Ouroboros: a simple, secure and efficient key exchange protocol based on coding theory

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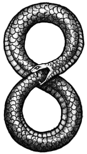


Joint work with:

P. Gaborit
University of Limoges

G. Zémor
University of Bordeaux

Motivations

[ME78]

Motivations

[ME78] $\longrightarrow$ [Nie86]

Motivations

[ME78] $\longrightarrow$ [Nie86] $\longrightarrow$

RS
BCH
Goppa
RM

$\Bigg\}$ 80's $\downarrow$ 00's

Motivations

Key Sizes

[ME78] ⟶ [Nie86] ⟶ RS, BCH, Goppa, RM } 80's ↓ 00's

Security reduction to a standard problem (random codes)

Motivations

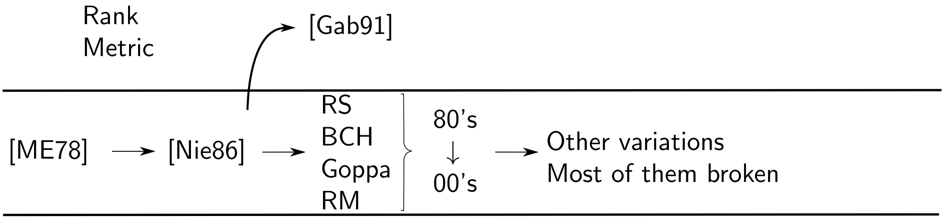
Key Sizes

[ME78] → [Nie86] → RS, BCH, Goppa, RM (80's ↓ 00's) → Other variations
Most of them broken

Security proof

Motivations

Key Sizes



Security proof

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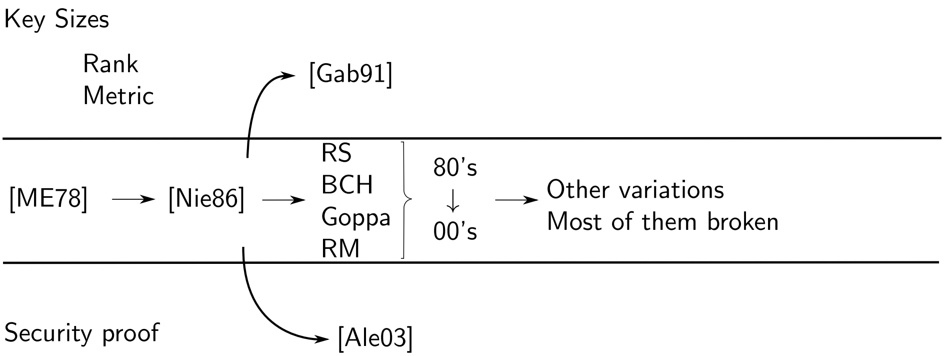

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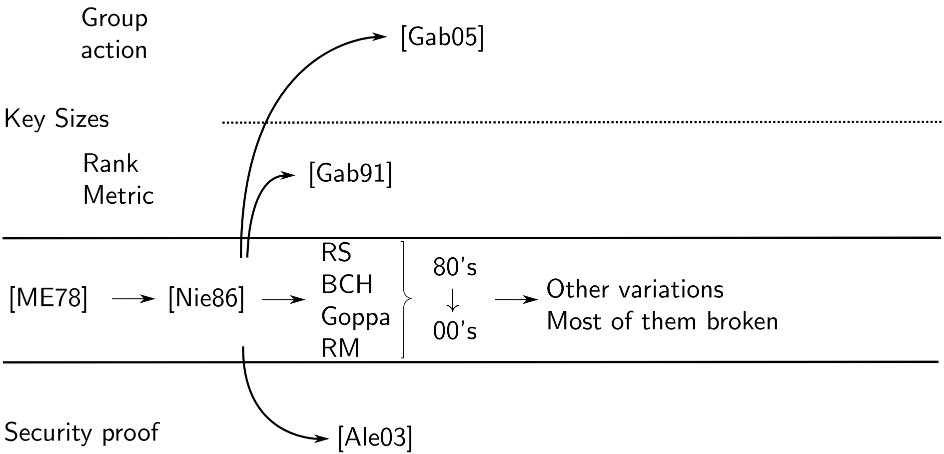

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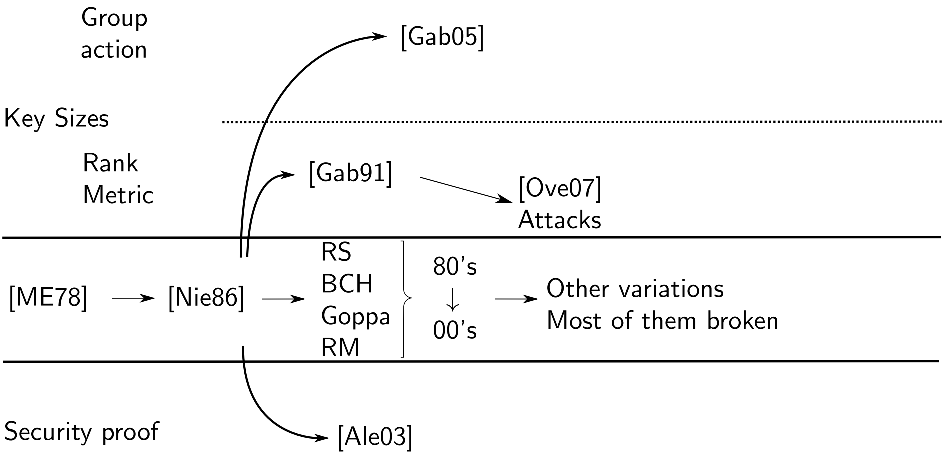

