, must be taken into account when selecting the tolerance level for manufacturing.

The standard has the same internal minimum effective space width and external maximum effective tooth thickness for all tolerance classes and has two types of fit. For tooth side fits, the minimum effective space width and the maximum effective tooth thickness are of equal value. This basic concept makes it possible to have interchangeable assembly between mating splines where they are made to this standard regardless of the tolerance class of the individual members. A tolerance class "mix" of mating members is thus allowed, which often is an advantage where one member is considerably less difficult to produce than its mate, and the "average" tolerance applied to the two units is such that it satisfies the design need. For instance, assigning a Class 5 tolerance to one member and Class 7 to its mate will provide an assembly tolerance in the Class 6 range. The maximum effective tooth thickness is less than the minimum effective space width for major diameter fits to allow for eccentricity variations.

In the event the fit as provided in this standard does not satisfy a particular design need and a specific amount of effective clearance or press fit is desired, the change should be made only to the external spline by a reduction or an increase in effective tooth thickness and a like change in actual tooth thickness. The minimum effective space width, in this standard, is always basic. The basic minimum effective space width should always be retained when special designs are derived from the concept of this standard.

Terms Applied to Involute Splines.—The following definitions of involute spline terms, here listed in alphabetical order, are given in the American National Standard. Some of these terms are illustrated in the diagram in Tables 6.

Active Spline Length (L_a) is the length of spline that contacts the mating spline. On sliding splines, it exceeds the length of engagement.

Actual Space Width (s) is the circular width on the pitch circle of any single space considering an infinitely thin increment of axial spline length.

Actual Tooth Thickness (t) is the circular thickness on the pitch circle of any single tooth considering an infinitely thin increment of axial spline length.

Alignment Variation is the variation of the effective spline axis with respect to the reference axis (see Fig. 1c).

Base Circle is the circle from which involute spline tooth profiles are constructed.

Base Diameter (D_b) is the diameter of the base circle.

Basic Space Width is the basic space width for 30-degree pressure angle splines; half the circular pitch. The basic space width for 37.5- and 45-degree pressure angle splines, however, is greater than half the circular pitch. The teeth are proportioned so that the external tooth, at its base, has about the same thickness as the internal tooth at the form diameter. This proportioning results in greater minor diameters than those of comparable involute splines of 30-degree pressure angle.

Circular Pitch (p) is the distance along the pitch circle between corresponding points of adjacent spline teeth.

Depth of Engagement is the radial distance from the minor circle of the internal spline to the major circle of the external spline, minus corner clearance and/or chamfer depth.

Diametral Pitch (*P*) is the number of spline teeth per inch of pitch diameter. The diametral pitch determines the circular pitch and the basic space width or tooth thickness. In conjunction with the number of teeth, it also determines the pitch diameter. (See also Pitch.)

Effective Clearance (c_v) is the effective space width of the internal spline minus the effective tooth thickness of the mating external spline.

Effective Space Width (S_v) of an internal spline is equal to the circular tooth thickness on the pitch circle of an imaginary perfect external spline that would fit the internal spline without looseness or interference considering engagement of the entire axial length of the spline. The minimum effective space width of the internal spline is always basic, as shown in Table 3. Fit variations may be obtained by adjusting the tooth thickness of the external spline.

Three types of involute spline variations

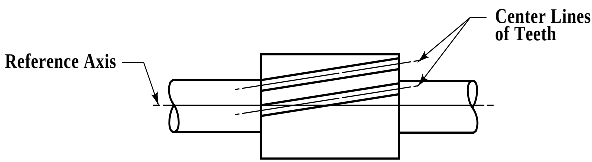


Fig. 1a. Lead Variation

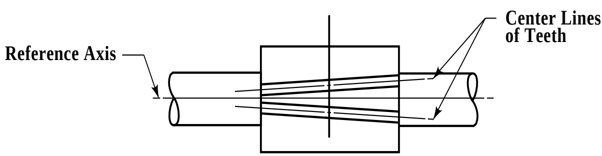


Fig. 1b. Parallelism Variation

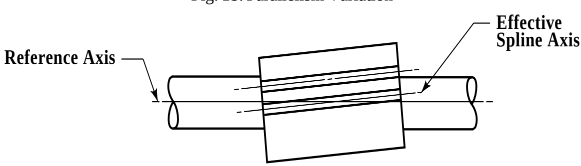


Fig. 1c. Alignment Variation

Effective Tooth Thickness (t_v) of an external spline is equal to the circular space width on the pitch circle of an imaginary perfect internal spline that would fit the external spline without looseness or interference, considering engagement of the entire axial length of the spline.

Effective Variation is the accumulated effect of the spline variations on the fit with the mating part.

External Spline is a spline formed on the outer surface of a cylinder.

Fillet is the concave portion of the tooth profile that joins the sides to the bottom of the space.

Fillet Root Splines are those in which a single fillet in the general form of an arc joins the sides of adjacent teeth.

Flat Root Splines are those in which fillets join the arcs of major or minor circles to the tooth sides.

Form Circle is the circle which defines the deepest points of involute form control of the tooth profile. This circle along with the tooth tip circle (or start of chamfer circle) determines the limits of tooth profile requiring control. It is located near the major circle on the internal spline and near the minor circle on the external spline.

Form Clearance (c_F) is the radial depth of involute profile beyond the depth of engagement with the mating part. It allows for looseness between mating splines and for eccentricities between the minor circle (internal), the major circle (external), and their respective pitch circles.

Form Diameter (D_{Fe}, D_{Fi}) the diameter of the form circle.

Internal Spline is a spline formed on the inner surface of a cylinder.

Involute Spline is one having teeth with involute profiles.

Lead Variation is the variation of the direction of the spline tooth from its intended direction parallel to the reference axis, also including parallelism and alignment variations (see Fig. 1a). *Note:* Straight (nonhelical) splines have an infinite lead.

Length of Engagement (L_q) is the axial length of contact between mating splines.

Machining Tolerance (m) is the permissible variation in actual space width or actual tooth thickness.

Major Circle is the circle formed by the outermost surface of the spline. It is the outside circle (tooth tip circle) of the external spline or the root circle of the internal spline.

Major Diameter (D_o, D_{ri}) is the diameter of the major circle.

Minor Circle is the circle formed by the innermost surface of the spline. It is the root circle of the external spline or the inside circle (tooth tip circle) of the internal spline.

Minor Diameter (D_{re}, D_i) is the diameter of the minor circle.

Nominal Clearance is the actual space width of an internal spline minus the actual tooth thickness of the mating external spline. It does not define the fit between mating members, because of the effect of variations.

Out of Roundness is the variation of the spline from a true circular configuration.

Parallelism Variation is the variation of parallelism of a single spline tooth with respect to any other single spline tooth (see Fig. 1b).

Pitch (P/P_s) is a combination number of a one-to-two ratio indicating the spline proportions; the upper or first number is the diametral pitch, the lower or second number is the stub pitch and denotes, as that fractional part of an inch, the basic radial length of engagement, both above and below the pitch circle.

Pitch Circle is the reference circle from which all transverse spline tooth dimensions are constructed.

Pitch Diameter (D) is the diameter of the pitch circle.

Pitch Point is the intersection of the spline tooth profile with the pitch circle.

Pressure Angle (ϕ) is the angle between a line tangent to an involute and a radial line through the point of tangency. Unless otherwise specified, it is the standard pressure angle.

Profile Variation is any variation from the specified tooth profile normal to the flank.

Spline is a machine element consisting of integral keys (spline teeth) or keyways (spaces) equally spaced around a circle or portion thereof.

Standard (Main) Pressure Angle (ϕ_D) is the pressure angle at the specified pitch diameter.

Stub Pitch (P_s) is a number used to denote the radial distance from the pitch circle to the major circle of the external spline and from the pitch circle to the minor circleof the internal spline. The stub pitch for splines in this standard is twice the diametral pitch.

Total Index Variation is the greatest difference in any two teeth (adjacent or otherwise) between the actual and the perfect spacing of the tooth profiles.

Total Tolerance ($m + \lambda$) is the machining tolerance plus the variation allowance.

Variation Allowance (λ) is the permissible effective variation.

Tooth Proportions.—There are 17 pitches: 2.5⁄5, 3⁄6, 4⁄8,5⁄10, 6⁄12, 8⁄16, 10⁄20, 12⁄24, 16⁄32, 20⁄40, 24⁄48, 32⁄64, 40⁄80, 48⁄96, 64⁄128, 80⁄160, and 128⁄256. The numerator in this fractional designation is known as the diametral pitch and controls the pitch diameter; the denominator, which is always double the numerator, is known as the stub pitch and controls the tooth depth. For convenience in calculation, only the numerator is used in the formulas given and is designated as *P*. Diametral pitch, as in gears, means the number of teeth per inch of pitch diameter.

Table 1 shows the symbols and Table 2 the formulas for basic tooth dimensions of involute spline teeth of various pitches. Basic dimensions are given in Table 3.

Table 1. American National Standard Involute Spline Symbols
ANSI B92.1-1970, R1993

c_v	effective clearance	M_i	measurement between pins, internal spline
c_F	form clearance	N	number of teeth
D	pitch diameter	P	diametral pitch
D_b	base diameter	P_s	stub pitch
D_{ci}	pin contact diameter, internal spline	p	circular pitch
D_{ce}	pin contact diameter, external spline	r_f	fillet radius
D_{Fe}	form diameter, external spline	s	actual space width, circular
D_{Fi}	form diameter, internal spline	s_v	effective space width, circular
D_i	minor diameter, internal spline	s_c	allowable compressive stress, psi
D_o	major diameter, external spline	s_s	allowable shear stress, psi
D_{re}	minor diameter, external spline (root)	t	actual tooth thickness, circular
D_{ri}	major diameter, internal spline (root)	t_v	effective tooth thickness, circular
d_e	diameter of measuring pin for external spline	λ	variation allowance
d_i	diameter of measuring pin for internal spline	$\in$	involute roll angle
K_e	change factor for external spline	ϕ	pressure angle
K_i	change factor for internal spline	ϕ_D	standard pressure angle
L	spline length	ϕ_{ci}	pressure angle at pin contact diameter, internal spline
L_a	active spline length	ϕ_{ce}	pressure angle at pin contact diameter, external spline
L_g	length of engagement	ϕ_i	pressure angle at pin center, internal spline
m	machining tolerance	ϕ_e	pressure angle at pin center, external spline
M_e	measurement over pins, external spline	ϕ_F	pressure angle at form diameter

Table 2. Formulas for Basic Dimensions *ANSI B92.1-1970, R1993*

Term	Symbol	Formula				
		30 deg ϕ_D			37.5 deg ϕ_D	45 deg ϕ_D
		Flat Root Side Fit	Flat Root Major Dia Fit	Fillet Root Side Fit	Fillet Root Side Fit	Fillet Root Side Fit
		2.5⁄5–32⁄64 Pitch	3⁄6–16⁄32 Pitch	2.5⁄5–48⁄96 Pitch	2.5⁄5–48⁄96 Pitch	10⁄20–128⁄256 Pitch
Stub Pitch	P_s	$2P$	$2P$	$2P$	$2P$	$2P$
Pitch Diameter	D	$\frac{N}{P}$	$\frac{N}{P}$	$\frac{N}{P}$	$\frac{N}{P}$	$\frac{N}{P}$
Base Diameter	D_b	$D \cos \phi_D$	$D \cos \phi_D$	$D \cos \phi_D$	$D \cos \phi_D$	$D \cos \phi_D$
Circular Pitch	p	$\frac{\pi}{P}$	$\frac{\pi}{P}$	$\frac{\pi}{P}$	$\frac{\pi}{P}$	$\frac{\pi}{P}$
Minimum Effective Space Width	s_v	$\frac{\pi}{2P}$	$\frac{\pi}{2P}$	$\frac{\pi}{2P}$	$\frac{0.5\pi + 0.1}{P}$	$\frac{0.5\pi + 0.2}{P}$
Major Diameter, Internal	D_{ri}	$\frac{N + 1.35}{P}$	$\frac{N + 1}{P}$	$\frac{N + 1.8}{P}$	$\frac{N + 1.6}{P}$	$\frac{N + 1.4}{P}$
Major Diameter, External	D_o	$\frac{N + 1}{P}$	$\frac{N + 1}{P}$	$\frac{N + 1}{P}$	$\frac{N + 1}{P}$	$\frac{N + 1}{P}$
Minor Diameter, Internal	D_i	$\frac{N - 1}{P}$	$\frac{N - 1}{P}$	$\frac{N - 1}{P}$	$\frac{N - 0.8}{P}$	$\frac{N - 0.6}{P}$

Table 2. *(Continued)* **Formulas for Basic Dimensions** *ANSI B92.1-1970, R1993*

Term		Symbol	Formula				
			30 deg ϕ_D			37.5 deg ϕ_D	45 deg ϕ_D
			Flat Root Side Fit	Flat Root Major Dia Fit	Fillet Root Side Fit	Fillet Root Side Fit	Fillet Root Side Fit
			2.5⁄5–32⁄64 Pitch	3⁄6–16⁄32 Pitch	2.5⁄5–48⁄96 Pitch	2.5⁄5–48⁄96 Pitch	10⁄20–128⁄256 Pitch
Minor Dia. Ext.	2.5⁄5 thru 12⁄24 pitch	D_{re}	$\frac{N-1.35}{P}$		$\frac{N-1.8}{P}$	$\frac{N-1.3}{P}$	…
	16⁄32 pitch and finer				$\frac{N-2}{P}$		
	10⁄20 16⁄32 pitch and finer				…		$\frac{N-1}{P}$
Form Diameter, Internal		D_{Fi}	$\frac{N+1}{P}+2c_F$	$\frac{N+0.8}{P}-0.004+2c_F$	$\frac{N+1}{P}+2c_F$	$\frac{N+1}{P}+2c_F$	$\frac{N+1}{P}+2c_F$
Form Diameter, External		D_{Fe}	$\frac{N-1}{P}-2c_F$	$\frac{N-1}{P}-2c_F$	$\frac{N-1}{P}-2c_F$	$\frac{N-0.8}{P}-2c_F$	$\frac{N-0.6}{P}-2c_F$
Form Clearance (Radial)		c_F	0.001 *D*, with max of 0.010, min of 0.002				

$\pi = 3.1415927$

Note: All spline specification table dimensions in the standard are derived from these basic formulas by application of tolerances.

Table 3. Basic Dimensions for Involute Splines *ANSI B92.1-1970, R1993*

Pitch, P/P_s	Circular Pitch, p	Min Effective Space Width (BASIC), S_v min			Pitch, P/P_s	Circular Pitch, p	Min Effective Space Width (BASIC), S_v min		
		30 deg ϕ	37.5 deg ϕ	45 deg ϕ			30 deg ϕ	37.5 deg ϕ	45 deg ϕ
2.5⁄5	1.2566	0.6283	0.6683	…	20⁄40	0.1571	0.0785	0.0835	0.0885
3⁄6	1.0472	0.5236	0.5569	…	24⁄48	0.1309	0.0654	0.0696	0.0738
4⁄8	0.7854	0.3927	0.4177	…	32⁄64	0.0982	0.0491	0.0522	0.0553
5⁄10	0.6283	0.3142	0.3342	…	40⁄80	0.0785	0.0393	0.0418	0.0443
6⁄12	0.5236	0.2618	0.2785	…	48⁄96	0.0654	0.0327	0.0348	0.0369
8⁄16	0.3927	0.1963	0.2088	…	64⁄128	0.0491	…	…	0.0277
10⁄20	0.3142	0.1571	0.1671	0.1771	80⁄160	0.0393	…	…	0.0221
12⁄24	0.2618	0.1309	0.1392	0.1476	128⁄256	0.0246	…	…	0.0138
16⁄32	0.1963	0.0982	0.1044	0.1107	…	…	…	…	…

Tooth Numbers.—The American National Standard covers involute splines having tooth numbers ranging from 6 to 60 with a 30- or 37.5-degree pressure angle and from 6 to 100 with a 45-degree pressure angle. In selecting the number of teeth for a given spline application, it is well to keep in mind that there are no advantages to be gained by using odd numbers of teeth and that the diameters of splines with odd tooth numbers, particularly internal splines, are troublesome to measure with pins since no two tooth spaces are diametrically opposite each other.

Types and Classes of Involute Spline Fits.—Two types of fits are covered by the American National Standard for involute splines, the side fit, and the major diameter fit. Dimensional data for flat root side fit, flat root major diameter fit, and fillet root side fit splines are tabulated in this standard for 30-degree pressure angle splines; but for only the fillet root side fit for 37.5- and 45-degree pressure angle splines.

Side Fit: In the side fit, the mating members contact only on the sides of the teeth; major and minor diameters are clearance dimensions. The tooth sides act as drivers and centralize the mating splines.

Major Diameter Fit: Mating parts for this fit contact at the major diameter for centralizing. The sides of the teeth act as drivers. The minor diameters are clearance dimensions.

The major diameter fit provides a minimum effective clearance that will allow for contact and location at the major diameter with a minimum amount of location or centralizing effect by the sides of the teeth. The major diameter fit has only one space width and tooth thickness tolerance which is the same as side fit Class 5.

A fillet root may be specified for an external spline, even though it is otherwise designed to the flat root side fit or major diameter fit standard. An internal spline with a fillet root can be used only for the side fit.

Classes of Tolerances.—This standard includes four classes of tolerances on space width and tooth thickness. This has been done to provide a range of tolerances for selection to suit a design need. The classes are variations of the former single tolerance which is now Class 5 and are based on the formulas shown in the footnote of Table 4. All tolerance classes have the same minimum effective space width and maximum effective tooth thickness limits so that a mix of classes between mating parts is possible.

Table 4. Maximum Tolerances for Space Width and Tooth Thickness of Tolerance Class 5 Splines *ANSI B92.1-1970, R1993*
(Values shown in ten thousandths; 20 = 0.0020)

No. of Teeth	Pitch, P/P_s							
	2.5⁄5 and 3⁄6	4⁄8 and 5⁄10	6⁄12 and 8⁄16	10⁄20 and 12⁄24	16⁄32 and 20⁄40	24⁄48 thru 48⁄96	64⁄128 and 80⁄160	128⁄256
N	Machining Tolerance, *m*							
10	15.8	14.5	12.5	12.0	11.7	11.7	9.6	9.5
20	17.6	16.0	14.0	13.0	12.4	12.4	10.2	10.0
30	18.4	17.5	15.5	14.0	13.1	13.1	10.8	10.5
40	21.8	19.0	17.0	15.0	13.8	13.8	11.4	—
50	23.0	20.5	18.5	16.0	14.5	14.5	—	—
60	24.8	22.0	20.0	17.0	15.2	15.2	—	—
70	—	—	—	18.0	15.9	15.9	—	—
80	—	—	—	19.0	16.6	16.6	—	—
90	—	—	—	20.0	17.3	17.3	—	—
100	—	—	—	21.0	18.0	18.0	—	—
N	Variation Allowance, λ							
10	23.5	20.3	17.0	15.7	14.2	12.2	11.0	9.8
20	27.0	22.6	19.0	17.4	15.4	13.4	12.0	10.6
30	30.5	24.9	21.0	19.1	16.6	14.6	13.0	11.4
40	34.0	27.2	23.0	21.6	17.8	15.8	14.0	—
50	37.5	29.5	25.0	22.5	19.0	17.0	—	—
60	41.0	31.8	27.0	24.2	20.2	18.2	—	—
70	—	—	—	25.9	21.4	19.4	—	—
80	—	—	—	27.6	22.6	20.6	—	—
90	—	—	—	29.3	23.8	21.8	—	—
100	—	—	—	31.0	25.0	23.0	—	—
N	Total Index Variation							
10	20	17	15	15	14	12	11	10
20	24	20	18	17	15	13	12	11
30	28	22	20	19	16	15	14	13
40	32	25	22	20	18	16	15	—
50	36	27	25	22	19	17	—	—
60	40	30	27	24	20	18	—	—
70	—	—	—	26	21	20	—	—
80	—	—	—	28	22	21	—	—
90	—	—	—	29	24	23	—	—
100	—	—	—	31	25	24	—	—
N	Profile Variation							
All	+7 −10	+6 −8	+5 −7	+4 −6	+3 −5	+2 −4	+2 −4	+2 −4

Lead Variation												
L_g, in.	0.3	0.5	1	2	3	4	5	6	7	8	9	10
Variation	2	3	4	5	6	7	8	9	10	11	12	13

For other tolerance classes: Class 4 = 0.71 × Tabulated value

Class 5 = As tabulated in table

Class 6 = 1.40 × Tabulated value

Class 7 = 2.00 × Tabulated value

Fillets and Chamfers.—Spline teeth may have either a flat root or a rounded fillet root.

Flat Root Splines: are suitable for most applications. The fillet that joins the sides to the bottom of the tooth space, if generated, has a varying radius of curvature. Specification of this fillet is usually not required. It is controlled by the form diameter, which is the diameter at the deepest point of the desired true involute form (sometimes designated as TIF).

When flat root splines are used for heavily loaded couplings that are not suitable for fillet root spline application, it may be desirable to minimize the stress concentration in the flat root type by specifying an approximate radius for the fillet.

Because internal splines are stronger than external splines due to their broad bases and high pressure angles at the major diameter, broaches for flat root internal splines are normally made with the involute profile extending to the major diameter.

Fillet Root Splines: are recommended for heavy loads because the larger fillets provided reduce the stress concentrations. The curvature along any generated fillet varies and cannot be specified by a radius of any given value.