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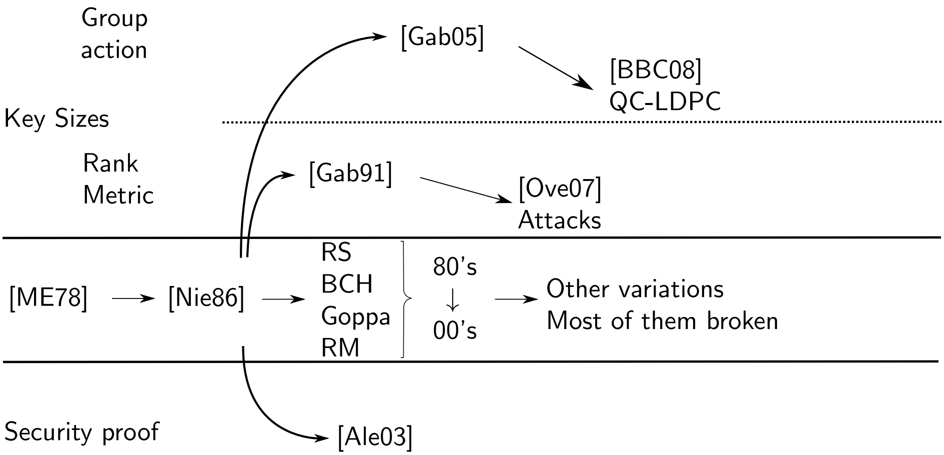

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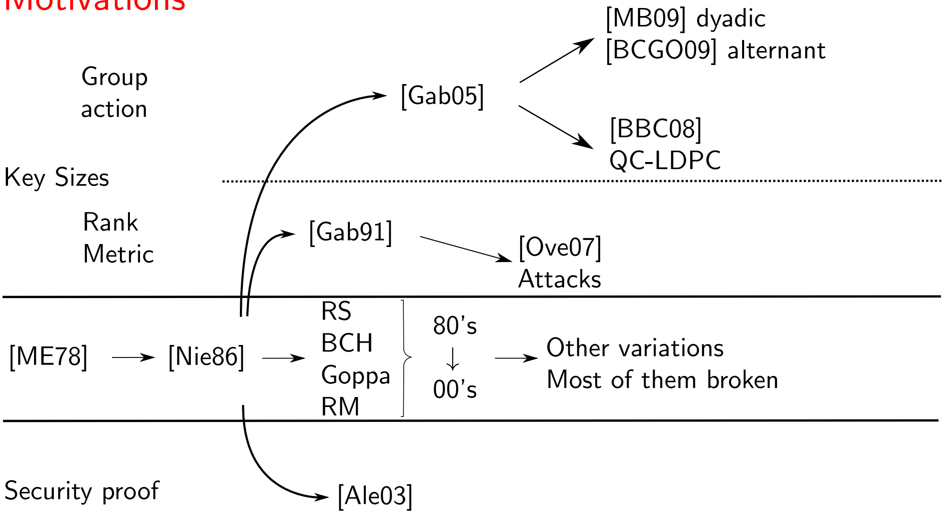

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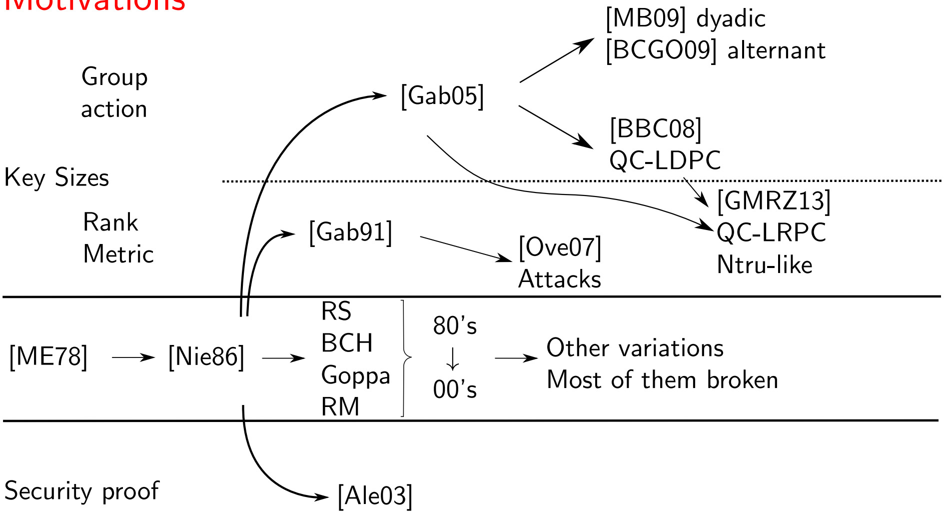

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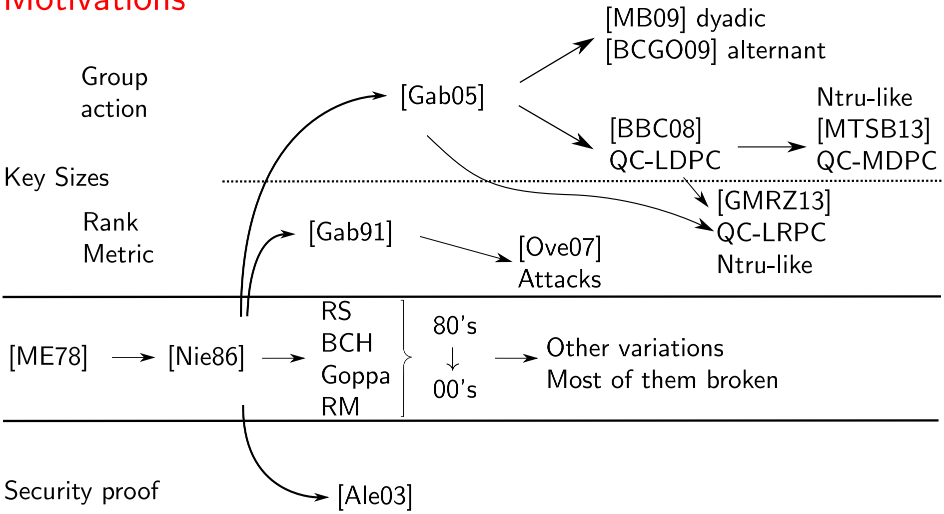

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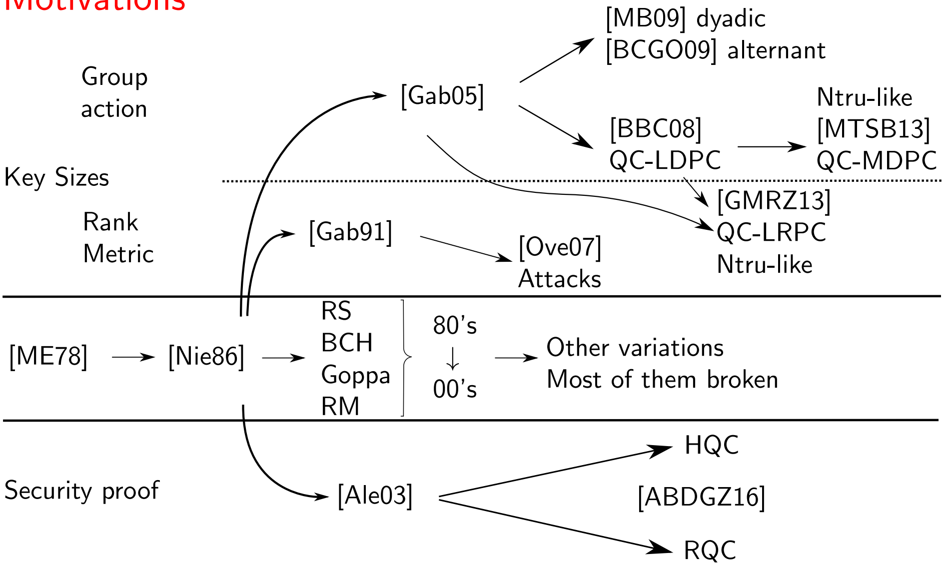

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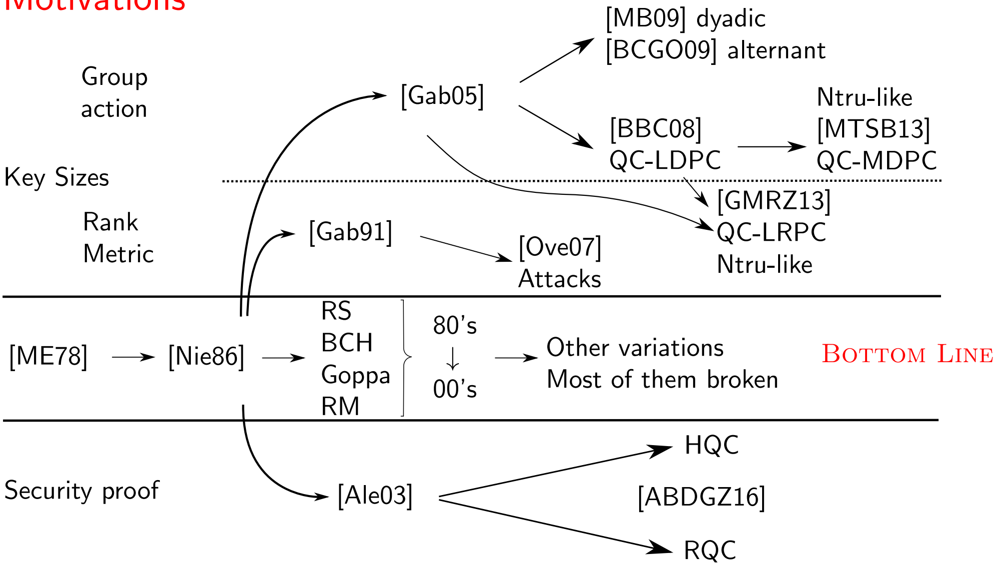