External splines may be produced by generating with a pinion-type shaper cutter or with a hob, or by cutting with no generating motion using a tool formed to the contour of a tooth space. External splines are also made by cold forming and are usually of the fillet root design. Internal splines are usually produced by broaching, by form cutting, or by generating with a shaper cutter. Even when full-tip radius tools are used, each of these cutting methods produces a fillet contour with individual characteristics. Generated spline fillets are curves related to the prolate epicycloid for external splines and the prolate hypocycloid for internal splines. These fillets have a minimum radius of curvature at the point where the fillet is tangent to the external spline minor diameter circle or the internal spline major diameter circle and a rapidly increasing radius of curvature up to the point where the fillet comes tangent to the involute profile.

Chamfers and Corner Clearance: In major diameter fits, it is always necessary to provide corner clearance at the major diameter of the spline coupling. This clearance is usually effected by providing a chamfer on the top corners of the external member. This method may not be possible or feasible because of the following:

A) If the external member is roll formed by plastic deformation, a chamfer cannot be provided by the process.

B) A semitopping cutter may not be available.

C) When cutting external splines with small numbers of teeth, a semitopping cutter may reduce the width of the top land to a prohibitive point.

In such conditions, the corner clearance can be provided on the internal spline, as shown in Fig. 2.

When this option is used, the form diameter may fall in the protuberance area.

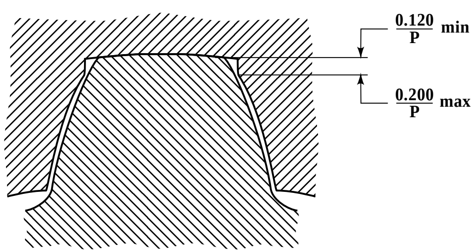


Fig. 2. Internal corner clearance.

Spline Variations.—The maximum allowable variations for involute splines are listed in Table 4.

Profile Variation: The reference profile, from which variations occur, passes through the point used to determine the actual space width or tooth thickness. This is either the pitch point or the contact point of the standard measuring pins.

Profile variation is positive in the direction of the space and negative in the direction of the tooth. Profile variations may occur at any point on the profile for establishing effective fits and are shown in Table 4.

Lead Variations: The lead tolerance for the total spline length applies also to any portion thereof unless otherwise specified.

Out of Roundness: This condition may appear merely as a result of index and profile variations given in Table 4 and requires no further allowance. However, heat treatment and deflection of thin sections may cause out of roundness, which increases index and profile variations. Tolerances for such conditions depend on many variables and are therefore not tabulated. Additional tooth and/or space width tolerance must allow for such conditions.

Eccentricity: Eccentricity of major and minor diameters in relation to the effective diameter of side fit splines should not cause contact beyond the form diameters of the mating splines, even under conditions of maximum effective clearance. This standard does not establish specific tolerances.

Eccentricity of major diameters in relation to the effective diameters of major diameter fit splines should be absorbed within the maximum material limits established by the tolerances on major diameter and effective space width or effective tooth thickness.

If the alignment of mating splines is affected by eccentricity of locating surfaces relative to each other and/or the splines, it may be necessary to decrease the effective and actual tooth thickness of the external splines in order to maintain the desired fit condition. This standard does not include allowances for eccentric location.

Effect of Spline Variations.—Spline variations can be classified as index variations, profile variations, or lead variations.

Index Variations: These variations cause the clearance to vary from one set of mating tooth sides to another. Because the fit depends on the areas with minimum clearance, index variations reduce the effective clearance.

Profile Variations: Positive profile variations affect the fit by reducing effective clearance. Negative profile variations do not affect the fit but reduce the contact area.

Lead Variations: These variations will cause clearance variations and therefore reduce the effective clearance.

Variation Allowance: The effect of individual spline variations on the fit (effective variation) is less than their total, because areas of more than minimum clearance can be altered without changing the fit. The variation allowance is 60 percent of the sum of twice the positive profile variation, the total index variation and the lead variation for the length of engagement. The variation allowances in Table 4 are based on a lead variation for an assumed length of engagement equal to one-half the pitch diameter. Adjustment may be required for a greater length of engagement.

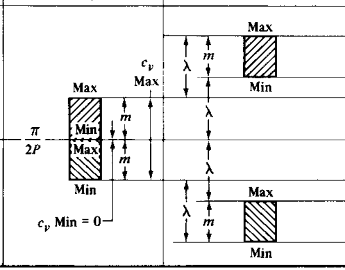
Effective and Actual Dimensions.—Although each space of an internal spline may have the same width as each tooth of a perfect mating external spline, the two may not fit because of variations of index and profile in the internal spline. To allow the perfect external spline to fit in any position, all spaces of the internal spline must then be widened by the amount of interference. The resulting width of these tooth spaces is the *actual* space width of the internal spline. The *effective* space width is the tooth thickness of the perfect mating external spline. The same reasoning applied to an external spline that has variations of index and profile when mated with a perfect internal spline leads to the concept of effective tooth thickness, which exceeds the actual tooth thickness by the amount of the effective variation.

The effective space width of the internal spline minus the effective tooth thickness of the external spline is the effective clearance and defines the fit of the mating parts. (This statement is strictly true only if high points of mating parts come into contact.) Positive effective clearance represents looseness or backlash. Negative effective clearance represents tightness or interference.

Space Width and Tooth Thickness Limits.—The variation of actual space width and actual tooth thickness within the machining tolerance causes corresponding variations of effective dimensions, so that there are four limit dimensions for each component part.

These variations are shown diagrammatically in Table 5.

Table 5. Specification Guide for Space Width and Tooth Thickness
ANSI B92.1-1970, R1993



Dimension	Dimension of Variations, Clearances, and Tolerances on Part (Effective / Actual)	Dimensioning Method: Standard	Alternatives: A	Alternatives: B
Space Width of Internal Spline	Actual Max	Required	Required	Ref.
	Actual Min	Ref.	Ref.	Ref.
	Effective Max	Ref.	Required	Required
(Basic)	$\frac{\pi}{2P}$ (Effective Min / Effective Max)	Required	Required	Required
Tooth Thickness of External Spline	Effective Min	Ref.	Required	Required
	Actual Max	Ref.	Ref.	Ref.
	Actual Min	Required	Required	Ref.

The minimum effective space width is always basic. The maximum effective tooth thickness is the same as the minimum effective space width except for the major diameter fit. The major diameter fit maximum effective tooth thickness is less than the minimum effective space width by an amount that allows for eccentricity between the effective spline and the major diameter. The permissible variation of the effective clearance is divided between the internal and external splines to arrive at the maximum effective space width and the minimum effective tooth thickness. Limits for the actual space width and actual tooth thickness are constructed from suitable variation allowances.

Use of Effective and Actual Dimensions.—Each of the four dimensions for space width and tooth thickness shown in Table 5 has a definite function.

Minimum Effective Space Width and Maximum Effective Tooth Thickness: These dimensions control the minimum effective clearance, and must always be specified.

Minimum Actual Space Width and Maximum Actual Tooth Thickness: These dimensions cannot be used for acceptance or rejection of parts. If the actual space width is less than the minimum without causing the effective space width to be undersized, or if the actual tooth thickness is more than the maximum without causing the effective tooth thickness to be oversized, the effective variation is less than anticipated; such parts are desirable and not defective. The specification of these dimensions as processing reference dimensions is optional. They are also used to analyze undersize effective space width or oversize effective tooth thickness conditions to determine whether or not these conditions are caused by excessive effective variation.

Maximum Actual Space Width and Minimum Actual Tooth Thickness: These dimensions control machining tolerance and limit the effective variation. The spread between these dimensions, reduced by the effective variation of the internal and external spline, is the maximum effective clearance. Where the effective variation obtained in machining is appreciably less than the variation allowance, these dimensions must be adjusted in order to maintain the desired fit.

Maximum Effective Space Width and Minimum Effective Tooth Thickness: These dimensions define the maximum effective clearance but they do not limit the effective variation. They may be used, in addition to the maximum actual space width and minimum actual tooth thickness, to prevent the increase of maximum effective clearance due to reduction of effective variations. The notation "inspection optional" may be added where maximum effective clearance is an assembly requirement, but does not need absolute control. It will indicate, without necessarily adding inspection time and equipment, that the actual space width of the internal spline must be held below the maximum, or the actual tooth thickness of the external spline above the minimum, if machining methods result in less than the allowable variations. Where effective variation needs no control or is controlled by laboratory inspection, these limits may be substituted for maximum actual space width and minimum actual tooth thickness.

Combinations of Involute Spline Types.—Flat root side fit internal splines may be used with fillet root external splines where the larger radius is desired on the external spline for control of stress concentrations. This combination of fits may also be permitted as a design option by specifying for the minimum root diameter of the external, the value of the minimum root diameter of the fillet root external spline and noting this as "optional root."

A design option may also be permitted to provide either flat root internal or fillet root internal by specifying for the maximum major diameter, the value of the maximum major diameter of the fillet root internal spline and noting this as "optional root."

Interchangeability.—Splines made to this standard may interchange with splines made to older standards. Exceptions are listed below.

External Splines: These external splines will mate with older internal splines as follows:

Year	Major Dia. Fit	Flat Root Side Fit	Fillet Root Side Fit
1946	Yes	No (A)[a]	No (A)
1950[b]	Yes (B)	Yes (B)	Yes (C)
1950[c]	Yes (B)	No (A)	Yes (C)
1957 SAE	Yes	No (A)	Yes (C)
1960	Yes	No (A)	Yes (C)

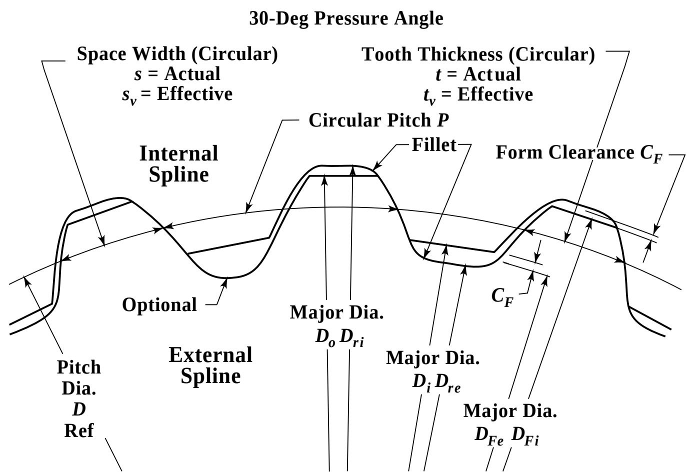
[a] For exceptions A, B, C, see the paragraph on *Exceptions* that follows.
[b] Full dedendum.
[c] Short dedendum.

Internal Splines: These will mate with older external splines as follows:

Year	Major Dia. Fit	Flat Root Side Fit	Fillet Root Side Fit
1946	No (D)[a]	No (E)	No (D)
1950	Yes (F)	Yes	Yes (C)
1957 SAE	Yes (G)	Yes	Yes
1960	Yes (G)	Yes	Yes

[a] For exceptions C, D, E, F, G, see the paragraph on *Exceptions* that follows.

Table 6. Spline Terms, Symbols, and Drawing Data, 30-Degree Pressure Angle, Flat Root Side Fit *ANSI B92.1-1970, R1993*



The fit shown is used in restricted areas (as with tubular parts with wall thickness too small to permit use of fillet roots, and to allow hobbing closer to shoulders, etc.) and for economy (when hobbing, shaping, etc., and using shorter broaches for the internal member).

Press fits are not tabulated because their design depends on the degree of tightness desired and must allow for such factors as the shape of the blank, wall thickness, materila, hardness, thermal expansion, etc. Close tolerances or selective size grouping may be required to limit fit variations.

Drawing Data

Internal Involute Spline Data		External Involute Spline Data	
Flat Root Side Fit		Flat Root Side Fit	
Number of Teeth	xx	Number of Teeth	xx
Pitch	xx/xx	Pitch	xx/xx
Pressure Angle	30°	Pressure Angle	30°
Base Diameter	x.xxxxxx Ref	Base Diameter	x.xxxxxx Ref
Pitch Diameter	x.xxxxxx Ref	Pitch Diameter	x.xxxxxx Ref
Major Diameter	x.xxx max	Major Diameter	x.xxx/x.xxx
Form Diameter	x.xxx	Form Diameter	x.xxx
Minor Diameter	x.xxx/x.xxx	Minor Diameter	x.xxx min
Circular Space Width		Circular Tooth Thickness	
Max Actual	x.xxxx	Max Effective	x.xxxx
Min Effective	x.xxxx	Min Actual	x.xxxx
The following information may be added as required:		The following information may be added as required:	
Max Measurement Between Pins	x.xxx Ref	Min Measurement Over Pins	x.xxxx Ref
Pin Diameter	x.xxxx	Pin Diameter	x.xxxx

The above drawing data are required for the spline specifications. The standard system is shown; for alternate systems, see Table 5. Number of x's indicates number of decimal places normally used.

Exceptions:

A) The external major diameter, unless chamfered or reduced, may interfere with the internal form diameter on flat root side fit splines. Internal splines made to the 1957 and 1960 standards had the same dimensions as shown for the major diameter fit splines in this standard.

B) For 15 teeth or less, the minor diameter of the internal spline, unless chamfered, will interfere with the form diameter of the external spline.

C) For 9 teeth or less, the minor diameter of the internal spline, unless chamfered, will interfere with form diameter of the external spline.

D) The internal minor diameter, unless chamfered, will interfere with the external form diameter.

E) The internal minor diameter, unless chamfered, will interfere with the external form diameter.

F) For 10 teeth or less, the minimum chamfer on the major diameter of the external spline may not clear the internal form diameter.

G) Depending upon the pitch of the spline, the minimum chamfer on the major diameter may not clear the internal form diameter.

Drawing Data.—It is important that uniform specifications be used to show complete information on detail drawings of splines. Much misunderstanding will be avoided by following the suggested arrangement of dimensions and data as given in Table 6. The number of x's indicates the number of decimal places normally used. With this tabulated type of spline specifications, it is usually not necessary to show a graphic illustration of the spline teeth.

Spline Data and Reference Dimensions.—Spline data are used for engineering and manufacturing purposes. Pitch and pressure angle are not subject to individual inspection.

As used in this standard, *reference* is an added notation or modifier to a dimension, specification, or note when that dimension, specification, or note is:

1) Repeated for drawing clarification.

2) Needed to define a nonfeature datum or basis from which a form or feature is generated.

3) Needed to define a nonfeature dimension from which other specifications or dimensions are developed.

4) Needed to define a nonfeature dimension at which toleranced sizes of a feature are specified.

5) Needed to define a nonfeature dimension from which control tolerances or sizes are developed or added as useful information.

Any dimension, specification, or note that is noted “REF” should not be used as a criterion for part acceptance or rejection.

Estimating Key and Spline Sizes and Lengths.—Fig. 1 may be used to estimate the size of American Standard involute splines required to transmit a given torque. It also may be used to find the outside diameter of shafts used with single keys. After the size of the shaft is found, the proportions of the key can be determined from Table 1 on page 2342.

Curve A is for flexible splines with teeth hardened to Rockwell C 55–65. For these splines, lengths are generally made equal to or somewhat greater than the pitch diameter for diameters below 1¼ inches; on larger diameters, the length is generally one-third to two-thirds the pitch diameter. Curve A also applies for a single key used as a fixed coupling, the length of the key being one to one and one-quarter times the shaft diameter. The stress in the shaft, neglecting stress concentration at the keyway, is about 7500 pounds per square inch. See also *Effect of Keyways on Shaft Strength* starting on page 283.

Curve B represents high-capacity single keys used as fixed couplings for stresses of 9500 pounds per square inch, neglecting stress concentration. Key-length is one to one and one-quarter times shaft diameter and both shaft and key are of moderately hard heat-treated steel. This type of connection is commonly used to key commercial flexible couplings to motor or generator shafts.

Curve C is for multiple-key fixed splines with lengths of three-quarters to one and one-quarter times pitch diameter and shaft hardness of 200–300 BHN.

Curve D is for high-capacity splines with lengths one-half to one times the pitch diameter. Hardnesses up to Rockwell C 58 are common and in aircraft applications the shaft is generally hollow to reduce weight.

Curve E represents a solid shaft with 65,000 pounds per square inch shear stress. For hollow shafts with inside diameter equal to three-quarters of the outside diameter the shear stress would be 95,000 pounds per square inch.

Length of Splines: Fixed splines with lengths of one-third the pitch diameter will have the same shear strength as the shaft, assuming uniform loading of the teeth; however, errors in spacing of teeth result in only half the teeth being fully loaded. Therefore, for balanced strength of teeth and shaft the length should be two-thirds the pitch diameter. If weight is not important, however, this may be increased to equal the pitch diameter. In the case of flexible splines, long lengths do not contribute to load carrying capacity when there is misalignment to be accommodated. Maximum effective length for flexible splines may be approximated from Fig. 2.

Formulas for Torque Capacity of Involute Splines.—The formulas for torque capacity of 30-degree involute splines given in the following paragraphs are derived largely from an article “When Splines Need Stress Control” by D. W. Dudley, *Product Engineering*, Dec. 23, 1957.

In the formulas that follow the symbols used are as defined on page 2130 with the following additions: D_h = inside diameter of hollow shaft, inches; K_a = application factor from Table 1; K_m = load distribution factor from Table 2; K_f = fatigue life factor from Table 3; K_w = wear life factor from Table 4; L_e = maximum effective length from Fig. 2, to be used in stress formulas even though the actual length may be greater; T = transmitted torque, pound-inches. For fixed splines without helix modification, the effective length L_e should never exceed $5000\,D^{3.5} \div T$.

Table 1. Spline Application Factors, K_a

Power Source	Type of Load			
	Uniform (Generator-Fan)	Light Shock (Oscillating Pumps, etc.)	Intermittent Shock (Actuating Pumps, etc.)	Heavy Shock (Punches, Shears, etc.)
	Application Factor, K_a			
Uniform (Turbine, Motor)	1.0	1.2	1.5	1.8
Light Shock (Hydraulic Motor)	1.2	1.3	1.8	2.1
Medium Shock (Internal Combustion, Engine)	2.0	2.2	2.4	2.8

Table 2. Load Distribution Factors, K_m, for Misalignment of Flexible Splines

Misalignment, inches per inch	Load Distribution Factor, K_m [a]			
	½-in. Face Width	1-in. Face Width	2-in. Face Width	4-in. Face Width
0.001	1	1	1	1 ½
0.002	1	1	1 ½	2
0.004	1	1 ½	2	2 ½
0.008	1 ½	2	2 ½	3

[a] For fixed splines, K_m=1.

For fixed splines, $K_m = 1$.

Table 3. Fatigue-Life Factors, K_f, for Splines

No. of Torque Cycles[a]	Fatigue-Life Factor, K_f	
	Unidirectional	Fully-reversed
1,000	1.8	1.8
10,000	1.0	1.0
100,000	0.5	0.4
1,000,000	0.4	0.3
10,000,000	0.3	0.2

[a] A torque cycle consists of one start and one stop, not the number of revolutions.

Table 4. Wear Life Factors, K_w, for Flexible Splines

Number of Revolutions of Spline	Life Factor, K_w	Number of Revolutions of Spline	Life Factor, K_w
10,000	4.0	100,000,000	1.0
100,000	2.8	1,000,000,000	0.7
1,000,000	2.0	10,000,000,000	0.5
10,000,000	1.4	…	…

Wear life factors, unlike fatigue life factors given in Table 3, are based on the total number of revolutions of the spline, since each revolution of a flexible spline results in a complete cycle of rocking motion which contributes to spline wear.

Definitions: A *fixed* spline is one which is either shrink fitted or loosely fitted but piloted with rings at each end to prevent rocking of the spline which results in small axial movements that cause wear. A *flexible* spline permits some rocking motion such as occurs when the shafts are not perfectly aligned. This flexing or rocking motion causes axial movement and consequently wear of the teeth. Straight-toothed flexible splines can accommodate only small angular misalignments (less than 1 deg.) before wear becomes a serious problem. For greater amounts of misalignment (up to about 5 deg.), crowned splines are preferable to reduce wear and end-loading of the teeth.

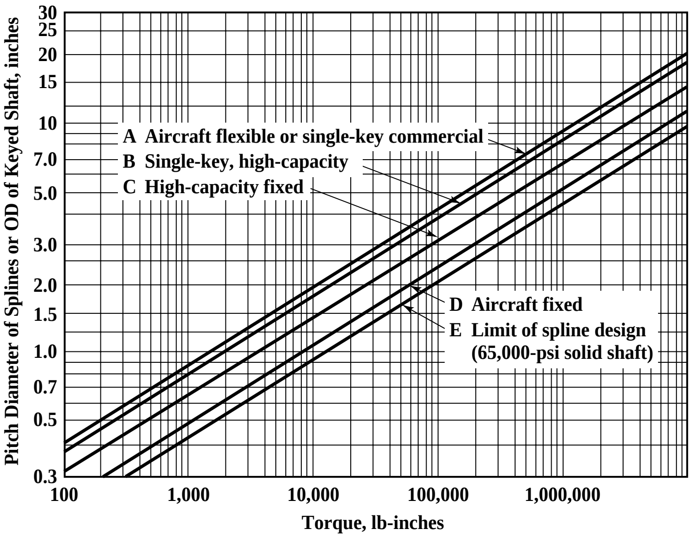


Fig. 1. Chart for Estimating Involute Spline Size Based on Diameter-Torque Relationships

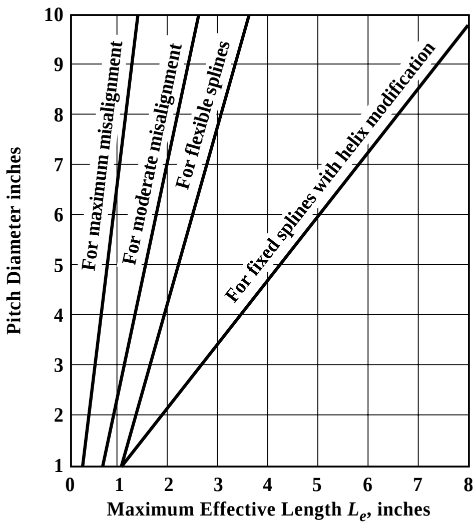


Fig. 2. Maximum Effective Length for Fixed and Flexible Splines

Shear Stress Under Roots of External Teeth: For a transmitted torque T, the torsional shear stress induced in the shaft under the root diameter of an external spline is:

$$S_s = \frac{16TK_a}{\pi D_{re}^3 K_f} \quad \text{for a solid shaft} \tag{1}$$

$$S_s = \frac{16TD_{re}K_a}{\pi(D_{re}^4 - D_h^4)K_f} \quad \text{for a hollow shaft} \tag{2}$$

The computed stress should not exceed the values in Table 5.