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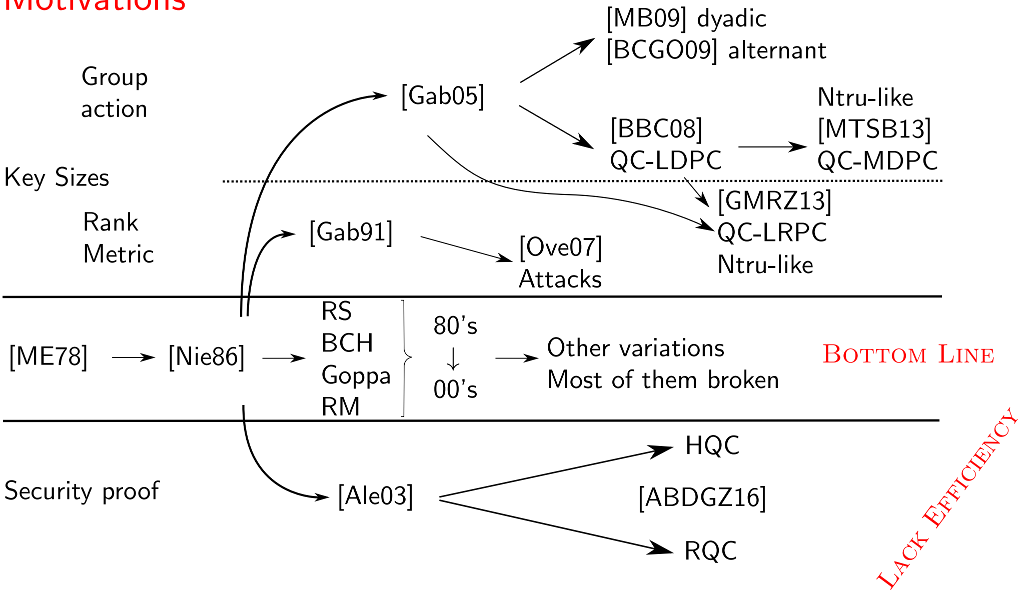

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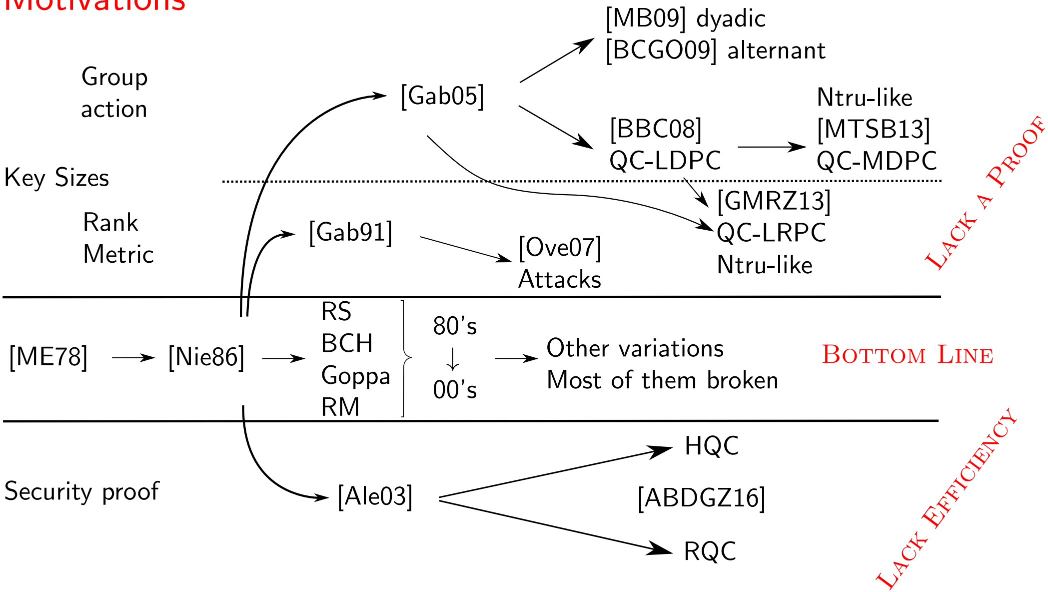

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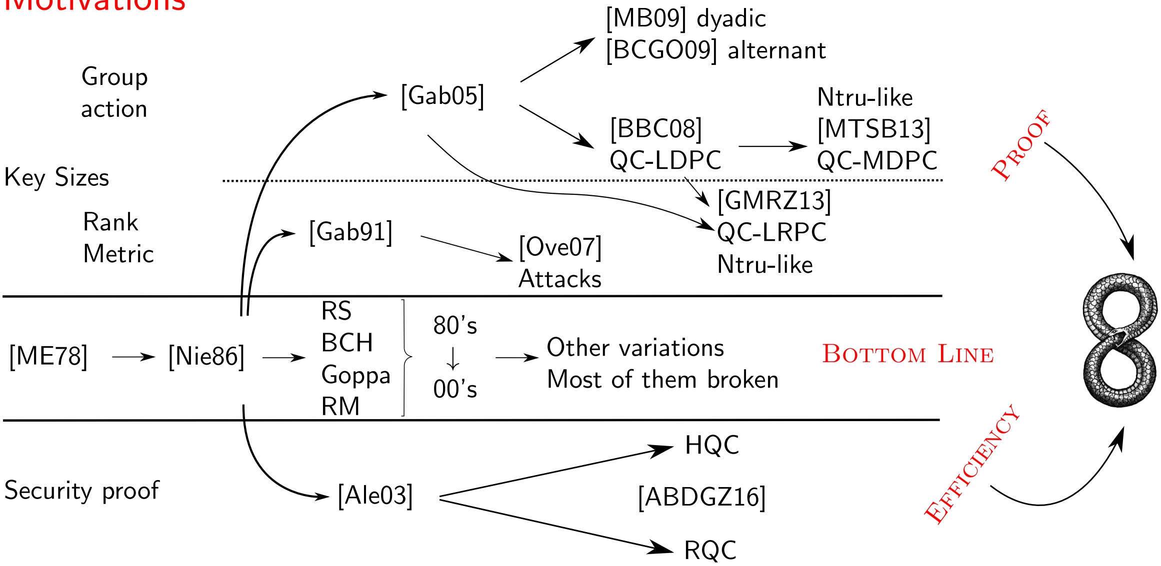

Outline

1. Reminders on HQC
2. Presentation of the Ouroboros protocol
3. Security
4. Parameters

HQC Encryption Scheme [ABD+16]

Encryption scheme in **H**amming metric, using **Q**uasi-**C**yclic Codes

- Notation: Secret data - Public data - One-time Randomness
- **G** is the generator matrix of some public code $\mathcal{C}$.

Alice		Bob
$\mathsf{seed}_\mathbf{h} \xleftarrow{\$} \{0,1\}^\lambda$, $\mathbf{h} \xleftarrow{\mathsf{seed}_\mathbf{h}} \mathbb{F}_2^n$ $\mathbf{x}, \mathbf{y} \xleftarrow{\$} \mathcal{S}_w^n(\mathbb{F}_2)$, $\mathbf{s} \leftarrow \mathbf{x} + \mathbf{h}\mathbf{y}$	$\xrightarrow{\mathsf{seed}_\mathbf{h}, \mathbf{s}}$	
	$\xleftarrow{\mathbf{v}, \rho}$	$\mathbf{r}_1, \mathbf{r}_2 \xleftarrow{\$} \mathcal{S}_w^n(\mathbb{F}_2)$, $\epsilon \xleftarrow{\$} \mathcal{S}_{cw}^n(\mathbb{F}_2)$ $\mathbf{v} \leftarrow \mathbf{r}_1 + \mathbf{h}\mathbf{r}_2$, $\rho \leftarrow \mu\mathbf{G} + \mathbf{s}\mathbf{r}_2 + \epsilon$
$\mu \leftarrow \mathcal{C}.\mathsf{Decode}(\rho - \mathbf{v}\mathbf{y})$		

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Correctness

Correctness Property

$$\text{Decrypt}(\text{sk}, \text{Encrypt}(\text{pk}, \boldsymbol{\mu}, \theta)) = \boldsymbol{\mu}$$

$\mathcal{C}$.Decode correctly decodes $\rho - \mathbf{v} \cdot \mathbf{y}$ whenever

the error term is **not too big**

$$\omega(\mathbf{s} \cdot \mathbf{r}_2 - \mathbf{v} \cdot \mathbf{y} + \epsilon) \leq \delta$$

$$\omega((\mathbf{x} + \mathbf{h} \cdot \mathbf{y}) \cdot \mathbf{r}_2 - (\mathbf{r}_1 + \mathbf{h} \cdot \mathbf{r}_2) \cdot \mathbf{y} + \epsilon) \leq \delta$$

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Error distribution analysis $\rightarrow$ Decryption failure probability better understood

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2 Presentation of the Ouroboros protocol
- Cyclic Error Decoding
- BitFlipping algorithm
- Description of the protocol

3 Security

4 Parameters

A particular decoding

- HQC requires $\mathbf{x} \cdot \mathbf{r}_2 - \mathbf{r}_1 \cdot \mathbf{y} + \epsilon$ to be "small" to correctly decode
- Ouroboros further exploits the shape of the error

Cyclic Error Decoding (CED) Problem

- Let $\mathbf{x}, \mathbf{y}, \mathbf{r}_1, \mathbf{r}_2 \xleftarrow{\$} \mathcal{S}_w^n(\mathbb{F}_2)$ with $w = \mathcal{O}(\sqrt{n})$, and $\mathbf{e} \xleftarrow{\$} \mathcal{S}_{cw}^n(\mathbb{F}_2)$ a random error vector.
- Given $(\mathbf{x}, \mathbf{y}) \in (\mathcal{S}_w^n(\mathbb{F}_2))^2$ and $\mathbf{e}_c \leftarrow \mathbf{x}\mathbf{r}_2 - \mathbf{y}\mathbf{r}_1 + \mathbf{e}$ such that $\omega(\mathbf{r}_1) = \omega(\mathbf{r}_2) = w$, find $(\mathbf{r}_1, \mathbf{r}_2)$.