Table 5. Allowable Shear Stresses for Splines

Material	Hardness		Max. Allowable Shear Stress, psi
	Brinell	Rockwell C	
Steel	160–200	—	20,000
Steel	230–260	—	30,000
Steel	302–351	33–38	40,000
Surface-hardened Steel	—	48–53	40,000
Case-hardened Steel	—	58–63	50,000
Through-hardened Steel (Aircraft Quality)	—	42–46	45,000

Shear Stress at the Pitch Diameter of Teeth: The shear stress at the pitch line of the teeth for a transmitted torque T is:

$$S_s = \frac{4TK_aK_m}{DNL_etK_f} \tag{3}$$

The factor of 4 in (3) assumes that only half the teeth will carry the load because of spacing errors. For poor manufacturing accuracies, change the factor to 6.

The computed stress should not exceed the values in Table 5.

Compressive Stresses on Sides of Spline Teeth: Allowable compressive stresses on splines are very much lower than for gear teeth since non-uniform load distribution and misalignment result in unequal load sharing and end loading of the teeth.

$$\text{For flexible splines,} \quad S_c = \frac{2TK_mK_a}{DNL_ehK_w} \tag{4}$$

$$\text{For fixed splines,} \quad S_c = \frac{2TK_mK_a}{9DNL_ehK_f} \tag{5}$$

In these formulas, h is the depth of engagement of the teeth, which for flat root splines is $0.9/P$ and for fillet root splines is $1/P$, approximately.

The stresses computed from Formulas (4) and (5) should not exceed the values in Table 6.

Table 6. Allowable Compressive Stresses for Splines

Material	Hardness		Max. Allowable Compressive Stress, psi	
	Brinell	Rockwell C	Straight	Crowned
Steel	160–200	—	1,500	6,000
Steel	230–260	—	2,000	8,000
Steel	302–351	33–38	3,000	12,000
Surface-hardened Steel	—	48–53	4,000	16,000
Case-hardened Steel	—	58–63	5,000	20,000

Bursting Stresses on Splines: Internal splines may burst due to three kinds of tensile stress: 1) tensile stress due to the radial component of the transmitted load; 2) centrifugal tensile stress; and 3) tensile stress due to the tangential force at the pitch line causing bending of the teeth.

$$\text{Radial load tensile stress,}\ S_1 = \frac{T\tan\phi}{\pi D t_w L} \quad (6)$$

where t_w = wall thickness of internal spline = outside diameter of spline sleeve minus spline major diameter, all divided by 2. L = full length of spline.

$$\text{Centrifugal tensile stress,}\ S_2 = \frac{1.656 \times (\text{rpm})^2 (D_{oi}^2 + 0.212 D_{ri}^2)}{1,000,000} \quad (7)$$

where D_{oi} = outside diameter of spline sleeve.

$$\text{Beam loading tensile stress,}\ S_3 = \frac{4T}{D^2 L_e Y} \quad (8)$$

In this equation, Y is the Lewis form factor obtained from a tooth layout. For internal splines of 30-deg. pressure angle a value of Y = 1.5 is a satisfactory estimate. The factor 4 in (8) assumes that only half the teeth are carrying the load.

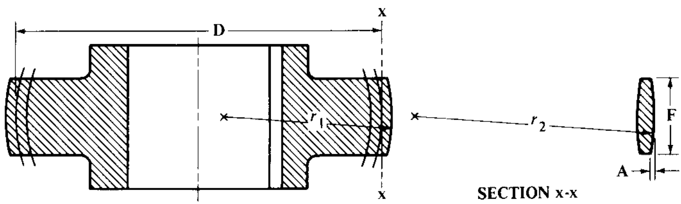
The total tensile stress tending to burst the rim of the external member is: $S_t = [K_a K_m (S_1 + S_3) + S_2]/K_f$; and should be less than those in Table 7.

Table 7. Allowable Tensile Stresses for Splines

Material	Hardness		Max. Allowable Stress, psi
	Brinell	Rockwell C	
Steel	160–200	—	22,000
Steel	230–260	—	32,000
Steel	302–351	33–38	45,000
Surface-hardened Steel	—	48–53	45,000
Case-hardened Steel	—	58–63	55,000
Through-hardened Steel	—	42–46	50,000

Crowned Splines for Large Misalignments.—As mentioned on page 2142, crowned splines can accommodate misalignments of up to about 5 degrees. Crowned splineshave considerably less capacity than straight splines of the same size if both are operating with precise alignment. However, when large misalignments exist, the crowned spline has greater capacity.

American Standard tooth forms may be used for crowned external members so that they may be mated with straight internal members of Standard form.



The accompanying diagram of a crowned spline shows the radius of the crown r_1; the radius of curvature of the crowned tooth, r_2; the pitch diameter of the spline, D; the face width, F; and the relief or crown height A at the ends of the teeth. The crown height A should always be made somewhat greater than one-half the face width multiplied by the tangent of the misalignment angle. For a crown height A, the approximate radius of curvature r_2 is $F^2 \div 8A$, and $r_1 = r_2 \tan \phi$, where ϕ is the pressure angle of the spline.

For a torque T, the compressive stress on the teeth is:

$$S_c = 2290\sqrt{2T \div DNhr_2};$$

and should be less than the value in Table 6.

Fretting Damage to Splines and Other Machine Elements.—Fretting is wear that occurs when cyclic loading, such as vibration, causes two surfaces in intimate contact to undergo small oscillatory motions with respect to each other. During fretting, high points or asperities of the mating surfaces adhere to each other and small particles are pulled out, leaving minute, shallow pits and a powdery debris. In steel parts exposed to air, the metallic debris oxidizes rapidly and forms a red, rustlike powder or sludge; hence, the coined designation “fretting corrosion.”

Fretting is mechanical in origin and has been observed in most materials, including those that do not oxidize, such as gold, platinum, and nonmetallics; hence, the corrosion accompanying fretting of steel parts is a secondary factor.

Fretting can occur in the operation of machinery subject to motion or vibration or both. It can destroy close fits; the debris may clog moving parts; and fatigue failure may be accelerated because stress levels to initiate fatigue in fretted parts are much lower than for undamaged material. Sites for fretting damage include interference fits; splined, bolted, keyed, pinned, and riveted joints; between wires in wire rope; flexible shafts and tubes; between leaves in leaf springs; friction clamps; small amplitude-of-oscillation bearings; and electrical contacts.

Vibration or cyclic loadings are the main causes of fretting. If these factors cannot be eliminated, greater clamping force may reduce movement but, if not effective, may actually worsen the damage. Lubrication may delay the onset of damage; hard plating or surface hardening methods may be effective, not by reducing fretting, but by increasing the fatigue strength of the material. Plating soft materials having inherent lubricity onto contacting surfaces is effective until the plating wears through.

Involute Spline Inspection Methods.—Spline gages are used for routine inspection of production parts.

Analytical inspection, which is the measurement of individual dimensions and variations, may be required:

A) To supplement inspection by gages, for example, where NOT GO composite gages are used in place of NOT GO sector gages and variations must be controlled.

B) To evaluate parts rejected by gages.

C) For prototype parts or short runs where spline gages are not used.

D) To supplement inspection by gages where each individual variation must be restrained from assuming too great a portion of the tolerance between the minimum material actual and the maximum material effective dimensions.

Inspection with Gages.—A variety of gages is used in the inspection of involute splines.

Types of Gages: A composite spline gage has a full complement of teeth. A sector spline gage has two diametrically opposite groups of teeth. A sector plug gage with only two teeth per sector is also known as a "paddle gage." A sector ring gage with only two teeth per sector is also known as a "snap ring gage." A progressive gage is a gage consisting of two or more adjacent sections with different inspection functions. Progressive GO gages are physical combinations of GO gage members that check consecutively first one feature or one group of features, then their relationship to other features. GO and NOT GO gages may also be combined physically to form a progressive gage.

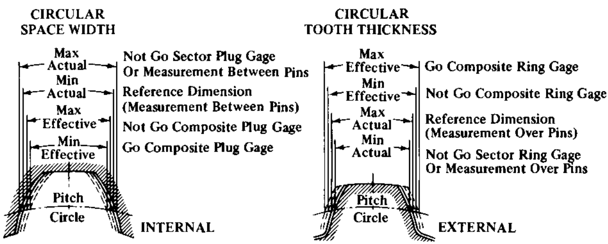


Fig. 3. Space width and tooth-thickness inspection.

GO and NOT GO Gages: GO gages are used to inspect maximum material conditions (maximum external, minimum internal dimensions). They may be used to inspect an individual dimension or the relationship between two or more functional dimensions. They control the minimum looseness or maximum interference.

NOT GO gages are used to inspect minimum material conditions (minimum external, maximum internal dimensions), thereby controlling the maximum looseness or minimum interference. Unless otherwise agreed upon, a product is acceptable only if the NOT GO gage does not enter or go on the part. A NOT GO gage can be used to inspect only one dimension. An attempt at simultaneous NOT GO inspection of more than one dimension could result in failure of such a gage to enter or go on (acceptance of part), even though all but one of the dimensions were outside product limits. In the event all dimensions are outside the limits, their relationship could be such as to allow acceptance.

Effective and Actual Dimensions: The effective space width and tooth thickness are inspected by means of an accurate mating member in the form of a composite spline gage.

The actual space width and tooth thickness are inspected with sector plug and ring gages, or by measurements with pins.

Measurements with Pins.—The actual space width of internal splines, and the actual tooth thickness of external splines, may be measured with pins. These measurements do not determine the fit between mating parts, but may be used as part of the analytic inspection of splines to evaluate the effective space width or effective tooth thickness by approximation.

Formulas for 2-Pin Measurement: For measurement *between* pins of internal splines using the symbols given on page 2130:

1) Find involute of pressure angle at pin center:

$$\text{inv}\,\phi_i = s/D + \text{inv}\,\phi_d - d_i/D_b$$

2) Find the value of ϕ_i, in degrees, in the involute function tables beginning on page 98. Find sec ϕ_i = 1/cosine ϕ_i in the trig tables, pages 94 through 96, using interpolation to obtain higher accuracy.

3) Compute measurement, M_i, between pins:

For even numbers of teeth: $M_i = D_b \sec\phi_i - d_i$

For odd numbers of teeth: $M_i = (D_b \cos 90°/N) \sec\phi_i - d_i$

where: $d_i = 1.7280/P$ for 30° and 37.5° standard pressure angle (ϕ_D) splines

$d_i = 1.9200/P$ for 45° pressure angle splines

For measurement *over* pins of external splines:

1) Find involute of pressure angle at pin center:

$$\text{inv}\,\phi_e = t/D + \text{inv}\,\phi_D + d_e/D_b - \pi/N$$

2) Find the value of ϕ_e and sec ϕ_e from the involute function tables beginning on page 98.

3) Compute measurement, M_e, over pins:

For even numbers of teeth: $M_e = D_b \sec\phi_e + d_e$

For odd numbers of teeth: $M_i = (D_b \cos 90°/N) \sec\phi_e - d_e$

where $d_e = 1.9200/P$ for all external splines

Example: Find the measurement between pins for *maximum* actual space width of an internal spline of 30° pressure angle, tolerance class 4, $^3/_6$ diametral pitch, and 20 teeth.

The maximum actual space width to be substituted for *s* in Step 1 above is obtained as follows: In Table 5, page 2137, the maximum actual space width is the sum of the minimum effective space width (second column) and $\lambda + m$ (third column). The minimum effective space width s_v from Table 2, page 2131, is $\pi/2P = \pi/(2 \times 3)$. The values of λ and *m* from Table 4, page 2134, are, for a class 4 fit, 3⁄6 diametral pitch, 20-tooth spline: $\lambda = 0.0027 \times 0.71 = 0.00192$; and $m = 0.00176 \times 0.71 = 0.00125$, so that $s = 0.52360 + 0.00192 + 0.00125 = 0.52677$.

Other values required for Step 1 are:

$D = N/P = 20/3 = 6.66666$

$inv\phi_D$ = inv 30° = 0.053751 from a calculator

$d_i = 1.7280/3 = 0.57600$

$D_b = D \cos\phi_D = 6.66666 \times 0.86603 = 5.77353$

The computation is made as follows:

1) $\text{inv}\,\phi_i = 0.52677/6.66666 + 0.053751 - 0.57600/5.77353 = 0.03300$

2) From a calculator, $\phi_i = 25°46.18'$ and $\sec\phi_i = 1.11044$

3) $M_i = 5.77353 \times 1.11044 - 0.57600 = 5.8352$ inches

American National Standard Metric Module Splines.—ANSI B92.2M-1980 (R1989) is the American National Standards Institute version of the International Standards Organization involute spline standard. It is not a "soft metric" conversion of any previous, inch-based, standard,* and splines made to this hard metric version are not intended for use with components made to the B92.1 or other, previous standards. The ISO 4156 Standard from

* A "soft" conversion is one in which dimensions in inches, when multiplied by 25.4 will, after being appropriately rounded off, provide equivalent dimensions in millimeters. In a "hard" system the tools of production, such as hobs, do not bear a usable relation to the tools in another system; i.e., a 10 diametral pitch hob calculates to be equal to a 2.54 module hob in the metric module system, a hob that does not exist in the metric standard.

which this one is derived is the result of a cooperative effort between the ANSI B92 committee and other members of the ISO/TC 14-2 involute spline committee.

Many of the features of the previous standard, ANSI B92.1-1970 (R1993), have been retained such as: 30-, 37.5-, and 45-degree pressure angles; flat root and fillet root side fits; the four tolerance classes 4, 5, 6, and 7; tables for a single class of fit; and the effective fit concept.

Among the major differences are: use of modules of from 0.25 through 10 mm in place of diametral pitch; dimensions in millimeters instead of inches; the “basic rack”; removal of the major diameter fit; and use of ISO symbols in place of those used previously. Also, provision is made for calculating three defined clearance fits.

The Standard recognizes that proper assembly between mating splines is dependent only on the spline being within effective specifications from the tip of the tooth to the form diameter. Therefore, the internal spline major diameter is shown as a maximum dimension and the external spline minor diameter is shown as a minimum dimension. The minimum internal major diameter and the maximum external minor diameter must clear the specified form diameter and thus require no additional control. All dimensions are for the finished part; any compensation that must be made for operations that take place during processing, such as heat treatment, must be considered when selecting the tolerance level for manufacturing.

The Standard provides the same internal minimum effective space width and external maximum effective tooth thickness for all tolerance classes. This basic concept makes possible interchangeable assembly between mating splines regardless of the tolerance class of the individual members, and permits a tolerance class “mix” of mating members. This arrangement is often an advantage when one member is considerably less difficult to produce than its mate, and the “average” tolerance applied to the two units is such that it satisfies the design need. For example, by specifying Class 5 tolerance for one member and Class 7 for its mate, an assembly tolerance in the Class 6 range is provided.

If a fit given in this Standard does not satisfy a particular design need, and a specific clearance or press fit is desired, the change shall be made only to the external spline by a reduction of, or an increase in, the effective tooth thickness and a like change in the actual tooth thickness. The minimum effective space width is always basic and this basic width should always be retained when special designs are derived from the concept of this Standard.

Spline Terms and Definitions: The spline terms and definitions given for American National Standard ANSI B92.1-1970 (R1993) described in the preceding section, may be used in regard to ANSI B92.2M-1980 (R1989). The 1980 Standard utilizes ISO symbols in place of those used in the 1970 Standard; these differences are shown in Table 1.

Table 1. Comparison of Symbols Used in ***ANSI B92.2M-1980 (R1989)*** **and Those in** ***ANSI B92.1-1970, R1993***

Symbol			Symbol		
B92.2M	B92.1	Meaning of Symbol	B92.2M	B92.1	Meaning of Symbol
c	…	theoretical clearance	m	…	module
c_v	c_v	effective clearance	…	P	diametral pitch
c_F	c_F	form clearance	…	P_s	stub pitch = 2*P*
D	D	pitch diameter	P_b	…	base pitch
DB	D_b	base diameter	p	p	circular pitch
d_{ce}	D_{ce}	pin contact diameter, external spline	π	π	3.141592654
d_{ci}	D_{ci}	pin contact diameter, internal spline	rfe	r_f	fillet rad., ext. spline
DEE	D_o	major diam., ext. spline	rfi	r_f	fillet rad., int. spline
DEI	D_{ri}	major diam., int. spline	E_{bsc}	s_v min	basic circular space width
DFE	D_{Fe}	form diam., ext. spline	E_{max}	s	max. actual circular space width
DFI	D_{Fi}	form diam., int. spline	E_{min}	s	min. actual circular space width
DIE	D_{re}	minor diam., ext. spline	EV	s_v	effective circular space width
DII	D_i	minor diam., int. spline	S_{bsc}	t_v max	basic circular tooth thickness
DRE	d_e	pin diam., ext. spline	S_{max}	t	max. actual circular tooth thick.
DRI	d_i	pin diam., int. spline	S_{min}	t	min. actual circular tooth thick.
h_s	…	see Figs. 1a, 1b, 1c, and 1d	SV	t_v	effective circular tooth thick.
λ	λ	effective variation	α	ϕ	pressure angle
INV α	…	involute $\alpha=\tan\alpha - \text{arc}\ \alpha$	α_D	ϕ_D	standard pressure angle
KE	K_e	change factor, ext. spline	α_{ci}	ϕ_{ci}	press. angle at pin contact diameter, internal spline
KI	K_i	change factor, int. spline	α_{ce}	ϕ_{ce}	press. angle at pin contact diameter, external spline
g	L	spline length	α_i	ϕ_i	press. angle at pin center, internal spline
g_w	…	active spline length	α_e	ϕ_e	press. angle at pin center, external spline
$g\gamma$	…	length of engagement	α_{Fe}	ϕ_F	press. angle at form diameter, external spline
T	m	machining tolerance	α_{Fi}	ϕ_F	press. angle at form diameter, internal spline
MRE	M_e	meas. over 2 pins, ext. spline	es	…	ext. spline cir. tooth thick.modification for required fit class=c_v min (Table 3)
MRI	M_i	meas. bet. 2 pins, int. spline	h, f, e, or d	…	tooth thick, size modifiers (called fundamental deviation in ISO R286), Table 3
Z	N	number of teeth	H	…	space width size modifier (called fundamental deviation in ISO R286), Table 3

Dimensions and Tolerances: Dimensions and tolerances of splines made to the 1980 Standard may be calculated using the formulas given in Table 2. These formulas are for metric module splines in the range of from 0.25 to 10 mm metric module of side-fit design and having pressure angles of 30-, 37.5-, and 45-degrees. The standard modules in the system are: 0.25; 0.5; 0.75; 1; 1.25; 1.5; 1.75; 2; 2.5; 3; 4; 5; 6; 8; and 10. The range of from 0.5 to 10 module applies to all splines except 45-degree fillet root splines; for these, the range of from 0.25 to 2.5 module applies.