- This is essentially a *noisy* SD problem

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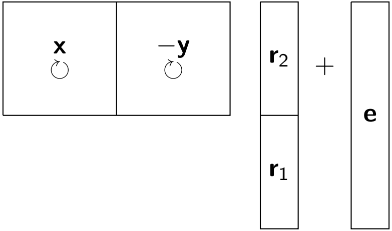

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Hard Decision Decoding: BitFlipping

- Introduced by Gallager in 1962
- Iterative decoding for **L**ow **D**ensity **P**arity **C**heck codes
- Decoding capacity increase linearly with the code length

Intuition

1. Compute the number of unsatisfied parity-check equations for each bit of the message
2. If this number is greater than some *threshold*, flip the bit and go to 1.
3. Stop when the syndrome is null (or after a certain number of iterations).

- Easy to understand
- Easy to implement
- Pretty efficient
- The threshold value is crucial [CS16]

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Ouroboros

- Requires a hash function Hash : $\{0,1\}^* \longrightarrow \mathcal{S}^n_{cw}(\mathbb{F}_2)$ [Sen05]
- ϵ of HQC plays the role of the exchanged secret in Ouroboros
- CE-Decoder is a modified BitFlipping algorithm to solve the CED problem

Alice		Bob
$\mathsf{seed}_{\mathbf{h}} \xleftarrow{\$} \{0,1\}^\lambda$, $\mathbf{h} \xleftarrow{\mathsf{seed}_{\mathbf{h}}} \mathbb{F}_2^n$		
$\mathbf{x}, \mathbf{y} \xleftarrow{\$} \mathcal{S}^n_w(\mathbb{F}_2)$, $\mathbf{s} \leftarrow \mathbf{x} + \mathbf{h}\mathbf{y}$	$\xrightarrow{\mathbf{h},\mathbf{s}}$	$\mathbf{r}_1, \mathbf{r}_2 \xleftarrow{\$} \mathcal{S}^n_w(\mathbb{F}_2)$
		$\mathbf{e}_r \leftarrow \mathrm{Hash}(\mathbf{r}_1, \mathbf{r}_2)$, $\epsilon \xleftarrow{\$} \mathcal{S}^n_{w_e}(\mathbb{F}_2)$
	$\xleftarrow{\mathbf{s}_r, \mathbf{s}_e}$	$\mathbf{s}_r \leftarrow \mathbf{r}_1 + \mathbf{h}\mathbf{r}_2$, $\mathbf{s}_e \leftarrow \mathbf{s}\mathbf{r}_2 + \mathbf{e}_r + \epsilon$
$\mathbf{e}_c \leftarrow \mathbf{s}_e - \mathbf{y}\mathbf{s}_r = \mathbf{x}\mathbf{r}_2 - \mathbf{y}\mathbf{r}_1 + \epsilon'$		
$(\mathbf{r}_1, \mathbf{r}_2) \leftarrow \text{CE-Decoder}(\mathbf{x}, \mathbf{y}, \mathbf{e}_c, t, w, w_e)$		
$\epsilon \leftarrow \mathbf{e}_c - \mathbf{x}\mathbf{r}_2 + \mathbf{y}\mathbf{r}_1 - \mathrm{Hash}(\mathbf{r}_1, \mathbf{r}_2)$		
ϵ	SHARED SECRET	ϵ

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$\mathbf{e_c} \leftarrow \mathbf{s_e} - \mathbf{ys_r} = \mathbf{xr}_2 - \mathbf{yr}_1 + \epsilon'$		
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$\mathbf{e_c} \leftarrow \mathbf{s_e} - \mathbf{y}\mathbf{s_r} = \mathbf{x}\mathbf{r}_2 - \mathbf{y}\mathbf{r}_1 + \epsilon'$ $(\mathbf{r}_1, \mathbf{r}_2) \leftarrow \text{CE-Decoder}(\mathbf{x}, \mathbf{y}, \mathbf{e_c}, t, w, w_e)$ $\epsilon \leftarrow \mathbf{e_c} - \mathbf{x}\mathbf{r}_2 + \mathbf{y}\mathbf{r}_1 - \mathrm{Hash}(\mathbf{r}_1, \mathbf{r}_2)$	$\xleftarrow{\mathbf{s_r},\mathbf{s_e}}$	
$\boxed{\epsilon}$	SHARED SECRET	$\boxed{\epsilon}$

Ouroboros

- Requires a hash function Hash : $\{0,1\}^* \longrightarrow \mathcal{S}^n_{cw}(\mathbb{F}_2)$ [Sen05]
- ϵ of HQC plays the role of the exchanged secret in Ouroboros
- CE-Decoder is a modified BitFlipping algorithm to solve the CED problem

Alice

$\mathsf{seed}_\mathbf{h} \xleftarrow{\$} \{0,1\}^\lambda$, $\mathbf{h} \xleftarrow{\mathsf{seed}_\mathbf{h}} \mathbb{F}_2^n$

$\mathbf{x}, \mathbf{y} \xleftarrow{\$} \mathcal{S}^n_w(\mathbb{F}_2)$, $\mathbf{s} \leftarrow \mathbf{x} + \mathbf{h}\mathbf{y}$

$\xrightarrow{\mathbf{h}, \mathbf{s}}$

Bob

$\mathbf{r}_1, \mathbf{r}_2 \xleftarrow{\$} \mathcal{S}^n_w(\mathbb{F}_2)$

$\mathbf{e}_r \leftarrow \mathrm{Hash}(\mathbf{r}_1, \mathbf{r}_2)$, $\epsilon \xleftarrow{\$} \mathcal{S}^n_{w_e}(\mathbb{F}_2)$

$\mathbf{s}_r \leftarrow \mathbf{r}_1 + \mathbf{h}\mathbf{r}_2$, $\mathbf{s}_e \leftarrow \mathbf{s}\mathbf{r}_2 + \mathbf{e}_r + \epsilon$

$\xleftarrow{\mathbf{s}_r, \mathbf{s}_e}$

Alice

$\mathbf{e}_c \leftarrow \mathbf{s}_e - \mathbf{y}\mathbf{s}_r = \mathbf{x}\mathbf{r}_2 - \mathbf{y}\mathbf{r}_1 + \epsilon'$

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Outline

1. Reminders on HQC
2. Presentation of the Ouroboros protocol
3. Security
   - Security Model and Hybrid Argument
   - Ouroboros Security
4. Parameters

Security Model and Hybrid Argument

- Key exchange as an encryption scheme
- Same as Ding *et al.* [Din12, DXL12], Peikert's [Pei14], BCNS [BCNS15] and NewHope [ADPS16]
- Usual game:

Exp$_{\mathcal{E},\mathcal{A}}^{\text{ind}-b}(\lambda)$
1. param ← Setup(1^λ)
2. (pk, sk) ← KeyGen(param)
3. $(\epsilon_0, \epsilon_1) \leftarrow \mathcal{A}(\texttt{FIND} : \text{pk})$
4. $\mathbf{c}^* \leftarrow \text{Encrypt}(\text{pk}, \epsilon_b, \theta)$
5. $b' \leftarrow \mathcal{A}(\texttt{GUESS} : \mathbf{c}^*)$
6. RETURN b'

- Hybrid argument:
  1. Construct a sequence of games transitioning from Enc(ϵ_0) to Enc(ϵ_1)
  2. Prove they are indistinguishable one from another

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Security

Definition (SD Distribution)

For positive integers, n, k, and w, the *SD(n, k, w) Distribution* chooses $\mathbf{H} \stackrel{\$}{\leftarrow} \mathbb{F}^{(n-k)\times n}$ and $\mathbf{x} \stackrel{\$}{\leftarrow} \mathbb{F}^n$ such that $\omega(\mathbf{x}) = w$, and outputs $(\mathbf{H}, \mathbf{H}\mathbf{x}^\top)$.

Definition (Decisional s-QCSD Problem)

For positive integers n, k, w, s, a random parity check matrix $\mathbf{H}$ of a QC code $\mathcal{C}$ and $\mathbf{y} \stackrel{\$}{\leftarrow} \mathbb{F}^n$, the *Decisional s-Quasi-Cyclic SD Problem s-DQCSD(n, k, w)* asks to decide with non-negligible advantage whether $(\mathbf{H}, \mathbf{y}^\top)$ came from the s-QCSD(n, k, w) distribution or the uniform distribution over $\mathbb{F}^{(n-k)\times n} \times \mathbb{F}^{n-k}$.

Theorem

Ouroboros is IND-CPA *under the 2-DQCSD and 3-DQCSD assumptions.* $\longrightarrow$ sketch of proof

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Reduction Compliant Parameters

	Ouroboros Parameters					
Instance	n	w	w_e	threshold	security	DFR
Low-I	$5,851$	47	94	30	80	$0.92 \cdot 10^{-5}$
Low-II	$5,923$	47	94	30	80	$2.3 \cdot 10^{-6}$
Medium-I	$13,691$	75	150	45	128	$0.96 \cdot 10^{-5}$
Medium-II	$14,243$	75	150	45	128	$1.09 \cdot 10^{-6}$
Strong-I	$40,013$	147	294	85	256	$4.20 \cdot 10^{-5}$
Strong-II	$40,973$	147	294	85	256	$< 10^{-6}$

Table : Parameter sets for Ouroboros

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Optimized Parameters wrt Best Know Attacks

	Ouroboros Optimized Parameters					
Instance	n	w	w_e	threshold	security	DFR
Low-I	$4,813$	41	123	27	80	$2.23 \cdot 10^{-5}$
Low-II	$5,003$	41	123	27	80	$2.60 \cdot 10^{-6}$
Medium-I	$10,301$	67	201	42	128	$1.01 \cdot 10^{-4}$
Medium-II	$10,837$	67	201	42	128	$< 10^{-7}$
Strong-I	$32,771$	131	393	77	256	$< 10^{-4}$
Strong-II	$33,997$	131	393	77	256	$< 10^{-7}$

Table : Optimized parameter sets for Ouroboros in Hamming metric

Conclusion

In this talk

- Ouroboros: a *secure*, *simple*, and *efficient* code-based key exchange protocol
- Efficient decoding through BitFlipping
- Competitive parameters

Further Improvements

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Thanks!