Table 2. Formulas for Dimensions and Tolerances for All Fit Classes—Metric Module Involute Splines

Term	Symbol	Formula			
		30-Degree Flat Root	30-Degree Fillet Root	37.5-Degree Fillet Root	45-Degree Fillet Root
		0.5 to 10 module	0.5 to 10 module	0.5 to 10 module	0.25 to 2.5 module
Pitch Diameter	D	mZ			
Base Diameter	DB	$mZ \cos \alpha_D$			
Circular Pitch	p	πm			
Base Pitch	p_b	$\pi m \cos \alpha_D$			
Tooth Thick Mod	es	According to selected fit class, H/h, H/f, H/e, or H/d (see Table 3)			
Min Maj. Diam. Int	DEI min	$m(Z + 1.5)$	$m(Z + 1.8)$	$m(Z + 1.4)$	$m(Z + 1.2)$
Max Maj Diam. Int.	DEI max	DEI min + $(T + \lambda)/\tan \alpha_D$ (see Note 1)			
Form Diam, Int.	DFI	$m(Z + 1) + 2c_F$	$m(Z + 1) + 2c_F$	$m(Z + 0.9) + 2c_F$	$m(Z + 0.8) + 2c_F$
Min Minor Diam, Int	DII min	$DFE + 2c_F$ (see Note 2)			
Max Minor Diam, Int	DII max	DII min + $(0.2m^{0.667} - 0.01m^{-0.5})$[a]			
Cir Space Width,					
Basic	E_{bsc}	$0.5\pi m$			
Min Effective	EV min	$0.5\pi m$			
Max Actual	E max	EV min + $(T + \lambda)$ for classes 4, 5, 6, and 7 (see Table 4 for $T + \lambda$)			
Min Actual	E min	EV min + λ (see text on page 2153 for λ)			
Max Effective	EV max	E max − λ (see text on page 2153 for λ)			
Max Major Diam, Ext	DEE max	$m(Z + 1) - es/\tan \alpha_D$[b]	$m(Z + 1) - es/\tan \alpha_D$[b]	$m(Z + 0.9) - es/\tan \alpha_D$[b]	$m(Z + 0.8) - es/\tan \alpha_D$[b]
Min Major Diam. Ext	DEE min	DEE max − $(0.2m^{0.667} - 0.01m^{-0.5})$[a]			
Form Diam, External	DFE	$2 \times \sqrt{(0.5DB)^2 + \left[0.5D\sin\alpha_D - \frac{h_s + \left(\frac{0.5es}{\tan\alpha_D}\right)}{\sin\alpha_D}\right]^2}$			
Max Minor Diam, Ext	DIE max	$m(Z - 1.5) - es/\tan \alpha_D$[b]	$m(Z - 1.8) - es/\tan \alpha_D$[b]	$m(Z - 1.4) - es/\tan \alpha_D$[b]	$m(Z - 1.2) - es/\tan \alpha_D$[b]

Table 2. Formulas for Dimensions and Tolerances for All Fit Classes—Metric Module Involute Splines

Term	Symbol	Formula			
		30-Degree Flat Root	30-Degree Fillet Root	37.5-Degree Fillet Root	45-Degree Fillet Root
		0.5 to 10 module	0.5 to 10 module	0.5 to 10 module	0.25 to 2.5 module
Min Minor Diam, Ext	DIE min	DIE max – $(T + \lambda)/\tan \alpha_D$ (see Note 1)			
Cir Tooth Thick,					
Basic	S_{bsc}	$0.5\pi m$			
Max Effective	SV max	$S_{bsc} - es$			
Min Actual	S min	SV max – $(T + \lambda)$ for classes 4, 5, 6, and 7 (see Table 4 for $T + \lambda$)			
Max Actual	S max	SV max – λ (see text on page 2153 for λ)			
Min Effective	SV min	S min + λ (see text on page 2153 for λ)			
Total Tolerance on Circular Space Width or Tooth Thickness	$(T + \lambda)$	See formulas in Table 4			
Machining Tolerance on Circular Space Width or Tooth Thickness	T	$T = (T + \lambda)$ from Table 4 – λ from text on page 2153.			
Effective Variation Allowed on Circular Space Width or Tooth Thickness	λ	See text on page 2153.			
Form Clearance	c_F	$0.1m$			
Rack Dimension	h_s	$0.6m$(see Fig. 1a)	$0.6m$(see Fig. 1b)	$0.55m$(see Fig. 1c)	$0.5m$(see Fig. 1d)

[a] Values of $(0.2m^{0.667} - 0.01m^{-0.5})$ are as follows: for 10 module, 0.93; for 8 module, 0.80; for 6 module, 0.66; for 5 module, 0.58; for 4 module, 0.50; for 3 module, 0.41; for 2.5 module, 0.36; for 2 module, 0.31; for 1.75 module, 0.28; for 1.5 module, 0.25; for 1.25 module, 0.22; for 1 module, 0.19; for 0.75 module, 0.15; for 0.5 module, 0.11; and for 0.25 module, 0.06.

[b] See Table 6 for values of $es/\tan \alpha_D$.

Note 1: Use $(T + \lambda)$ for class 7 from Table 4.

Note 2: For all types of fit, always use the DFE value corresponding to the H/h fit.

Fit Classes: Four classes of side fit splines are provided: spline fit class H/h having a minimum effective clearance, $c_v = es = 0$; classes H/f, H/e, and H/d having tooth thickness modifications, es, of f, e, and d, respectively, to provide progressively greater effective clearance c_v. The tooth thickness modifications h, f, e, and d in Table 3 are fundamental deviations selected from ISO R286, "ISO System of Limits and Fits." They are applied to the external spline by shifting the tooth thickness total tolerance below the basic tooth thickness by the amount of the tooth thickness modification to provide a prescribed minimum effective clearance c_v.

Table 3. Tooth Thickness Modification, *es*, for Selected Spline Fit Classes

Pitch Diameter in mm, *D*	External Splines[a] Selected Fit Class d	e	f	h	Pitch Diameter in mm, *D*	External Splines[a] Selected Fit Class d	e	f	h
	Tooth Thickness Modification (Reduction) Relative to Basic Tooth Thickness at Pitch Diameter, *es*, in mm					Tooth Thickness Modification (Reduction) Relative to Basic Tooth Thickness at Pitch Diameter, *es*, in mm			
≤ 3	0.020	0.014	0.006	0	> 120 to 180	0.145	0.085	0.043	0
> 3 to 6	0.030	0.020	0.010	0	> 180 to 250	0.170	0.100	0.050	0
> 6 to 10	**0.040**	0.025	0.013	0	> 250 to 315	0.190	0.110	0.056	0
> 10 to 18	0.050	0.032	0.016	0	> 315 to 400	0.210	**0.125**	**0.062**	0
> 18 to 30	0.065	0.040	0.020	0	> 400 to 500	0.230	0.135	**0.068**	0
> 30 to 50	0.080	0.050	0.025	0	> 500 to 630	0.260	0.145	**0.076**	0
> 50 to 80	0.100	0.060	0.030	0	> 630 to 800	0.290	0.160	**0.080**	0
> 80 to 120	0.120	**0.072**	0.036	0	> 800 to 1000	0.320	**0.170**	**0.086**	0

[a] Internal splines are fit class H and have space width modification from basic space width equal to zero; thus, an H/h fit class has effective clearance $c_v = 0$.

Note: The values listed in this table are taken from ISO R286 and have been computed on the basis of the geometrical mean of the size ranges shown. Values in **boldface** type do not comply with any documented rule for rounding but are those used by ISO R286; they are used in this table to comply with established international practice.

Basic Rack Profiles: The basic rack profile for the standard pressure angle splines are shown in see Fig. 1a, 1b, 1c, and 1d. The dimensions shown are for maximum material condition and for fit class H/h.

Spline Machining Tolerances and Variations.—The total tolerance $(T + \lambda)$, Table 4, is the sum of Effective Variation, λ, and a Machining Tolerance, T.

Table 4. Space Width and Tooth Thickness Total Tolerance, $(T + \lambda)$, in Millimeters

Spline Tolerance Class	Formula for Total Tolerance, $(T + \lambda)$	Spline Tolerance Class	Formula for Total Tolerance, $(T + \lambda)$	In these formulas, i* and i** are tolerance units based upon pitch diameter and tooth thickness, respectively:
4	10i* + 40i**	6	25i* + 100i**	$i^* = 0.001(0.45\sqrt[3]{D} + 0.001D)$ for $D \le 500$ mm
5	16i* + 64i**	7	40i* + 160i**	$= 0.001(0.004D + 2.1)$ for $D > 500$ mm
				$i^{**} = 0.001(0.45\sqrt[3]{S_{bsc}} + 0.001S_{bsc})$

Effective Variation: The effective variation, λ, is the combined effect that total index variation, positive profile variation, and tooth alignment variation has on the effective fit of mating involute splines. The effect of the individual variations is less than the sum of the allowable variations because areas of more than minimum clearance can have profile, tooth alignment, or index variations without changing the fit. It is also unlikely that these variations would occur in their maximum amounts simultaneously on the same spline. For this reason, total index variation, total profile variation, and tooth alignment variation are used to calculate the combined effect by the following formula:

$$\lambda = 0.6\sqrt{(F_p)^2 + (f_f)^2 + (F_\beta)^2} \text{ millimeters}$$

The above variation is based upon a length of engagement equal to one-half the pitch diameter of the spline; adjustment of λ may be required for a greater length of engagement. Formulas for values of F_p, f_f, and F_β used in the above formula are given in Table 5.

Table 5. Formulas for F_p, f_f, and F_β used to calculate λ

Spline Tolerance Class	Total Index Variation, in mm, F_p	Total Profile Variation, in mm, f_f	Total Lead Variation, in mm, F_β
4	$0.001(2.5\sqrt{mZ\pi/2} + 6.3)$	$0.001\,[1.6m(1 + 0.0125Z) + 10]$	$0.001(0.8\sqrt{g} + 4)$
5	$0.001(3.55\sqrt{mZ\pi/2} + 9)$	$0.001\,[2.5m(1 + 0.0125Z) + 16]$	$0.001(1.0\sqrt{g} + 5)$
6	$0.001(5\sqrt{mZ\pi/2} + 12.5)$	$0.001\,[4m(1 + 0.0125Z) + 25]$	$0.001(1.25\sqrt{g} + 6.3)$
7	$0.001(7.1\sqrt{mZ\pi/2} + 18)$	$0.001\,[6.3m(1 + 0.0125Z) + 40]$	$0.001(2\sqrt{g} + 10)$

g = length of spline in millimeters.

Table 6. Reduction, $es/\tan \alpha_D$, of External Spline Major and Minor Diameters Required for Selected Fit Classes

Pitch Diameter D in mm	Standard Pressure Angle, in Degrees									
	30	37.5	45	30	37.5	45	30	37.5	45	All
	Classes of Fit									
	d			e			f			h
	$es/\tan \alpha_D$ in millimeters									
≤3	0.035	0.026	0.020	0.024	0.018	0.014	0.010	0.008	0.006	0
>3 to 6	0.052	0.039	0.030	0.035	0.026	0.020	0.017	0.013	0.010	0
> 6 to 10	0.069	0.052	0.040	0.043	0.033	0.025	0.023	0.017	0.013	0
> 10 to 18	0.087	0.065	0.050	0.055	0.042	0.032	0.028	0.021	0.016	0
> 18 to 30	0.113	0.085	0.065	0.069	0.052	0.040	0.035	0.026	0.020	0
> 30 to 50	0.139	0.104	0.080	0.087	0.065	0.050	0.043	0.033	0.025	0
> 50 to 80	0.173	0.130	0.100	0.104	0.078	0.060	0.052	0.039	0.030	0
> 80 to 120	0.208	0.156	0.120	0.125	0.094	0.072	0.062	0.047	0.036	0
> 120 to 180	0.251	0.189	0.145	0.147	0.111	0.085	0.074	0.056	0.043	0
> 180 to 250	0.294	0.222	0.170	0.173	0.130	0.100	0.087	0.065	0.050	0
> 250 to 315	0.329	0.248	0.190	0.191	0.143	0.110	0.097	0.073	0.056	0
> 315 to 400	0.364	0.274	0.210	0.217	0.163	0.125	0.107	0.081	0.062	0
> 400 to 500	0.398	0.300	0.230	0.234	0.176	0.135	0.118	0.089	0.068	0
> 500 to 630	0.450	0.339	0.260	0.251	0.189	0.145	0.132	0.099	0.076	0
> 630 to 800	0.502	0.378	0.290	0.277	0.209	0.160	0.139	0.104	0.080	0
> 800 to 1000	0.554	0.417	0.320	0.294	0.222	0.170	0.149	0.112	0.086	0

These values are used with the applicable formulas in Table 2.

Machining Tolerance: A value for machining tolerance may be obtained by subtracting the effective variation, λ, from the total tolerance ($T + \lambda$). Design requirements or specific processes used in spline manufacture may require a different amount of machining tolerance in relation to the total tolerance.

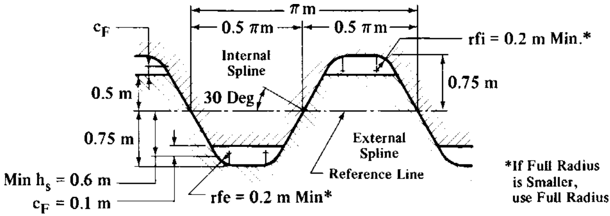


Fig. 1a. Profile of Basic Rack for 30° Flat Root Spline

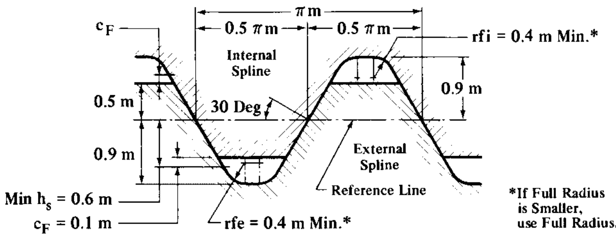


Fig. 1b. Profile of Basic Rack for 30° Fillet Root Spline

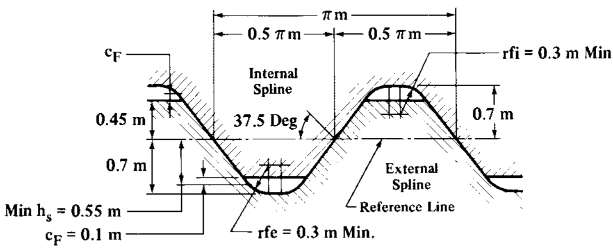


Fig. 1c. Profile of Basic Rack for 37.5° Fillet Root Spline

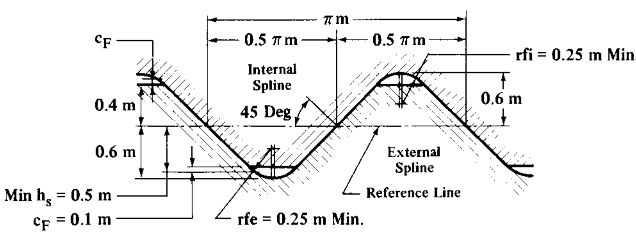


Fig. 1d. Profile of Basic Rack for 45° Fillet Root Spline

British Standard Striaght Splines.—British Standard BS 2059:1953, “Straight-sided Splines and Serrations”, was introduced because of the widespread development and use of splines and because of the increasing use of involute splines it was necessary to provide a separate standard for straight-sided splines. BS 2059 was prepared on the hole basis, the hole being the constant member, and provide for different fits to be obtained by varying the size of the splined or serrated shaft. Part 1 of the standard deals with 6 splines only, irrespective of the shaft diameter, with two depths termed shallow and deep. The splines are bottom fitting with top clearance.

The standard contains three different grades of fit, based on the principle of variations in the diameter of the shaft at the root of the splines, in conjunction with variations in the widths of the splines themselves. Fit 1 represents the condition of closest fit and is designed for minimum backlash. Fit 2 has a positive allowance and is designed for ease of assembly, and Fit 3 has a larger positive allowance for applications that can accept such clearances. all these splines allow for clearance on the sides of the splines (the widths), but in Fit 1, the minor diameters of the hole and the shaft may be of identical size.

Assembly of a splined shaft and hole requires consideration of the designed profile of each member, and this consideration should concentrate on the maximum diameter of the shafts and the widths of external splines, in association with the minimum diameter of the hole and the widths of the internal splineways. In other words, both internal and external splines are in the maximum metal condition. The accuracy of spacing of the splines will affect the quality of the resultant fit. If angular positioning is inaccurate, or the splines are not parallel with the axis, there will be interference between the hole and the shaft.

Part 2 of the Standard deals with straight-sided 90° serrations having nominal diameters from 0.25 to 6.0 inches. Provision is again made for three grades of fits, the basic constant being the serrated hole size. Variations in the fits of these serrations is obtained by varying the sizes of the serrations on the shaft, and the fits are related to flank bearing, the depth of engagement being constant for each size and allowing positive clearance at crest and root.

Fit 1 is an interference fit intended for permanent or semi-permanent ass emblies. Heating to expand the internally-serrated member is needed for assembly. Fit 2 is a transition fit intended for assemblies that require accurate location of the serrated members, but must allow disassembly. In maximum metal conditions, heating of the outside member may be needed for assembly. Fit. 3 is a clearance or sliding fit, intended for general applications.

Maximum and minimum dimensions for the various features are shown in the Standard for each class of fit. Maximum metal conditions presupposes that there are no errors of form such as spacing, alignment, or roundness of hole or shaft. Any compensation needed for such errors may require reduction of a shaft diameter or enlargement of a serrated bore, but the measured effective size must fall within the specified limits.

British Standard BS 3550:1963, “Involute Splines”, is complementary to BS 2059, and the basic dimensions of all the sizes of splines are the same as those in the ANSI/ASME B5.15-1960, for major diameter fit and side fit. The British Standard uses the same terms and symbols and provides data and guidance for design of straight involute splines of 30° pressure angle, with tables of limiting dimensions. The standard also deals with manufacturing errors and their effect on the fit between mating spline elements. The range of splines covered is:

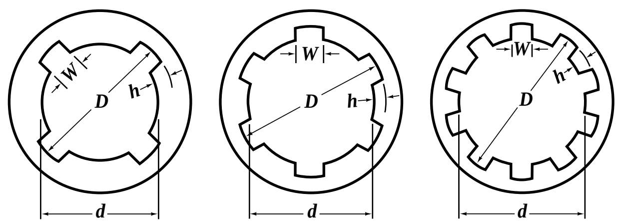
Side fit, flat root, 2.5/5.0 to 32/64 pitch, 6 to 60 splines.

Major diameter, flat root, 3.0/6.0 to 16/32 pitch, 6 to 60 splines.

Side fit, fillet root, 2.5/5.0 to 48/96 pitch, 6 to 60 splines.

British Standard BS 6186, Part 1:1981, “Involute Splines, Metric Module, Side Fit” is identical with sections 1 and 2 of ISO 4156 and with ANSI/ASME B92.2M-1980 (R1989) “Straight Cylindrical Involute Splines, Metric Module, Side Fit – Generalities, Dimensions and Inspection”.

Table 1. S.A.E. Standard Splined Fittings



4-Spline Fittings

Nom. Diam	For All Fits D Min.	D Max.	W Min.	W Max.	4A—Permanent Fit d Min.	d Max.	h Min.	h Max.	T[a]
¾	0.749	0.750	0.179	0.181	0.636	0.637	0.055	0.056	78
⅞	0.874	0.875	0.209	0.211	0.743	0.744	0.065	0.066	107
1	0.999	1.000	0.239	0.241	0.849	0.850	0.074	0.075	139
1⅛	1.124	1.125	0.269	0.271	0.955	0.956	0.083	0.084	175
1¼	1.249	1.250	0.299	0.301	1.061	1.062	0.093	0.094	217
1⅜	1.374	1.375	0.329	0.331	1.168	1.169	0.102	0.103	262
1½	1.499	1.500	0.359	0.361	1.274	1.275	0.111	0.112	311
1⅝	1.624	1.625	0.389	0.391	1.380	1.381	0.121	0.122	367
1¾	1.749	1.750	0.420	0.422	1.486	1.487	0.130	0.131	424
2	1.998	2.000	0.479	0.482	1.698	1.700	0.148	0.150	555
2¼	2.248	2.250	0.539	0.542	1.910	1.912	0.167	0.169	703
2½	2.498	2.500	0.599	0.602	2.123	2.125	0.185	0.187	865
3	2.998	3.000	0.720	0.723	2.548	2.550	0.223	0.225	1249

Nom. Diam.	4-Spline Fittings, 4B—To Slide—No Load d Min.	d Max.	h Min.	h Max.	T[a]	6-Spline Fittings, For All Fits D Min.	D Max.	W Min.	W Max.
¾	0.561	0.562	0.093	0.094	123	0.749	0.750	0.186	0.188
⅞	0.655	0.656	0.108	0.109	167	0.874	0.875	0.217	0.219
1	0.749	0.750	0.124	0.125	219	0.999	1.000	0.248	0.250
1⅛	0.843	0.844	0.140	0.141	277	1.124	1.125	0.279	0.281
1¼	0.936	0.937	0.155	0.156	341	1.249	1.250	0.311	0.313
1⅜	1.030	1.031	0.171	0.172	414	1.374	1.375	0.342	0.344
1½	1.124	1.125	0.186	0.187	491	1.499	1.500	0.373	0.375
1⅝	1.218	1.219	0.202	0.203	577	1.624	1.625	0.404	0.406
1¾	1.311	1.312	0.218	0.219	670	1.749	1.750	0.436	0.438
2	1.498	1.500	0.248	0.250	875	1.998	2.000	0.497	0.500
2¼	1.685	1.687	0.279	0.281	1106	2.248	2.250	0.560	0.563
2½	1.873	1.875	0.310	0.312	1365	2.498	2.500	0.622	0.625
3	2.248	2.250	0.373	0.375	1969	2.998	3.000	0.747	0.750

[a] See note at end of Table 4.

Table 2. S.A.E. Standard Splined Fittings

6-Spline Fittings									
Nom. Diam.	6A—Permanent Fit			6B—To Slide—No Load			6C—To Slide Under Load		
	d			*d*			*d*		
	Min.	Max.	*T*[a]	Min.	Max.	*T*[a]	Min.	Max.	*T*[a]
3⁄4	0.674	0.675	80	0.637	0.638	117	0.599	0.600	152
7⁄8	0.787	0.788	109	0.743	0.744	159	0.699	0.700	207
1	0.899	0.900	143	0.849	0.850	208	0.799	0.800	270
1 1⁄8	1.012	1.013	180	0.955	0.956	263	0.899	0.900	342
1 1⁄4	1.124	1.125	223	1.062	1.063	325	0.999	1.000	421
1 3⁄8	1.237	1.238	269	1.168	1.169	393	1.099	1.100	510
1 1⁄2	1.349	1.350	321	1.274	1.275	468	1.199	1.200	608
1 5⁄8	1.462	1.463	376	1.380	1.381	550	1.299	1.300	713
1 3⁄4	1.574	1.575	436	1.487	1.488	637	1.399	1.400	827
2	1.798	1.800	570	1.698	1.700	833	1.598	1.600	1080
2 1⁄4	2.023	2.025	721	1.911	1.913	1052	1.798	1.800	1367
2 1⁄2	2.248	2.250	891	2.123	2.125	1300	1.998	2.000	1688
3	2.698	2.700	1283	2.548	2.550	1873	2.398	2.400	2430

[a] See note at end of Table 4.

10-Spline Fittings							
Nom. Diam.	For All Fits				10A—Permanent Fit		
	D		*W*		*d*		
	Min.	Max.	Min.	Max.	Min.	Max.	*T*[a]
3⁄4	0.749	0.750	0.115	0.117	0.682	0.683	120
7⁄8	0.874	0.875	0.135	0.137	0.795	0.796	165
1	0.999	1.000	0.154	0.156	0.909	0.910	215
1 1⁄8	1.124	1.125	0.174	0.176	1.023	1.024	271
1 1⁄4	1.249	1.250	0.193	0.195	1.137	1.138	336
1 3⁄8	1.374	1.375	0.213	0.215	1.250	1.251	406
1 1⁄2	1.499	1.500	0.232	0.234	1.364	1.365	483
1 5⁄8	1.624	1.625	0.252	0.254	1.478	1.479	566
1 3⁄4	1.749	1.750	0.271	0.273	1.592	1.593	658
2	1.998	2.000	0.309	0.312	1.818	1.820	860
2 1⁄4	2.248	2.250	0.348	0.351	2.046	2.048	1088
2 1⁄2	2.498	2.500	0.387	0.390	2.273	2.275	1343
3	2.998	3.000	0.465	0.468	2.728	2.730	1934
3 1⁄2	3.497	3.500	0.543	0.546	3.182	3.185	2632
4	3.997	4.000	0.621	0.624	3.637	3.640	3438
4 1⁄2	4.497	4.500	0.699	0.702	4.092	4.095	4351
5	4.997	5.000	0.777	0.780	4.547	4.550	5371
5 1⁄2	5.497	5.500	0.855	0.858	5.002	5.005	6500
6	5.997	6.000	0.933	0.936	5.457	5.460	7735

Table 3. S.A.E. Standard Splined Fittings

10-Spline Fittings						
	10B—To Slide—No Load			10C—To Slide Under Load		
Nom. Diam.	*d*			*d*		
	Min.	Max.	*T*[a]	Min.	Max.	*T*[a]
3/4	0.644	0.645	183	0.607	0.608	241
7/8	0.752	0.753	248	0.708	0.709	329
1	0.859	0.860	326	0.809	0.810	430
1 1/8	0.967	0.968	412	0.910	0.911	545
1 1/4	1.074	1.075	508	1.012	1.013	672
1 3/8	1.182	1.183	614	1.113	1.114	813
1 1/2	1.289	1.290	732	1.214	1.215	967
1 5/8	1.397	1.398	860	1.315	1.316	1135
1 3/4	1.504	1.505	997	1.417	1.418	1316
2	1.718	1.720	1302	1.618	1.620	1720
2 1/4	1.933	1.935	1647	1.821	1.823	2176
2 1/2	2.148	2.150	2034	2.023	2.025	2688
3	2.578	2.580	2929	2.428	2.430	3869
3 1/2	3.007	3.010	3987	2.832	2.835	5266
4	3.437	3.440	5208	3.237	3.240	6878
4 1/2	3.867	3.870	6591	3.642	3.645	8705
5	4.297	4.300	8137	4.047	4.050	10746
5 1/2	4.727	4.730	9846	4.452	4.455	13003
6	5.157	5.160	11718	4.857	4.860	15475

[a] See note at end of Table 4.

16-Spline Fittings							
	For All Fits				16A—Permanent Fit		
Nom. Diam.	*D*		*W*		*d*		
	Min.	Max.	Min.	Max.	Min.	Max.	*T*[a]
2	1.997	2.000	0.193	0.196	1.817	1.820	1375
2 1/2	2.497	2.500	0.242	0.245	2.273	2.275	2149
3	2.997	3.000	0.291	0.294	2.727	2.730	3094
3 1/2	3.497	3.500	0.340	0.343	3.182	3.185	4212
4	3.997	4.000	0.389	0.392	3.637	3.640	5501
4 1/2	4.497	4.500	0.438	0.441	4.092	4.095	6962
5	4.997	5.000	0.487	0.490	4.547	4.550	8595
5 1/2	5.497	5.500	0.536	0.539	5.002	5.005	10395
6	5.997	6.000	0.585	0.588	5.457	5.460	12377

Table 4. S.A.E. Standard Splined Fittings

Nom. Diam.	16-Spline Fittings					
	16B—To Slide—No Load			16C—To Slide Under Load		
	d			*d*		
	Min.	Max.	*T*[a]	Min.	Max.	*T*[a]
2	1.717	1.720	2083	1.617	1.620	2751
2½	2.147	2.150	3255	2.022	2.025	4299
3	2.577	2.580	4687	2.427	2.430	6190
3½	3.007	3.010	6378	2.832	2.835	8426
4	3.437	3.440	8333	3.237	3.240	11005
4½	3.867	3.870	10546	3.642	3.645	13928
5	4.297	4.300	13020	4.047	4.050	17195
5½	4.727	4.730	15754	4.452	4.455	20806
6	5.157	5.160	18749	4.857	4.860	24760

[a] *Torque Capacity of Spline Fittings:* The torque capacities of the different spline fittings are given in the columns headed "*T*." The torque capacity, per inch of bearing length at 1000 pounds pressure per square inch on the sides of the spline, may be determined by the following formula, in which *T* = torque capacity in inch-pounds per inch of length, *N* = number of splines, *R* = mean radius or radial distance from center of hole to center of spline, *h* = depth of spline: $T = 1000NRh$

Table 5. Formulas for Determining Dimensions of S.A.E. Standard Splines

No. of Splines	*W* For All Fits	*A* Permanent Fit		*B* To Slide Without Load		*C* To Slide Under Load	
		h	*d*	*h*	*d*	*h*	*d*
Four	0.241*D*[a]	0.075*D*	0.850*D*	0.125*D*	0.750*D*	…	…
Six	0.250*D*	0.050*D*	0.900*D*	0.075*D*	0.850*D*	0.100*D*	0.800*D*
Ten	0.156*D*	0.045*D*	0.910*D*	0.070*D*	0.860*D*	0.095*D*	0.810*D*
Sixteen	0.098*D*	0.045*D*	0.910*D*	0.070*D*	0.860*D*	0.095*D*	0.810*D*

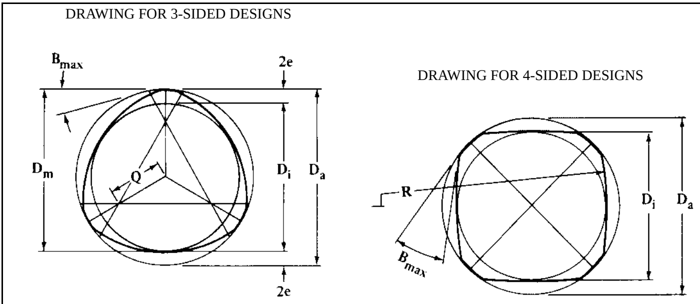
[a] Four splines for fits *A* and *B* only.

The formulas in the table above give the maximum dimensions for *W*, *h*, and *d*, as listed in Tables 1 through 4 inclusive.

S.A.E. Standard Spline Fittings.—The S.A.E. spline fittings (Tables 1 through 4 inclusive) have become an established standard for many applications in the agricultural, automotive, machine tool, and other industries. The dimensions given, in inches, apply only to soft broached holes. The tolerances given may be readily maintained by usual broaching methods. The tolerances selected for the large and small diameters may depend upon whether the fit between the mating part, as finally made, is on the large or the small diameter. The other diameter, which is designed for clearance, may have a larger manufactured tolerance. If the final fit between the parts is on the sides of the spline only, larger tolerances are permissible for both the large and small diameters. The spline should not be more than 0.006 inch per foot out of parallel with respect to the shaft axis. No allowance is made for corner radii to obtain clearance. Radii at the corners of the spline should not exceed 0.015 inch.

Polygon-Type Shaft Connections.— Involute-form and straight-sided splines are used for both fixed and sliding connections between machine members such as shafts and gears. Polygon-type connections, so called because they resemble regular polygons but with curved sides, may be used similarly. German DIN Standards 32711 and 32712 include data for three- and four-sided metric polygon connections. Data for 11 of the sizes shown in those Standards, but converted to inch dimensions by Stoffel Polygon Systems, are given in the accompanying table.

Dimensions of Three- and Four-Sided Polygon-type Shaft Connections



Three-Sided Designs					Four-Sided Designs				
Nominal Sizes			Design Data		Nominal Sizes			Design Data	
D_A (in.)	D_1 (in.)	e (in.)	Area (in.^{2})	Z_P (in.^{3})	D_A (in.)	D_1 (in.)	e (in.)	Area (in.^{2})	Z_P (in.^{3})
0.530	0.470	0.015	0.194	0.020	0.500	0.415	0.075	0.155	0.014
0.665	0.585	0.020	0.302	0.039	0.625	0.525	0.075	0.250	0.028
0.800	0.700	0.025	0.434	0.067	0.750	0.625	0.125	0.350	0.048
0.930	0.820	0.027	0.594	0.108	0.875	0.725	0.150	0.470	0.075
1.080	0.920	0.040	0.765	0.153	1.000	0.850	0.150	0.650	0.12
1.205	1.045	0.040	0.977	0.224	1.125	0.950	0.200	0.810	0.17
1.330	1.170	0.040	1.208	0.314	1.250	1.040	0.200	0.980	0.22
1.485	1.265	0.055	1.450	0.397	1.375	1.135	0.225	1.17	0.29
1.610	1.390	0.055	1.732	0.527	1.500	1.260	0.225	1.43	0.39
1.870	1.630	0.060	2.378	0.850	1.750	1.480	0.250	1.94	0.64
2.140	1.860	0.070	3.090	1.260	2.000	1.700	0.250	2.60	0.92

Dimensions Q and R shown on the diagrams are approximate and used only for drafting purposes: $Q \approx 7.5e$; $R \approx D_1/2 + 16e$.

Dimension $D_M = D_1 + 2e$. Pressure angle B_{max} is approximately $344e/D_M$ degrees for three sides, and $299e/D_M$ degrees for four sides.

Tolerances: ISO H7 tolerances apply to bore dimensions. For shafts, g6 tolerances apply for sliding fits; k7 tolerances for tight fits.

Choosing Between Three- and Four-Sided Designs: Three-sided designs are best for applications in which no relative movement between mating components is allowed while torque is transmitted. If a hub is to slide on a shaft while under torque, four-sided designs, which have larger pressure angles B_{max} than those of three-sided designs, are better suited to sliding even though the axial force needed to move the sliding member is approximately 50 percent greater than for comparable involute spline connections.

Strength of Polygon Connections: In the formulas that follow,

H_w = hub width, inches

H_t = hub wall thickness, inches

M_b = bending moment, lb-inch

M_t = torque, lb-inch

Z = section modulus, bending, in.^{3}

$= 0.098 D_M^4/D_A$ for three sides

$= 0.15 D_I^3$ for four sides

Z_P = polar section modulus, torsion, in.^{3}

$= 0.196 D_M^4/D_A$ for three sides

$=0.196D_I^3$ for four sides

D_A and D_M. See table footnotes.

S_b = bending stress, allowable, lb/in.^{2}

S_s = shearing stress, allowable, lb/in.^{2}

S_t = tensile stress, allowable, lb/in.^{2}

For shafts,

$$M_t\,(\text{maximum}) = S_s Z_p;$$

$$M_b\,(\text{maximum}) = S_b Z$$

For bores,

$$H_t(\text{minimum}) = K\sqrt{\frac{M_t}{S_t H_w}}$$

in which $K = 1.44$ for three sides except that if D_M is greater than 1.375 inches, then $K = 1.2$; $K = 0.7$ for four sides.

Failure may occur in the hub of a polygon connection if the hoop stresses in the hub exceed the allowable tensile stress for the material used. The radial force tending to expand the rim and cause tensile stresses is calculated from

$$\text{Radial Force, lb} = \frac{2M_t}{D_I n \tan(B_{max} + 11.3)}$$

This radial force acting at n points may be used to calculate the tensile stress in the hub wall using formulas from strength of materials.

Manufacturing: Polygon shaft profiles may be produced using conventional machining processes such as hobbing, shaping, contour milling, copy turning, and numerically controlled milling and grinding. Bores are produced using broaches, spark erosion, gear shapers with generating cutters of appropriate form, and, in some instances, internal grinders of special design. Regardless of the production methods used, points on both of the mating profiles may be calculated from the following equations:

$$X = (D_I/2 + e)\cos\alpha - e\cos\,n\alpha\,\cos\,\alpha - ne\,\sin$$

$$Y = (D_I/2 + e)\sin\alpha - e\cos\,n\alpha\,\sin\,\alpha + ne\,\sin$$

In these equations, α is the angle of rotation of the workpiece from any selected reference position; n is the number of polygon sides, either 3 or 4; D_I is the diameter of the inscribed circle shown on the diagram in the table; and e is the dimension shown on the diagram in the table and which may be used as a setting on special polygon grinding machines. The value of e determines the shape of the profile. A value of 0, for example, results in a circular shaft having a diameter of D_I. The values of e in the table were selected arbitrarily to provide suitable proportions for the sizes shown.

CAMS AND CAM DESIGN

Classes of Cams.—Cams may, in general, be divided into two classes: uniform motion cams and accelerated motion cams. The uniform motion cam moves the follower at the same rate of speed from the beginning to the end of the stroke; but as the movement is started from zero to the full speed of the uniform motion and stops in the same abrupt way, there is a distinct shock at the beginning and end of the stroke, if the movement is at all rapid. In machinery working at a high rate of speed, therefore, it is important that cams are so constructed that sudden shocks are avoided when starting the motion or when reversing the direction of motion of the follower.

The uniformly accelerated motion cam is suitable for moderate speeds, but it has the disadvantage of sudden changes in acceleration at the beginning, middle and end of the stroke. A cycloidal motion curve cam produces no abrupt changes in acceleration and is often used in high-speed machinery because it results in low noise, vibration and wear. The cycloidal motion displacement curve is so called because it can be generated from a cycloid which is the locus of a point of a circle rolling on a straight line.[*]

Cam Follower Systems.—The three most used cam and follower systems are radial and offset translating roller follower, Figs. 1a and 1b; and the swinging roller follower, Fig. 1c. When the cam rotates, it imparts a translating motion to the roller followers in Figs. 1a and 1b and a swinging motion to the roller follower in Fig. 1c. The motionof the follower is, of course, dependent on the shape of the cam; and the following section on displacement diagrams explains how a favorable motion is obtained so that the cam can rotate at high speed without shock.

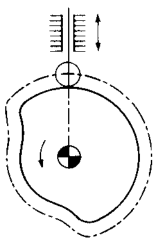


Fig. 1a. Radial Translating Roller Follower

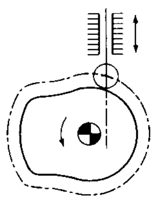


Fig. 1b. Offset Translating Roller Follower

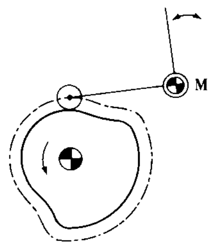


Fig. 1c. Swinging Roller Follower

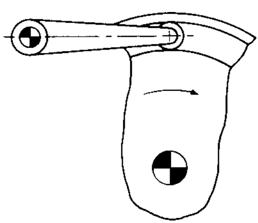


Fig. 2a. Closed-Track Cam

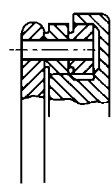


Fig. 2b. Closed-Track Cam With Two Rollers

The arrangements in Figs. 1a, 1b, and 1c show open-track cams. In Figs. 2a and 2b the roller is forced to move in a closed track. Open-track cams build smaller than closed-track

[*] Jensen, P. W., *Cam Design and Manufacture,* Industrial Press Inc.

cams but, in general, springs are necessary to keep the roller in contact with the cam at all times. Closed-track cams do not require a spring and have the advantage of positive drive throughout the rise and return cycle. The positive drive is sometimes required as in the case where a broken spring would cause serious damage to a machine.

Displacement Diagrams.—Design of a cam begins with the displacement diagram. A simple displacement diagram is shown in Fig. 3. One cycle means one whole revolution of the cam; i.e., one cycle represents 360°. The horizontal distances T_1, T_2, T_3, T_4 are expressed in units of time (seconds); or radians or degrees. The vertical distance, *h*, represents the maximum "rise" or stroke of the follower.

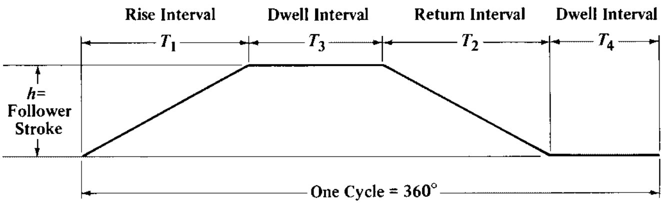


Fig. 3. A Simple Displacement Diagram

The displacement diagram of Fig. 3 is not a very favorable one because the motion from rest (the horizontal lines) to constant velocity takes place instantaneously and this means that accelerations become infinitely large at these transition points.

Types of Cam Displacement Curves: A variety of cam curves are available for moving the follower. In the following sections only the rise portions of the total time-displacement diagram are studied. The return portions can be analyzed in a similar manner. Complex cams are frequently employed which may involve a number of rise-dwell-return intervals in which the rise and return aspects are quite different. To analyze the action of a cam it is necessary to study its time-displacement and associated velocity and acceleration curves. The latter are based on the first and second time-derivatives of the equation describing the time-displacement curve:

$$y = \text{displacement} = f(t) \quad \text{or} \quad y = f(\phi)$$

$$v = \frac{dy}{dt} = \text{velocity} = \omega\frac{dy}{d\phi}$$

$$a = \frac{d^2y}{dt^2} = \text{acceleration} = \omega^2\frac{d^2y}{d\phi^2}$$

Meaning of Symbols and Equivalent Relations: y = displacement of follower, inch

h = maximum displacement of follower, inch

t = time for cam to rotate through angle ϕ, sec, = ϕ/ω, sec

T = time for cam to rotate through angle β, sec, = β/ω, or $\beta/6N$, sec

ϕ = cam angle rotation for follower displacement y, degrees

β = cam angle rotation for total rise h, degrees

v = velocity of follower, in./sec

a = follower acceleration, in./sec^2

$t/T = \phi/\beta$

N = cam speed, rpm

ω = angular velocity of cam, degrees/sec = $\beta/T = \phi/t = d\phi/dt = 6N$

ω_R = angular velocity of cam, radians/sec = $\pi\omega/180$

W = effective weight, lbs

g = gravitational constant = 386 in./sec^2

$f(t)$ = means a function of t

$f(\phi)$ = means a function of ϕ

R_{min} = minimum radius to the cam pitch curve, inch

R_{max} = maximum radius to the cam pitch curve, inch

r_f = radius of cam follower roller, inch

ρ = radius of curvature of cam pitch curve (path of center of roller follower), inch

R_c = radius of curvature of actual cam surface, in., $= \rho - r_f$ for convex surface; $= \rho + r_f$ for concave surface.

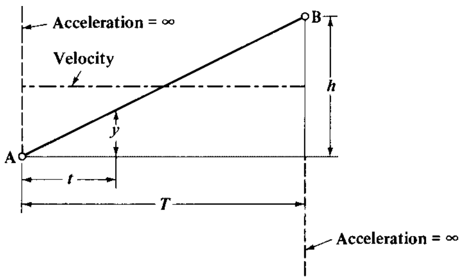


Fig. 4. Cam Displacement, Velocity, and Acceleration Curves for Constant Velocity Motion

Four displacement curves are of the greatest utility in cam design.

1. Constant-Velocity Motion: (Fig. 4)

$y = h\frac{t}{T}$ or $y = \frac{h\phi}{\beta}$	(1a)	
$v = \frac{dy}{dt} = \frac{h}{T}$ or $v = \frac{h\omega}{\beta}$	(1b)	} $0 < t < T$
$a = \frac{d^2y}{dt^2} = 0^*$	(1c)	

* Except at $t = 0$ and $t = T$ where the acceleration is theoretically infinite.

This motion and its disadvantages were mentioned previously. While in the unaltered form shown it is rarely used except in very crude devices, nevertheless, the advantage of uniform velocity is an important one and by modifying the start and finish of the follower stroke this form of cam motion can be utilized. Such modification is explained in the section Displacement Diagram Synthesis.

2. Parabolic Motion: (Fig. 5)

For $0 \le t \le T/2$ and $0 \le \phi \le \beta/2$		For $T/2 \le t \le T$ and $\beta/2 \le \phi \le \beta$	
$y = 2h(t/T)^2 = 2h(\phi/\beta)^2$	(2a)	$y = h[1 - 2(1 - t/T)^2] = h[1 - 2(1 - \phi/\beta)^2]$	(2d)
$v = 4ht/T^2 = 4h\omega\phi/\beta^2$	(2b)	$v = 4h/T(1 - t/T) = (4h\omega/\beta)(1 - \phi/\beta)$	(2e)
$a = 4h/T^2 = 4h(\omega/\beta)^2$	(2c)	$a = -4h/T^2 = -4h(\omega/\beta)^2$	(2f)

Examination of the above formulas shows that the velocity is zero when $t = 0$ and $y = 0$; and when $t = T$ and $y = h$.