Paper available @ http://unil.im/ouroboros

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Rank Metric Interlude (1/2)

Rank metric defined over (finite) extensions of finite fields

- $\mathbb{F}_q$ a finite field with q a power of a prime.
- $\mathbb{F}_{q^m}$ an extension of degree m of $\mathbb{F}_q$.
- $\mathbb{F}_{q^m}$ can be seen as a vector space on $\mathbb{F}_q$.
- $\mathcal{B} = (b_1, ..., b_m)$ a basis of $\mathbb{F}_{q^m}$ over $\mathbb{F}_q$.

Let $\mathbf{v} = (v_1, \ldots, v_n)$ be a word of length n in $\mathbb{F}_{q^m}$.

Any coordinate $v_j = \sum_{i=1}^{m} v_{ij} b_i$ with $v_{ij} \in \mathbb{F}_q$.

$$\mathbf{v} = (v_1, ..., v_n) \rightarrow \mathbf{V} = \begin{pmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mn} \end{pmatrix}$$

Rank weight of word

$\mathbf{v}$ has rank $r = \mathrm{rank}(\mathbf{v})$ iff the rank of $\mathbf{V} = (v_{ij})_{ij}$ is r.
Equivalently $\mathrm{rank}(\mathbf{v}) = r \Leftrightarrow v_j \in V_r \subset \mathbb{F}_{q^m}^n$ with $\dim(V_r)$=r.

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Rank Metric Interlude (2/2)

- Best Known Attacks have worse complexity in rank metric ($2^{\mathcal{O}(n^2)}$) than in Hamming metric ($2^{\mathcal{O}(n)}$)
- Consequence: worse attacks $\Rightarrow$ better parameters

Instance	Ouroboros-R Parameters						
	key size (bits)	n	m	q	w	security	decoding failure
Ouroboros-R-I	1,591	37	43	2	5	100	10^{-4}
Ouroboros-R-II	2,809	53	53	2	5	128	10^{-8}
Ouroboros-R-III	3, 953	59	67	2	6	192	10^{-7}
Ouroboros-R-IV	5, 293	67	79	2	7	256	10^{-5}
Ouroboros-R-V	5, 618	53	53	4	6	256	10^{-10}

Parameter sets for Ouroboros-R in rank metric.

back to conclusion

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Sketch of proof

Sequence of games from $\mathsf{Enc}(\epsilon_0)$ to $\mathsf{Enc}(\epsilon_1)$

$\mathsf{Enc}(\epsilon_0) \longrightarrow \mathsf{Enc}_{\mathbf{s}^\star}(\epsilon_0) \longrightarrow \mathsf{Enc}_{\mathbf{s}^\star,\mathbf{r}^\star}(\epsilon_0)$

$\downarrow$

$\mathsf{Enc}(\epsilon_1) \longleftarrow \mathsf{Enc}_{\mathbf{s}^\star}(\epsilon_1) \longleftarrow \mathsf{Enc}_{\mathbf{s}^\star,\mathbf{r}^\star}(\epsilon_1)$

$$\mathbf{Adv}^{\mathrm{ind}}_{\mathcal{E},\mathcal{A}}(\lambda) \leq 2 \cdot \left(\mathbf{Adv}^{\text{2-DQCSD}}(\lambda) + \mathbf{Adv}^{\text{3-DQCSD}}(\lambda)\right)$$

back to security

Sketch of proof

Sequence of games from $\text{Enc}(\epsilon_0)$ to $\text{Enc}(\epsilon_1)$

$\text{Enc}(\epsilon_0) \longrightarrow \text{Enc}_{\mathbf{s}^\star}(\epsilon_0) \longrightarrow \text{Enc}_{\mathbf{s}^\star,\mathbf{r}^\star}(\epsilon_0)$

$\downarrow$

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back to security

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back to security

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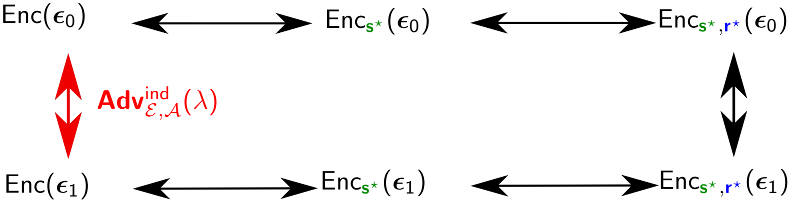
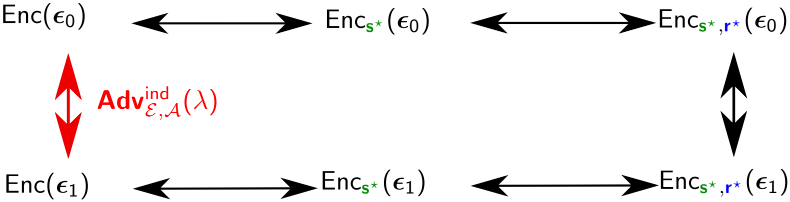


$$\mathbf{Adv}^{\mathsf{ind}}_{\mathcal{E},\mathcal{A}}(\lambda) \leq 2 \cdot \left(\mathbf{Adv}^{\text{2-DQCSD}}(\lambda) + \mathbf{Adv}^{\text{3-DQCSD}}(\lambda)\right)$$

back to security

Sketch of proof

Sequence of games from $\mathsf{Enc}(\epsilon_0)$ to $\mathsf{Enc}(\epsilon_1)$



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back to security