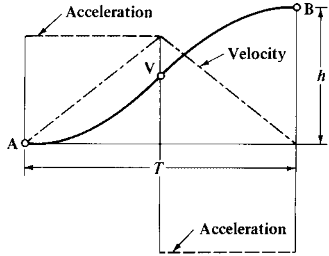


Fig. 5. Cam Displacement, Velocity, and Acceleration Curves for Parabolic Motion

The most important advantage of this curve is that for a given angle of rotation and rise it produces the smallest possible acceleration. However, because of the sudden changes in acceleration at the beginning, middle, and end of the stroke, shocks are produced. If the follower system were perfectly rigid with no backlash or flexibility, this would be of little significance. But such systems are mechanically impossible to build and a certain amount of impact is caused at each of these changeover points.

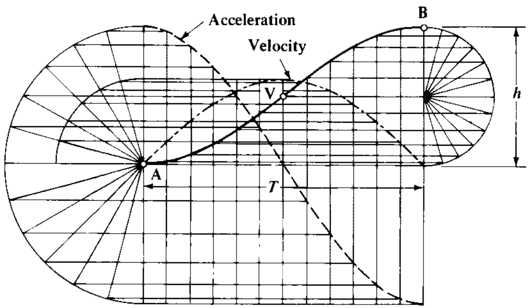


Fig. 6. Cam Displacement, Velocity, and Acceleration Curves for Simple Harmonic Motion

3. Simple Harmonic Motion: (Fig. 6)

$$y = \frac{h}{2}\left[1 - \cos\left(\frac{180°t}{T}\right)\right] \quad \text{or} \quad y = \frac{h}{2}\left[1 - \cos\left(\frac{180°\phi}{\beta}\right)\right] \quad \text{(3a)}$$

$$v = \frac{h}{2} \cdot \frac{\pi}{T}\sin\left(\frac{180°t}{T}\right) \quad \text{or} \quad v = \frac{h}{2} \cdot \frac{\pi\omega}{\beta}\sin\left(\frac{180°\phi}{\beta}\right) \quad \text{(3b)} \quad \} \quad 0 \le t \le T$$

$$a = \frac{h}{2} \cdot \frac{\pi^2}{T^2}\cos\left(\frac{180°t}{T}\right) \quad \text{or} \quad a = \frac{h}{2} \cdot \left(\frac{\pi\omega}{\beta}\right)^2 \cos\left(\frac{180°\phi}{\beta}\right) \quad \text{(3c)}$$

Smoothness in velocity and acceleration during the stroke is the advantage inherent in this curve. However, the instantaneous changes in acceleration at the beginning and end of the stroke tend to cause vibration, noise, and wear. As can be seen from Fig. 6, the maximum acceleration values occur at the ends of the stroke. Thus, if inertia loads are to be overcome by the follower, the resulting forces cause stresses in the members. These forces are in many cases much larger than the externally applied loads.

4. Cycloidal Motion: (Fig. 7)

$$y = h\left[\frac{t}{T} - \frac{1}{2\pi}\sin\left(\frac{360°t}{T}\right)\right] \quad \text{or} \quad y = h\left[\frac{\phi}{\beta} - \frac{1}{2\pi}\sin\left(\frac{360°\phi}{\beta}\right)\right] \quad (4a)$$

$$v = \frac{h}{T}\left[1 - \cos\left(\frac{360°t}{T}\right)\right] \quad \text{or} \quad v = \frac{h\omega}{\beta}\left[1 - \cos\left(\frac{360°\phi}{\beta}\right)\right] \quad (4b) \quad \} \quad 0 \le i \le T$$

$$a = \frac{2\pi h}{T^2}\sin\left(\frac{360°t}{T}\right) \quad \text{or} \quad a = \frac{2\pi h\omega^2}{\beta^2}\sin\left(\frac{360°\phi}{\beta}\right) \quad (4c)$$

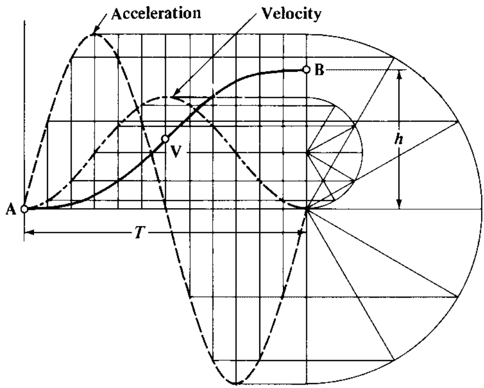


Fig. 7. Cam Displacement, Velocity, and Acceleration Curves for Cycloidal Motion

This time-displacement curve has excellent acceleration characteristics; there are no abrupt changes in its associated acceleration curve. The maximum value of the acceleration of the follower for a given rise and time is somewhat higher than that of the simple harmonic motion curve. In spite of this, the cycloidal curve is used often as a basis for designing cams for high-speed machinery because it results in low levels of noise, vibration, and wear.

Displacement Diagram Synthesis.—The straight-line graph shown in Fig. 3 has the important advantage of uniform velocity. This is so desirable that many cams based on this graph are used. To avoid impact at the beginning and end of the stroke, a modification is introduced at these points. There are many different types of modifications possible, ranging from a simple circular arc to much more complicated curves. One of the better curves used for this purpose is the parabolic curve given by Equation (2a). As seen from the derived time graphs, this curve causes the follower to begin a stroke with zero velocity but having a finite and constant acceleration. We must accept the necessity of acceleration, but effort should be made to hold it to a minimum.

Matching of Constant Velocity and Parabolic Motion Curves: By matching a parabolic cam curve to the beginning and end of a straight-line cam displacement diagram it is possible to reduce the acceleration from infinity to a finite constant value to avoid impact loads. As illustrated in Fig. 8, it can be shown that for any parabola the vertex of which is at *O*, the tangent to the curve at the point *P* intersects the line *OQ* at its midpoint. This means that the tangent at *P* represents the velocity of the follower at time X_0 as shown in Fig. 8. Since the tangent also represents the velocity of the follower over the constant velocity portion of the stroke, the transition from rest to the maximum velocity is accomplished with smoothness.

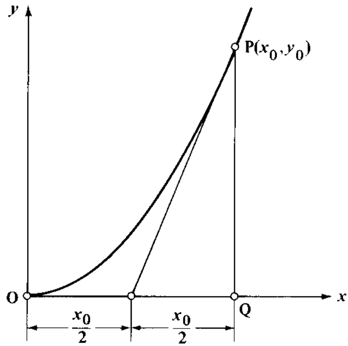


Fig. 8. The Tangent at *P* Bisects *OQ*, When Curve is a Parabola

Example: A cam follower is to rise ¼ in. with constant acceleration; 1¼ in. with constant velocity, over an angle of 50 degrees; and then ½ in. with constant deceleration.

In Fig. 9 the three rise distances are laid out, $y_1 = ¼$ in., $y_2 = 1¼$ in., $y_3 = ½$ in., and horizontals drawn. Next, an arbitrary horizontal distance ϕ_2 proportional to 50 degrees is measured off and points *A* and *B* are located. The line *AB* is extended to M_1 and M_2. By remembering that a tangent to a parabola, Fig. 8, will cut the abscissa axis at point $(X_0/2,0)$ where X_0 is the abscissa of the point of tangency, the two values $\phi_1 = 20°$ and $\phi_3 = 40°$ will be found. Analytically,

$$\frac{M_1E}{\phi_2} = \frac{y_1}{y_2} \qquad \frac{\frac{1}{2}\phi_1}{50°} = \frac{0.25}{1.25} \qquad \therefore \phi_1 = 20°$$

$$\frac{FM_2}{\phi_2} = \frac{y_3}{y_2} \qquad \frac{\frac{1}{2}\phi_3}{50°} = \frac{0.50}{1.25} \qquad \therefore \phi_3 = 40°$$

In Fig. 9, the portions of the parabola have been drawn in; the details of this operation are as follows:

Assume that accuracy to the nearest thousandth of one inch is desired, and it is decided to plot values for every 5 degrees of cam rotation.

The formula for the acceleration portion of the parabolic curve is:

$$y = \frac{2h}{T^2}t^2 = 2h\left(\frac{\phi}{\beta}\right)^2 \qquad (5)$$

Two different parabolas are involved in this example; one for accelerating the follower during a cam rotation of 20 degrees, the other for decelerating it in 40 degrees, these two being tangent, to opposite ends of the same line *AB*.

In Fig. 9 only the first half of a complete acceleration-deceleration parabolic curve is used to blend with the left end of the straight line *AB*. Therefore, in using the Formula (5) substitute $2y_1$ for *h* and $2\phi_1$ for β so that

$$y = \frac{2h\phi^2}{\beta^2} = \frac{(2)(2y_1)}{(2\phi_1)^2}\phi^2$$

For the right end of the straight line *AB*, the calculations are similar but, in using Formula (5), calculated *y* values are *subtracted* from the *total* rise of the cam $(y_1 + y_2 + y_3)$ to obtain the follower displacement.

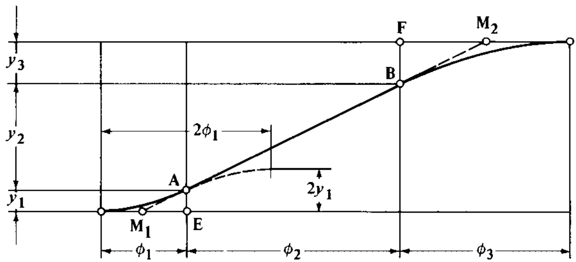


Fig. 9. Matching a Parabola at Each End of Straight Line Displacement Curve *AB* to Provide More Acceptable Acceleration and Deceleration

Table 1 shows the computations and resulting values for the cam displacement diagram described. The calculations are shown in detail so that if equations are programmed for a digital computer, the results can be verified easily. Obviously, the intermediate points are not needed to draw the straight line, but when the cam profile is later to be drawn or cut, these values will be needed since they are to be measured on radial lines.

The matching procedure when using cycloidal motion is exactly the same as for parabolic motion, because parabolic and cycloidal motion have the same maximum velocity for equal rise (or return) and lift angle (or return angle).

Cam Profile Determination.—In the cam constructions that follow an artificial device called an *inversion* is used. This represents a mental concept which is very helpful in performing the graphical work. The construction of a cam profile requires the drawing of many positions of the cam with the follower in each case in its related location. However, instead of revolving the cam, it is assumed that the follower rotates around the *fixed* cam. It requires the drawing of many follower positions, but since this is done more or less diagrammatically, it is relatively simple.

As part of the inversion process, the direction of rotation is important. In order to preserve the correct sequence of events, the artificial rotation of the follower must be the reverse of the cam's prescribed rotation. Thus, in Fig. 10 the cam rotation is counterclockwise, whereas the artificial rotation of the follower is clockwise.

Radial Translating Roller Follower: The time-displacement diagram for a camwith a radial translating roller follower is shown in Fig. 10(a). This diagram is read from left to right as follows: For 100 degrees of cam shaft rotation the follower rises *h* inches (*AB*), dwells in its upper position for 20 degrees (*BC*), returns over 180 degrees (*CD*), and finally dwells in its lowest position for 60 degrees (*DE*). Then the entire cycle is repeated.

Fig. 10(b) shows the cam construction layout with the cam pitch curve as a dot and dash line. To locate a point on this curve, take a point on the displacement curve, as 6′ at the 60-degree position, and project this horizontally to point 6″ on the 0-degree position of the cam construction diagram. Using the center of cam rotation, an arc is struck from point 6″ to intercept the 60-degree position radial line which gives point 6‴ on the cam pitch curve. It will be seen that the smaller circle in the cam construction layout has a radius R_{min} equal to the smallest distance from the center of cam rotation to the pitch curve and, similarly, the larger circle has a radius R_{max} equal to the largest distance to the pitch curve. Thus, the difference in radii of these two circles is equal to the maximum rise *h* of the follower.

The cam pitch curve is also the actual profile or working surface when a knife-edged follower is used. To get the profile or working surface for a cam with a roller follower, a series of arcs with centers on the pitch curve and radii equal to the radius of the roller are drawn and the inner envelope drawn tangent to these arcs is the cam working surface or profile shown as a solid line in Fig. 10(b).

Table 1. Development of Modified Constant Velocity Cam with Parabolic Matching

Rise Angle	ϕ Degrees	Computation	Follower Displacement *y*	Explanation
	0	0	0	$\beta = 40°, h = 0.500$
	5	0.000625×5^2	0.016	$y = \frac{(2)(0.500)}{(40)^2}\phi^2$
$\phi_1 =$ 20°	{ 10	0.00625×10^2	0.063	
	15	0.000625×15^2	0.141	$= 0.000625\phi^2$
	20	0.000625×20^2	0.250	
	25		0.375	
	30		0.500	
	35		0.625	
	40		0.750	
$\phi_2 =$ 50°	{ 45		0.875	1.250 in. divided into 10 uniform divisions
	50		1.000	
	55		1.125	
	60		1.250	
	65		1.375	
	70		1.500	
	75	$2.000 - (0.0003125 \times 35^2)$	1.617	$\beta = 80°, h = 1.000$
	80	$2.000 - (0.0003125 \times 30^2)$	1.617	$y = 2 - \frac{(2)(1.000)}{(80°)^2}(110° - \phi)^2$
	85	$2.000 - (0.0003125 \times 25^2)$	1.805	
	{ 90	$2.000 - (0.0003125 \times 20^2)$	1.875	$= 2 - 0.0003125(110° - \phi)^2$
$\phi_3 =$ 40°	95	$2.000 - (0.0003125 \times 15^2)$	1.930	
	100	$2.000 - (0.0003125 \times 10^2)$	1.969	
	105	$2.000 - (0.0003125 \times 5^2$	1.992	
	110	$2.000 - (0.0003125 \times 0^2)$	2.000	

Since the deceleration portion of a parabolic cam is the same shape as the acceleration portion, but inverted, Formula (5) may be used to calculate the *y* values by substituting $2y_3$ for *h* and for β and the result subtracted from the total rise $(y_1 + y_2 + y_3)$ to obtain the follower displacement.

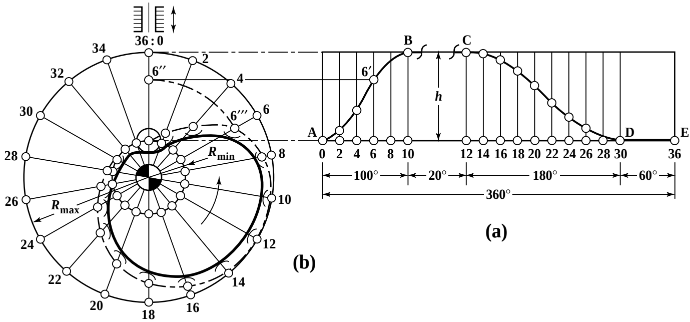


Fig. 10. (a) Time-Displacement Diagram for Cam to be Laid Out; (b) Construction of Contour of Cam With Radial Translating Roller Follower

Offset Translating Roller Follower: Given the time-displacement diagram Fig. 11(a) and an offset follower. The construction of the cam in this case is very similar to the foregoing case and is shown in Fig. 11(b). In this construction it will be noted that the angular position lines are not drawn radially from the cam shaft center but tangent to a circle having a radius equal to the amount of offset of the center line of the cam follower from the center of the cam shaft. For counterclockwise rotation of the cam, points 6′, 6″, and 6‴ are located in succession as indicated.

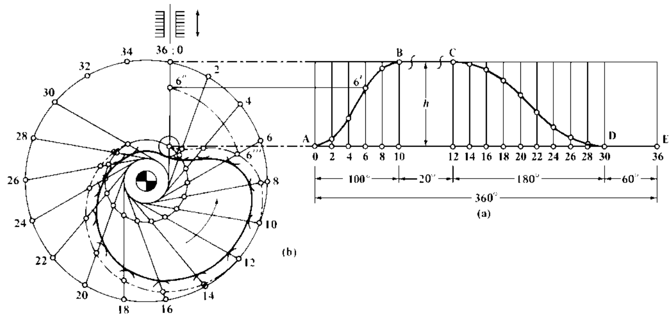


Fig. 11. (a) Time-Displacement Diagram for Cam to be Laid Out; (b) Construction of Contour of Cam With Offset Translating Roller Follower

Swinging Roller Follower: Given the time-displacement diagram Fig. 12(a) and the length of the swinging follower arm L_f, it is required that the displacement of the follower center along the circular arc that it describes be equal to the corresponding displacements in the time-displacement diagram. If ϕ_0 is known, the displacement h of Fig. 12(a) would be found from the formula $h = \pi\phi_0 L_f / 180°$: otherwise the maximum rise h of the follower is stepped off on the arc drawn with M as a center and starting at a point on the R_{min} circle. Point M is the actual position of the pivot center of the swinging follower with respect to the cam shaft center. It is again required that the rotation of the cam be counterclockwise and therefore M is considered to have been rotated clockwise around the cam shaft center, whereby the points 2, 4, 6, etc., are obtained as shown in Fig. 12(b). Around each of the pivot points, 2, 4, 6, etc., circular arcs whose radii equal L_f are drawn between the R_{min} and

R_{max} circles giving the points 2′, 4′, 6′, etc. The R_{min} circle with center at the cam shaft center is drawn through the lowest position of the center of the roller follower and the R_{max} circle through the highest position as shown. The different points on the pitch curve are now located. Point 6‴, for instance, is found by stepping off the y_6 ordinate of the displacement diagram on arc 6′ starting at the R_{min} circle.

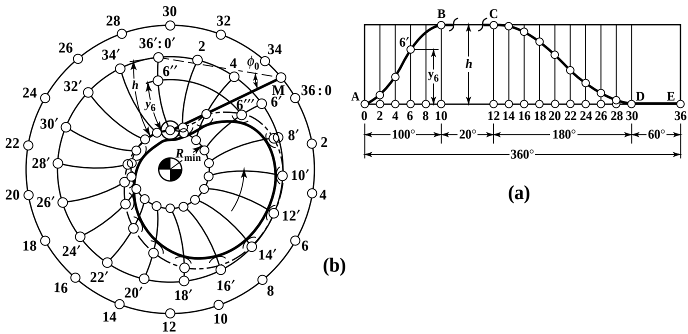


Fig. 12. (a) Time-Displacement Diagram for Cam to be Laid Out; (b) Construction of Contour of Cam With Swinging Roller Follower

Pressure Angle and Radius of Curvature.—The pressure angle at any point on the profile of a cam may be defined as the angle between the direction where the follower wants to go at that point and where the cam wants to push it. It is the angle between the tangent to the path of follower motion and the line perpendicular to the tangent of the cam profile at the point of cam-roller contact.

The size of the pressure angle is important because:

1) Increasing the pressure angle increases the side thrust and this increases the forces exerted on cam and follower.

2) Reducing the pressure angle increases the cam size and often this is not desirable because:

A) The size of the cam determines, to a certain extent, the size of the machine.

B) Larger cams require more precise cutting points in manufacturing and, therefore, an increase in cost.

C) Larger cams have higher circumferential speed and small deviations from the theoretical path of the follower cause additional acceleration, the size of which increases with the square of the cam size.

D) Larger cams mean more revolving weight and in high-speed machines this leads to increased vibrations in the machine.

E) The inertia of a large cam may interfere with quick starting and stopping.

The maximum pressure angle α_m should, in general, be kept at or below 30 degrees for translating-type followers and at or below 45 degrees for swinging-type followers.These values are on the conservative side and in many cases may be increased considerably, but beyond these limits trouble could develop and an analysis is necessary.

In the following, graphical methods are described by which a cam mechanism can be designed with translating or swinging roller followers having specified maximum pressure angles for rise and return. These methods are applicable to any kind of time-displacement diagram.

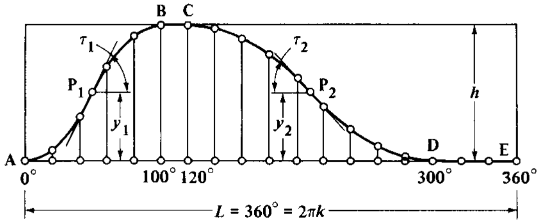


Fig. 13. Displacement Diagram

Determination of Cam Size for a Radial or an Offset Translating Follower.—Fig. 13 shows a time-displacement diagram. The maximum displacement is preferably made to scale, but the length of the abscissa, L, can be chosen arbitrarily. The distance L from 0 to 360 degrees is measured and is set equal to $2\pi k$ from which

$$k = \frac{L}{2\pi}$$

k is calculated and laid out as length E to M in Fig. 14.

In Fig. 13 the two points P_1 and P_2 having the maximum angles of slope, τ_1, and τ_2, are located by inspection. In this example y_1 and y_2 are of equal length.

Angles τ_1 and τ_2 are laid out as shown in Fig. 14, and the points of intersection with a perpendicular to EM at M determine Q_1 and Q_2. The measured distances

$$MQ_1 = k\tan\tau_1 \quad \text{and} \quad MQ_2 = k\tan\tau_2$$

are laid out in Fig. 15, which is constructed as follows:

Draw a vertical line R_uR_o of length h equal to the stroke of the roller follower, R_u being the lowest position and R_o the highest position of the center of the roller follower. From R_u lay out $R_uR_{y1} = y_1$ and $R_uR_{y2} = y_2$; these are equal lengths in this example. Next, if the rotation of the cam is counterclockwise, lay out k tan τ_1, to the left, k tan τ_2 to the right from points R_{y1} and R_{y2}, respectively, R_{y1} and R_{y2} being the same point in this case.

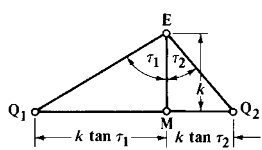


Fig. 14. Construction to Find k tan τ_1 and k tan τ_2

The specified maximum pressure angle α_1 is laid out at E_1 as shown, and a ray (line) E_1F_1 is determined. Any point on this ray chosen as the cam shaft center will proportion the cam so that the pressure angle at a point on the cam profile corresponding to point P_1, of the displacement diagram will be exactly α_1.

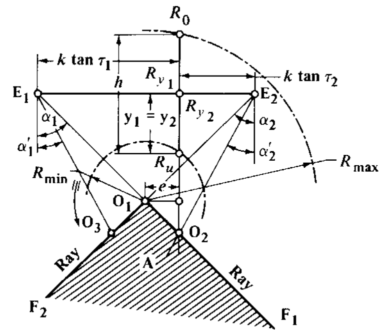


Fig. 15. Finding Proportions of Cam; Offset Translating Follower

The angle α_2 is laid out at E_2 as shown, and another ray E_2F_2 is determined. Similarly, any point on this ray chosen as the cam shaft center will proportion the cam so that the pressure angle at a point on the cam profile corresponding to point P_2 of the displacement diagram will be exactly α_2.

Any point chosen within the cross-hatched area A as the cam center will yield a cam whose pressure angles at points corresponding to P_1 and P_2 will not exceed the specified values α_1 and α_2 respectively. If O_1 is chosen as the cam shaft center, the pressure angles on the cam profile corresponding to points P_1 and P_2 are exactly α_1 and α_2, respectively. Selection of point O_1 also yields the smallest possible cam for the given requirements and requires an offset follower in which e is the offset distance.

If O_2 is chosen as the cam shaft center, a radial translating follower is obtained (zero offset). In that case, the pressure angle α_1 for the rise is unchanged, whereas the pressure angle for the return is changed from α_2 to α'_2. That is, the pressure angle on the return stroke is reduced at the point P_2. If point O_3 had been selected, then α_2 would remain unchanged but α_1 would be decreased and the offset, e, increased.

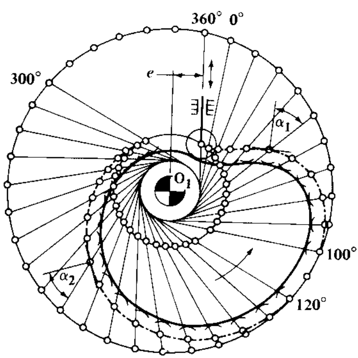


Fig. 16. Construction of Cam Contour; Offset Translating Follower

Fig. 16 shows the shape of the cam when O_1 from Fig. 15 is chosen as the cam shaft center, and it is seen that the pressure angle at a point on the cam profile corresponding to point P_1 is α_1 and at a point corresponding to point P_2 is α_2.

In the foregoing, a cam mechanism has been so proportioned that the pressure angles α_1 and α_2 at points on the cam corresponding to points P_1 and P_2 were obtained. Even though P_1 and P_2 are the points of greatest slope on the displacement diagram, the pressure angles produced at some other points on the actual cam may be slightly greater.

However, if the pressure angles α_1 and α_2 are not to be exceeded at any point — i.e., they are to be maximum pressure angles — then P_1 and P_2 must be selected to be at the locations where these maximum pressure angles occur. If these locations are not known, then the graphical procedure described must be repeated, letting P_1 take various positions on the curve for rise (*AB*) and P_2 various positions on the return curve (*CD*) and then setting R_{min} equal to the largest of the values determined from the various positions.

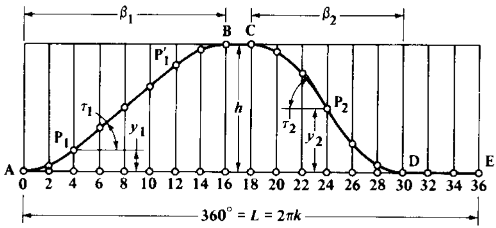


Fig. 17. Displacement Diagram

Determination of Cam Size for Swinging Roller Follower.—The proportioning of a cam with swinging roller follower having specific pressure angles at selected points follows the same procedure as that for a translating follower.

Example: Given the diagram for the roller displacement along its circular arc, Fig. 17 with h = 1.95 in., the periods of rise and fall, respectively, β_1 = 160° and β_2 = 120°, the length of the swinging follower arm L_f = 3.52 in., rotation of the cam away from pivot point *M*, and pressure angles $\alpha_1 = \alpha_2 = 45°$ (corresponding to the points P_1 and P_2 in the displacement diagram). Find the cam proportions.

Solution: Distances $k \tan \tau_1$ and $k \tan \tau_2$ are determined as in the previous example, Fig. 14. In Fig. 18, R_{y1} is determined by making the distance $R_u R_{y1} = y_1$ along the arc $R_u R_o$ and R_{y2} by making $R_u R_{y2} = y_2$. The arc $R_u R_o = h$ and R_u indicates thelowest position of the center of the swinging roller follower and R_o the highest position.

Because the cam (i.e., the surface of the cam as it passes under the follower roller) rotates away from pivot point *M*, $k \tan \tau_1$ is laid out away from *M*, that is, from R_{y1} to E_1 and $k \tan \tau_2$ is laid out toward *M* from R_{y2} to E_2. Angle α_1 at E_1 determines one ray and α_2 at E_2 another ray, which together subtend an area *A* having the property that if the cam shaft center is chosen inside this area, the pressure angles at the points of the cam corresponding to P_1 and P_2 in the displacement diagram will not exceed the given values α_1 and α_2, respectively. If the cam shaft center is chosen on the ray drawn from E_1 at an angle α_1 = 45°, the pressure angle α_1 on the cam profile corresponding to point P_1 will be exactly 45°, and if chosen on the ray from E_2, the pressure angle α_2 corresponding to P_2 will be exactly 45°. If another point, O_2 for example, is chosen as the cam shaft center, the pressure angle corresponding to P_1 will be α'_1 and that corresponding to P_2 will be α_2.

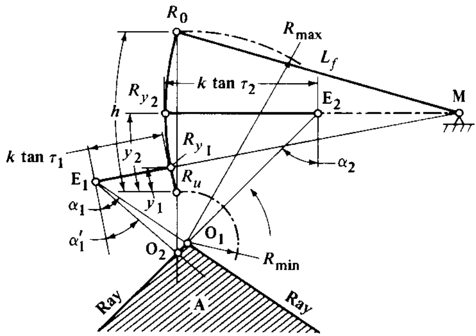


Fig. 18. Finding Proportions of Cam; Swinging Roller Follower (CCW Rotation)

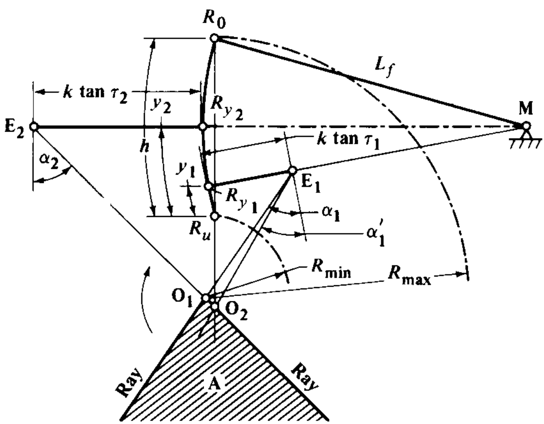


Fig. 19. Finding Proportions of Cam; Swinging Roller Follower (CW Rotation)

Fig. 19 shows the construction for rotation toward pivot point *M* (clockwise rotation of the cam in this case). The layout of the cam curve is made in a manner similar to that shown previously in Fig. 12.

In this example, the cam mechanism was so proportioned that the pressure angles at certain points (corresponding to P_1 and P_2) do not exceed certain specified values (namely α_1 and α_2).

To make sure that the pressure angle at *no point* along the cam profile exceeds the specified value, the previous procedure should be repeated for a series of points along the profile.

Formulas for Calculating Pressure Angles.—The graphical methods described previously are useful because they permit layout and measurement of pressure angles and radii of curvature of *any* cam profile. For cams of complicated profiles, and especially if the profile cannot be represented by a simple formula, the graphical method may be the only practical solution. However, for some of the standard cam profiles utilizing *radial* translating roller followers, the following formulas may be used to determine key cam dimensions before laying out the cam. These formulas enable the designer to specify the maximum pressure angle (usually 30° or less) and, using the specified value, to calculate the minimum cam size that will satisfy the requirement.

The following symbols are in addition to those starting on page 2164.

α_{max} = specified maximum pressure angle, degrees

$R_{\alpha max}$ = radius from cam center to point on pitch curve where α_{max} is located, inches

ϕ_p = rise angle, in degrees, corresponding to α_{max} and $R_{\alpha\,max}$

α = pressure angle at any selected point, degrees

R_α = radius from cam center to pitch curve at α, inches

ϕ = rise angle, in degrees, corresponding to α and R_α

For Uniform Velocity Motion: $\alpha = \arctan\left[\frac{180° h}{\pi\beta R_\alpha}\right]$ at radius R_α to the pitch curve (6a)

$\alpha_{max} = \arctan\left[\frac{180° h}{\pi\beta R_{min}}\right]$ at radius R_{min} of the pitch curve (ϕ=0°). (6b)

If α_{max} is specified, then the minimum radius to the lowest point on the pitch curve, R_{min}, is:

$R_{min} = \frac{180° h}{\pi\beta\tan\alpha_{max}}$ which corresponds to ϕ=0°. (6c)

For Parabolic Motion: $\alpha = \arctan\left[\frac{720° h\phi}{\pi\beta^2 R_\alpha}\right]$ at radius R_α to the pitch curve at angle ϕ, where $0 \le \phi \le \beta/2$ (7a)

$\alpha = \arctan\left[\frac{720° h(1-\phi/\beta)}{\pi\beta R_\alpha}\right]$ at radius R_α to the pitch curve at angle ϕ, where $\beta/2 \le \phi \le \beta$.

$\alpha = \arctan\left[\frac{360° h}{\pi\beta R_\alpha}\right]$ which occurs at $\phi=\beta/2$ and $R_\alpha = R_{min} + h/2$ (7b)

If α_{max} is specified, then the minimum radius to the lowest point of the pitch curve is:

$R_{min} = \left[\frac{360° h}{\pi\beta\tan\alpha_{max}} - \frac{h}{2}\right]$ which corresponds to ϕ=0°. (7c)

For Simple Harmonic Motion: $\alpha = \arctan\left[\frac{90° h}{\beta R_\alpha}\sin\left(\frac{180°\phi}{\beta}\right)\right]$ at radius R_α to the pitch curve at angle ϕ (8a)

$\phi_p = \left(\frac{\beta}{180°}\right)\left[\text{arccot}\left(\frac{\beta}{180°}\tan\alpha_{max}\right)\right]$ = value of ϕ where specified pressure angle α_{max} occurs (8b)

$R_{\alpha max} = \frac{h[\sin(180°\phi_p/\beta)]^2}{2\cos(180°\phi_p/\beta)}$ at point where $\alpha = \alpha_{max}$ and $\phi = \phi_{max}$ (8c)

$$R_{min} = R_{\alpha\ max} - \frac{h}{2}\left[1 - \cos\left(\frac{180°\phi_p}{\beta}\right)\right]$$ (8d)

For Cycloidal Motion: $\alpha = \arctan\left[\frac{180°}{\pi\beta R_\alpha}\left[1 - \cos\left(\frac{360°\phi}{\beta}\right)\right]\right]$ at radius R_α to the pitch curve at angle ϕ (9a)

$$\phi_p = \frac{\beta}{180°}\left[\operatorname{arccot}\left(\frac{\beta\tan\alpha_{max}}{360°}\right)\right]$$ ϕ_p=value of ϕ where specified pressure angle α_{max} occurs (9b)

$$R_{\alpha max} = \frac{h}{2\pi}\frac{[1 - \cos(360°\phi_p/\beta)]^2}{\sin(360°\phi_p/\beta)}$$ at point where $\alpha=\alpha_{max}$ and $\phi=\phi_p$ (9c)

$$R_{min} = R_{\alpha\ max} - h\left[\frac{\phi_p}{\beta} - \frac{1}{2\pi}\sin\left(\frac{360°\phi_p}{\beta}\right)\right]$$ (9d)

Radius of Curvature.—The minimum radius of curvature of a cam should be kept as large as possible (1) to prevent undercutting of the convex portion of the cam and (2) to prevent too high surface stresses. Figs. 20(a), (b) and (c) illustrate how undercutting occurs.

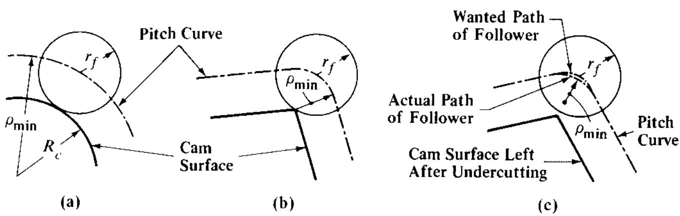


Fig. 20. (a) No Undercutting. (b) Sharp Corner on Cam. (c) Undercutting

In Fig. 20(a) the radius of curvature of the path of the follower is ρ_{min} and the cam will at that point have a radius of curvature $R_c = \rho_{min} - r_f$.

In Fig. 20(b) $\rho_{min} = r_f$ and $R_c = 0$. Therefore, the actual cam will have a sharp corner which in most cases will result in too high surface stresses.

In Fig. 20(c) is shown the case where $\rho_{min} < r_f$. This case is not possible because undercutting will occur and the actual motion of the roller follower will deviate from the desired one as shown.

Undercutting cannot occur at the *concave* portion of the cam profile (working surface), but caution should be exerted in not making the radius of curvature equal to the radius of the roller follower. This condition would occur if there is a cusp on the displacement diagram which, of course, should always be avoided. To enable milling or grinding of *concave* portions of a cam profile, the radius of curvature of concave portions of the cam, $R_c = \rho_{min} + r_f$, must be larger than the radius of the cutter to be used.

The radius of curvature is used in calculating surface stresses (see following section), and may be determined by measurement on the cam layout or, in the case of radial translating followers, may be calculated using the formulas that follow. Although these formulas are exact for radial followers, they may be used for offset and swinging followers to obtain an approximation.

Based upon polar coordinates, the radius of curvature is:

$$\rho = \frac{\left[r^2 + \left(\frac{dr}{d\phi}\right)^2\right]^{3/2}}{r^2 + 2\left(\frac{dr}{d\phi}\right)^2 - r\left(\frac{d^2r}{d\phi^2}\right)} \tag{10}$$

*Positive values (+) indicate convex curve; negative values (–), concave.

In Equation (10), $r = (R_{min} + y)$, where R_{min} is the smallest radius to the pitch curve (see Fig. 12) and y is the displacement of the follower from its lowest position given in terms of ϕ, the angle of cam rotation. The following formulas for r, $dr/d\phi$, and $d^2r/d\phi^2$ may be substituted into Equation (10) to calculate the radius of curvature at any point of the cam pitch curve; however, to determine the possibility of undercutting of the convex portion of the cam, it is the minimum radius of curvature on the convex portion, ρ_{min}, that is needed. The minimum radius of curvature occurs, generally, at the point of maximum *negative* acceleration.

Parabolic motion:

$$r = R_{min} + h - 2h\left(1 - \frac{\phi}{\beta}\right)^2 \tag{11a}$$

$$\frac{dr}{d\phi} = \frac{720°h}{\pi\beta}\left(1 - \frac{\phi}{\beta}\right) \tag{11b}$$

$$\frac{d^2r}{d\phi^2} = \frac{-4(180°)^2h}{\pi^2\beta^2} \tag{11c}$$

} $\frac{\beta}{2} \le \phi \le \beta$

These equations are for the deceleration portion of the curve as explained in the footnote to Table 1.

The minimum radius of curvature can occur at either $\phi = \beta/2$ or at $\phi = \beta$, depending on the magnitudes of h, R_{min}, and β. Therefore, to determine which is the case, make two calculations using Formula (10), one for $\phi = \beta/2$, and the other for $\phi = \beta$.

Simple harmonic motion:

$$r = R_{min} + \frac{h}{2}\left[1 - \cos\left(\frac{180°\phi}{\beta}\right)\right] \tag{12a}$$

$$\frac{dr}{d\phi} = \frac{180°h}{2\beta}\sin\left(\frac{180°\phi}{\beta}\right) \tag{12b}$$

$$\frac{d^2r}{d\phi^2} = \frac{(180°)^2h}{2\beta^2}\cos\left(\frac{180°\phi}{\beta}\right) \tag{12c}$$

} $0 \le \phi \le \beta$

The minimum radius of curvature can occur at either $\phi = \beta/2$ or at $\phi = \beta$, depending on the magnitudes of h, R_{min}, and β. Therefore, to determine which is the case, make two calculations using Formula (10), one for $\phi = \beta/2$, and the other for $\phi = \beta$.

Cycloidal motion:

$$r = R_{min} + h\left[\frac{\phi}{\beta} - \frac{1}{2\pi}\sin\left(\frac{360°\phi}{\beta}\right)\right] \tag{13a}$$

$$\frac{dr}{d\phi} = \frac{180°h}{\pi\beta}\left[1 - \cos\left(\frac{360°\phi}{\beta}\right)\right] \tag{13b}$$

} $0 \le \phi \le \beta$

$$\frac{d^2r}{d\phi^2} = \frac{2(180°)^2 h}{\pi\beta^2}\sin\left(\frac{360°\phi}{\beta}\right) \qquad (13c)$$

$$\rho_{min} = \frac{[(R_{min} + 0.91h)^2 + (180°h/\pi\beta)^2]^{3/2}}{(R_{min} + 0.91h)^2 + 2(180°h/\pi\beta)^2 + (R_{min} + 0.91h)[2(180°)^2 h/\pi\beta^2]} \qquad (13d)$$

(ρ_{min} occurs near $\phi = 0.75\beta$.)

Example: Given $h = 1$ in., $R_{min} = 2.9$ in., and $\beta = 60°$. Find ρ_{min} for parabolic motion, simple harmonic motion, and cycloidal motion.

Solution: $\rho_{min} = 2.02$ in. for parabolic motion, from Equation (10)
$\rho_{min} = 1.8$ in. for simple harmonic motion, from Equation (10)
$\rho_{min} = 1.6$ in. for cycloidal motion, from Equation (13d)

The value of ρ_{min} on *any* cam may also be obtained by measurement on the layout of the cam using a compass.

Cam Forces, Contact Stresses, and Materials.—After a cam and follower configuration has been determined, the forces acting on the cam may be calculated or otherwise determined. Next, the stresses at the cam surface are calculated and suitable materials to withstand the stress are selected. If the calculated maximum stress is too great, it will be necessary to change the cam design.

Such changes may include: 1) increasing the cam size to decrease pressure angle and increase the radius of curvature; 2) changing to an offset or swinging follower to reduce the pressure angle; 3) reducing the cam rotation speed to reduce inertia forces; 4) increasing the cam rise angle, β, during which the rise,*h*, occurs; 5) increasing the thickness of the cam, provided that deflections of the follower are small enough to maintain uniform loading across the width of the cam; and 6) using a more suitable cam curve or modifying the cam curve at critical points.

Although parabolic motion seems to be the best with respect to minimizing the calculated maximum acceleration and, therefore, also the maximum acceleration forces, nevertheless, in the case of high speed cams, cycloidal motion yields the lower maximum acceleration forces. Thus, it can be shown that owing to the sudden change in acceleration (called *jerk* or *pulse*) in the case of parabolic motion, the actual forces acting on the cam are doubled and sometimes even tripled at high speed, whereas with cycloidal motion, owing to the gradually changing acceleration, the actual dynamic forces are only slightly higher than the theoretical. Therefore, the calculated force due to acceleration should be multiplied by at least a factor of 2 for parabolic and 1.05 for cycloidal motion to provide an allowance for the load-increasing effects of elasticity and backlash.

The main factors influencing cam forces are: 1) displacement and cam speed (forces due to acceleration); 2) dynamic forces due to backlash and flexibility; 3) linkage dimensions which affect weight and weight distribution; 4) pressure angle and friction forces; and 5) spring forces.

The main factors influencing stresses in cams are: 1) radius of curvature for cam and roller; and 2) materials.

Acceleration Forces: The formula for the force acting on a translating body given an acceleration *a* is:

$$R = \frac{Wa}{g} = \frac{Wa}{386} \qquad (14)$$

In this formula, $g = 386$ inches/second squared, a = acceleration of W in inches/second squared; R = resultant of all the external forces (except friction) acting on the weight W. For cam analysis purposes, W, in pounds, consists of the weight of the follower, a portion of the weight of the return spring (⅓), and the weight of the members of the external mechanism against which the follower pushes, for example, the weight of a piston:

$$W = W_f + \tfrac{1}{3}W_s + W_e \tag{15}$$

where W = equivalent single weight; W_f = follower weight; W_s = spring weight; and W_e = external weight, all in pounds.

Spring Forces: The return spring, K_s, shown in Fig. 21a must be strong enough to hold the follower against the cam at all times. At high cam speeds the main force attempting to separate the follower from the cam surface is the acceleration force R at the point of maximum *negative* acceleration. Thus, at that point the spring must exert a force F_s,

$$F_s = R - W_f - F_e - F_f \tag{16}$$

where F_e = external force resisting motion of follower, and F_f = friction force from follower guide bushings and other sources.

When the follower is at its lowest position (R_{min} in Fig. 21a), it is usual practice to have the spring provide some estimated preload to account for "set" that takes place in a spring after repeated use and to prevent roller sliding at the start of movement.

The required spring constant, K_s, in pounds per inch of spring deflection is:

$$K_s = \frac{F_s - \text{preload}}{y_a} \tag{17}$$

where y_a = rise of cam from R_{min} to height at which maximum negative acceleration takes place.

The force, F_y, that the spring exerts at any height y above R_{min} is:

$$F_y = yK_s + \text{preload} \tag{18}$$

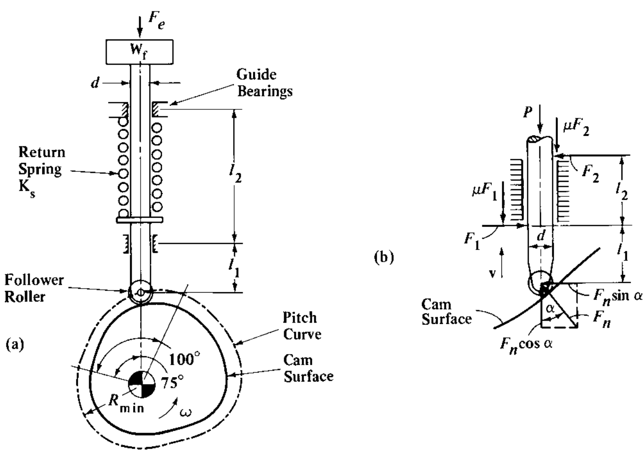


Fig. 21. (a) Radial Translating Follower and Cam System (b) Force Acting on a Translating Follower

Pressure Angle and Friction Forces: As shown in Fig. 21b, the pressure angle of the cam causes a sideways component $F_n \sin \alpha$ which produces friction forces μF_1 and μF_2 in the guide bushing. If the follower rod is too flexible, bending of the follower will increase these friction forces. The effect of the friction forces and the pressure angle are taken into account in the formula,

$$F_n = \frac{P}{\cos\alpha - \frac{\mu\sin\alpha}{l_2}(2l_1 + l_2 - \mu d)} \tag{19a}$$

where μ = coefficient of friction in bushing; l_1, l_2, and d are as shown in Fig. 21; and P = the sum of all the forces acting down against the upward motion of the follower (acceleration force + spring force + follower weight + external force)

$$P = \frac{W \times a}{386} + (yK_s + \text{preload}) + W_f + F_e \tag{19b}$$

Cam Torque: The follower pressing against the cam causes resisting torques during the rise period and assisting torques during the return period. The maximum value of the resisting torque determines the cam drive requirements. Instantaneous torque values may be calculated from

$$T_o = \frac{30vF_n\cos\alpha}{\pi N} = (R_{\min} + y)F_n\sin\alpha \tag{20}$$

in which T_o = instantaneous torque in pound-inches.

Exampleof Force Analysis: A radial translating follower system is shown in Fig. 21a. The follower is moved with cycloidal motion over a distance of 1 in. and an angle of lift β = 100°. Cam speed N = 900 rpm. The weight of the follower mass, W_f, is 2 pounds. Both the spring weight W_s and the external weight W_e are negligible. The follower stem diameter is 0.75 in., l_1 = 1.5 in., l_2 = 4 in., coefficient of friction μ = 0.05, external force F_e = 10 lbs, and the pressure angle is not to exceed 30°.

(a) What is the smallest radius $R_{\min}$ to the pitch curve?

From Formula (9b) the rise angle ϕ_p to where the maximum pressure angle $\alpha_{\max}$ exists is:

$$\phi_p = \frac{\beta}{180°}\left[\operatorname{arccot}\left(\frac{\beta\tan\alpha_{\max}}{360°}\right)\right] = \frac{100°}{180°}\left[\operatorname{arccot}\left(\frac{100° \times \tan 30°}{360°}\right)\right]$$
$$= 44.94^\circ = 45^\circ$$

From Formula (9c) the radius, $R_{\alpha\max}$, at which the angle of rise is ϕ_p is:

$$R_{\alpha\max} = \frac{h}{2\pi}\frac{[1 - \cos(360°\phi_p/\beta)]^2}{\sin(360°\phi_p/\beta)} = \frac{1}{2\pi}\frac{[1 - \cos[(360^\circ \times 45^\circ)/100^\circ]]^2}{\sin[(360^\circ \times 45^\circ)/100^\circ]} = 1.96 \text{ in.}$$

From Formula (9d), $R_{\min}$ is given by

$$R_{\min} = R_{\alpha\max} - h\left[\frac{\phi_p}{\beta} - \frac{1}{2\pi}\sin\left(\frac{360°\phi_p}{\beta}\right)\right]$$
$$= 1.96 - 1 \times \left[\frac{45°}{100°} - \frac{1}{2\pi}\sin\left(\frac{360° \times 45°}{100°}\right)\right] = 1.560 \text{ in.}$$

The same results could have been obtained graphically. If this $R_{\min}$ is too small, i.e., if the cam bore and hub require a larger cam, then $R_{\min}$ can be increased, in which case the maximum pressure angle will be less than 30°.

(b) If the return spring K_s is specified to provide a preload of 36 lbs when the follower is at $R_{\min}$, what is the spring constant required to hold the follower on the cam throughout the cycle?

The follower tends to leave the cam at the point of maximum *negative* acceleration. Fig. 7 shows this to be at $\phi = \frac{3}{4}\beta = 75°$.

From Formula (4c),

$$a = \frac{2\pi h\omega^2}{\beta^2}\sin\left(\frac{360°\phi}{\beta}\right) = \frac{2\pi \times 1 \times (6 \times 900)^2}{(100°)^2}\sin\left(\frac{360° \times 75°}{100°}\right) = -18{,}300 \text{ in./sec}^2$$

From Formulas (14) and (15),

$$R = \frac{Wa}{386} = \frac{(W_f + \frac{1}{3}W_s + W_e)a}{386} = \frac{(2+0+0)(-18{,}300)}{386} = 95 \text{ lbs (upward)}$$

Using Formula (16) to determine the spring force F_s to hold the follower on the cam,

$$F_s = R - W_f - F_e - F_f$$

as stated on page 2180, the value of R in the above formula should be multiplied by 1.05 for cycloidal motion to provide a factor of safety for dynamic pulses. Thus,

$$F_s = 1.05R - W_f - F_e - F_f = 1.05 \times 95 - 2 - 10 - 0 = 88 \text{ lbs (downward)}$$

The spring constant from Formula (17) is:

$$K_s = \frac{F_s - \text{preload}}{y_a} = \frac{88 - 36}{y_a}$$

and, from Formula (4a) y_a is:

$$y_a = h\left[\frac{\phi}{\beta} - \frac{1}{2\pi}\sin\left(\frac{360°\phi}{\beta}\right)\right] = 1 \times \left[\frac{75°}{100°} - \frac{1}{2\pi}\sin\left(\frac{360° \times 75°}{100°}\right)\right] = 0.909 \text{ in.}$$

so that $K_s = (88 - 36)/0.909 = 57$ lb/in.

(c) At the point where the pressure angle α_{max} is 30° ($\phi = 45°$) the rise of the follower is 1.96 – 1.56 = 0.40 in. What is the normal force, F_n, on the cam? From Formulas (19a) and (19b)

$$F_n = \frac{Wa/386 + yK_s + \text{preload} + W_f + F_e}{\cos\alpha - \frac{\mu\sin\alpha}{l_2}(2l_1 + l_2 - \mu d)}$$

using $\phi = 45°$, $h = 1$ in., $\beta = 100°$, and $\omega = 6 \times 900$ in Formula (4c) gives $a = 5660$ in./sec^2. So that, with $W = 2$ lbs, $y = 0.4$, $K_s = 57$, preload = 36 lbs, $W_f = 2$ lbs, $F_e = 10$ lbs, $\alpha = 30°$, $\mu = 0.05$, $l_1 = 1.5$, $l_2 = 4$, and $d = 0.75$,

$$F_n = \frac{(2 \times 5660)/386 + 0.4 \times 57 + 36 + 2}{\cos 30° - \frac{0.05 \times \sin 30°}{4}(2 \times 1.5 + 4 - 0.05 \times 0.75)} = 110 \text{ lbs}$$

Note: If the coefficient of friction had been assumed to be 0, then $F_n = 104$; on the other hand, if the follower is too flexible, so that sidewise bending occurs causing jamming in the bushing, the coefficient of friction may increase to, say, 0.5, in which case the calculated F_n = 200 lbs.

(d) Assuming that in the manufacture of this cam that an error or "bump" resulting from a chattermark or as a result of poor blending occurred, and that this "bump" rose to a height of 0.001 in. in a 1° rise of the cam in the vicinity of $\phi = 45°$. What effect would this bump have on the acceleration force R?

One formula that may be used to calculate the change in acceleration caused by such a cam error is:

$$\Delta a = \pm 2e\left(\frac{6N}{\Delta\phi}\right)^2 \qquad (21)$$

where Δa = change in acceleration,

e = error in inches,

$\Delta\phi$ = width of error in degrees. The plus (+) sign is used for a "bump" and the minus (−) sign for a dent or hollow in the surface

For $e = 0.001$, $\Delta\phi = 1°$, and $N = 900$ rpm,

$$\Delta a = +2 \times 0.001\left(\frac{6 \times 900}{1°}\right)^2 = 58{,}320 \text{ in./sec}^2$$

which is 10 times the acceleration calculated for a perfect cam and would cause sufficient force F_n to damage the cam surface. On high speed cams, therefore, accuracy is of considerable importance.

(e) What is the cam torque at $\phi = 45°$?

From Formula (20),

$$T_o = (R_{min} + y)F_n \sin\alpha$$
$$= (1.56 + 0.4) \times 110 \times \sin 30° = 108 \text{ in.-lbs}$$

(f) What is the radius of curvature at $\phi = 45°$?

From Formula (10),

$$\rho = \frac{\left[r^2 + \left(\frac{dr}{d\phi}\right)^2\right]^{3/2}}{r^2 + 2\left(\frac{dr}{d\phi}\right)^2 - r\left(\frac{d^2r}{d\phi^2}\right)}$$

$$r = R_{min} + y = 1.56 + 0.4 = 1.96$$

From Formula (13b),

$$\frac{dr}{d\phi} = \frac{180° h}{\pi\beta}\left[1 - \cos\left(\frac{360°\phi}{\beta}\right)\right] = \frac{180° \times 1}{\pi \times 100°}\left[1 - \cos\left(\frac{360° \times 45°}{100°}\right)\right]$$
$$= 1.12$$

From Formula (13c),

$$\frac{d^2r}{d\phi^2} = \frac{2(180°)^2 h}{\pi\beta^2}\sin\left(\frac{360°\phi}{\beta}\right) = \frac{2 \times (180°)^2 \times 1}{\pi \times (100°)^2}\sin\left(\frac{360° \times 45°}{100°}\right)$$
$$= 0.64$$

$$\rho = \frac{[(1.96)^2 + (1.12)^2]^{3/2}}{(1.96)^2 + 2(1.12)^2 - 1.96 \times 0.64} = 2.26 \text{ in.}$$

Calculation of Contact Stresses.—When a roller follower is loaded against a cam, the compressive stress developed at the surface of contact may be calculated from

$$S_c = 2290\sqrt{\frac{F_n}{b}\left(\frac{1}{r_f} \pm \frac{1}{R_c}\right)} \qquad (22)$$

for a steel roller against a steel cam. For a steel roller on a cast iron cam, use 1850 instead of 2290 in Equation (22).

S_c = maximum calculated compressive stress, psi

F_n = normal load, lb

b = width of cam, inch

R_c = radius of curvature of cam surface, inch

r_f = radius of roller follower, inch

The plus sign in (21) is used in calculating the maximum compressive stress when the roller is in contact with the convex portion of the cam profile and the minus sign is used when the roller is in contact with the concave portion. When the roller is in contact with the straight (flat) portion of the cam profile, $R_c = \infty$ and $1/R_c = 0$. In practice, the greatest compressive stress is most apt to occur when the roller is in contact with that part of the cam profile which is convex and has the smallest radius of curvature.

Example: Given the previous cam example, the radius of the roller $r_f = 0.25$ in., the convex radius of the cam $R_c = (2.26–0.25)$ in., the width of contact $b = 0.3$ in., and the normal load $F_n = 110$ lbs. Find the maximum surface compressive stress. From (21),

$$S_c = 2290\sqrt{\frac{110}{0.3}\left(\frac{1}{0.25} + \frac{1}{2.01}\right)} = 93{,}000 \text{ psi}$$

This calculated stress should be less than the allowable stress for the material selected from Table 2.

Cam Materials: In considering materials for cams it is difficult to select any single material as being the best for every application. Often the choice is based on custom or the machinability of the material rather than its strength. However, the failure of a cam or roller is commonly due to fatigue, so that an important factor to be considered is the limiting wear load, which depends on the surface endurance limits of the materials used and the relative hardnesses of the mating surfaces.

Table 2. Cam Materials

Cam Materials for Use with Roller of Hardened Steel	Maximum Allowable Compressive Stress, psi
Gray-iron casting, ASTM A 48-48, Class 20, 160–190 Bhn, phosphate-coated	58,000
Gray-iron casting, ASTM A 339-51T, Grade 20, 140–160 Bhn	51,000
Nodular-iron casting, ASTM A 339-51T, Grade 80-60-03, 207–241Bhn	72,000
Gray-iron casting, ASTM A 48–48, Class 30, 200–220 Bhn	65,000
Gray-iron casting, ASTM A 48–48, Class 35, 225–225 Bhn	78,000
Gray-iron casting, ASTM A 48-48, Class 30, heat treated (Austempered), 225–300 Bhn	90,000
SAE 1020 steel, 130–150 Bhn	82,000
SAE 4150 steel, heat treated to 270–300 Bhn, phosphate coated	20,000
SAE 4150 steel, heat treated to 270–300 Bhn	188,000
SAE 1020 steel, carburized to 0.045 in. depth of case, 50–58 Rc	226,000
SAE 1340 steel, induction hardened to 45–55 Rc	198,000
SAE 4340 steel, induction hardened to 50–55 Rc	226,000

Based on United Shoe Machinery Corp. data by Guy J. Talbourdet.

In Table 2 are given maximum permissible compressive stresses (surface endurance limits) for various cam materials when in contact with a roller of hardened steel. The stress values shown are based on 100,000,000 cycles or repetitions of stress for pure rolling. Where the repetitions of stress are considerably greater than 100,000,000, where there is appreciable misalignment, or where there is sliding, more conservative stress figures must be used.

Layout of Cylinder Cams.—In Fig. 22 is shown the development of a uniformly accelerated motion cam curve laid out on the surface of a cylindrical cam. This development is necessary for finding the projection on the cylindrical surface, as shown at *KL*. To construct the developed curve, first divide the base circle of the cylinder into, say, twelve equal parts. Set off these parts along line *ag*. Only one-half of the layout has been shown, as the other half is constructed in the same manner, except that the curve is here falling instead of rising. Divide line *aH* into the same number of divisions as the half circle, the divisions being in the proportion 1 : 3 : 5 : 5 : 3 : 1. Draw horizontal lines from these division points and vertical lines from *a*, *b*, *c*, etc. The intersections between the two sets of lines are points on the developed cam curve. These points are transferred to the cylindrical surface at the left by projection in the usual manner.

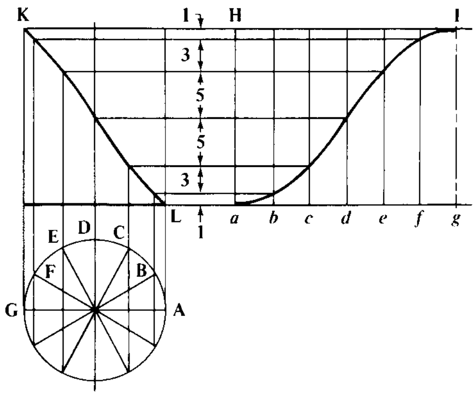


Fig. 22. Development of Cylindrical Cam

Shape of Rolls for Cylinder Cams.—The rolls for cylindrical cams working in a groove in the cam should be conical rather than cylindrical in shape, in order that they may rotate freely and without excessive friction. Fig. 23(a) shows a straight roll and groove, the action of which is faulty because of the varying surface speed at the top and bottom of the groove. Fig. 23(b) shows a roll with curved surface. For heavy work, however, the small bearing area is quickly worn down and the roll presses a groove into the side of the cam as well, thus destroying the accuracy of the movement and creating backlash. Fig. 23(c) shows the conical shape which permits a true rolling action in the groove. The amount of taper depends on the angle of spiral of the cam groove. As this angle, as a rule, is not constant for the whole movement, the roll and groove should be designed to meet the requirements on that section of the cam where the heaviest duty is performed. Frequently the cam groove is of a nearly even spiral angle for a considerable length. The method for determining the angle of the roll and groove to work correctly during the important part of the cycle is as follows:

In Fig. 23(d), *b* is the circumferential distance on the surface of the cam that includes the section of the groove for which correct rolling action is required. The throw of the cam for this circumferential movement is *a*. Line *OU* is the development of the movement of the cam roll during the given part of the cycle, and *c* is the movement corresponding to *b*, but on a circle the diameter of which is equal to that of the cam at the bottom of the groove. With the same throw *a* as before, the line *OV* will be the development of the cam at the bottom of the groove. *OU* then is the length of the helix traveled by the top of the roll, while *OV* is the travel at the bottom of the groove. If, then, the top width and bottom width of the groove be made proportional to *OU* and *OV*, the groove will be properly proportioned.

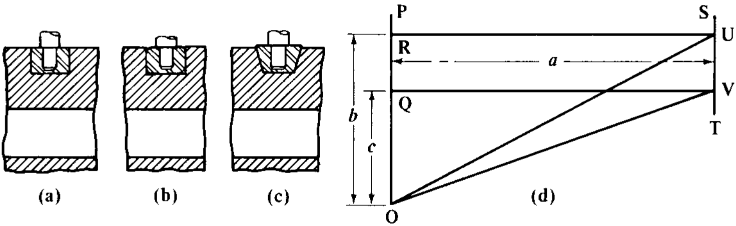


Fig. 23. Shape of Rolls for Cylinder Cams

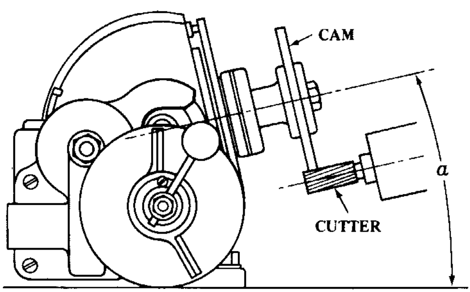
Cam Milling.—Plate cams having a constant rise, such as are used on automatic screw machines, can be cut in a universal milling machine, with the spiral head set at an angle α, as shown by the illustration. When the spiral head is set vertical, the "lead" of the cam (or its rise for one complete revolution) is the same as the lead for which the machine is geared; but when the spiral head and cutter are inclined, any lead or rise of the cam can be obtained, provided it is less than the lead for which the machine is geared, that is, less than the forward feed of the table for one turn of the spiral-head spindle. The cam lead, then, can be varied within certain limits by simply changing the inclination α of the spiral head and cutter. In the following formulas for determining this angle of inclination, for a given rise of cam and with the machine geared for a lead, *L*, selected from the tables beginning on page 1933, let

α = angle to which index head and milling attachment are set from horizontal as shown in the accompanying diagram

r = rise of cam in given part of circumference

L = spiral lead for which milling machine is geared

ϕ = angle in which rise is required, expressed in degrees



$$\sin\alpha = \frac{360^\circ \times r}{\phi \times L}$$

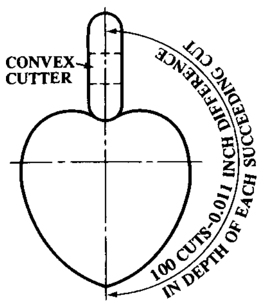
For example, suppose a cam is to be milled having a rise of 0.125 inch in 300 degrees and that the machine is geared for the smallest possible lead, or 0.670 inch; then:

$$\sin\alpha = \frac{360° \times 0.125}{300° \times 0.670} = 0.2239$$

which is the sine of 12° 56′. Therefore, to secure a rise of 0.125 inch with the machine geared for 0.670 inch lead, the spiral head is elevated to an angle of 12° 56′ and the vertical milling attachment is also swiveled around to locate the cutter in line with the spiral-head spindle, so that the edge of the finished cam will be parallel to its axis of rotation. In the example given, the lead used was 0.670. A larger lead, say 0.930, could have been selected from the table on page 1933. In that case, $\alpha = 9° 17′$.

When there are several lobes on a cam, having different leads, the machine can be geared for a lead somewhat in excess of the greatest lead on the cam, and then all the lobes can be milled without changing the spiral head gearing, by simply varying the angle of the spiral head and cutter to suit the different cam leads. Whenever possible, it is advisable to mill on the under side of the cam, as there is less interference from chips; moreover, it is easier to see any lines that may be laid out on the cam face. To set the cam for a new cut, it is first turned back by operating the handle of the table feed screw, after which the index crank is disengaged from the plate and turned the required amount.

Simple Method for Cutting Uniform Motion Cams.—Some cams are laid out with dividers, machined and filed to the line; but for a cam that must advance a certain number of thousandths per revolution of spindle this method is not accurate. Cams are easily and accurately cut in the following manner.



Let it be required to make the heart cam shown in the illustration. The throw of this cam is 1.1 inch. Now, by setting the index on the milling machine to cut 200 teeth and also dividing 1.1 inch by 100, we find that we have 0.011 inch to recede from or advance towards the cam center for each cut across the cam. Placing the cam securely on an arbor, and the latter between the centers of the milling machine, and using a convex cutter set the proper distance from the center of the arbor, make the first cut across the cam. Then, by lowering the milling machine knee 0.011 inch and turning the index pin the proper number of holes on the index plate, take the next cut and so on.